

1992 Paper II Question 8

Let  $f(x) = xe^{-x^2}$  for  $x \in \mathbf{R}$ .

a. Find  $f'(x)$  and  $f''(x)$ . (2 marks)

b. Determine the values of  $x$  for each of the following cases:

(i)  $f'(x) = 0$ ,

(ii)  $f'(x) > 0$ ,

(iii)  $f'(x) < 0$ ,

(iv)  $f''(x) = 0$ ,

(v)  $f''(x) > 0$ ,

(vi)  $f''(x) < 0$ .

(3 marks)

c. Find all relative extrema and points of inflexion of  $f(x)$ . (3 marks)

d. Find the asymptote of the graph of  $f(x)$ . (1 mark)

e. Sketch the graph of  $f(x)$ . (3 marks)

f. Hence sketch the curve  $x + y = (x - y)e^{-\frac{1}{2}(x-y)^2}$ . (3 marks)