1993 Paper 2 Question 11(a)(i)

a. Let f and g be real-valued functions defined on (a, ∞) where $a > 0$, and f be twice differentiable satisfying the following conditions:

A. g is decreasing,

B. $g(t) \geq 0$ and $f''(t) \geq 0$ for all $t \in (a, \infty)$,

C. $\lim_{t \to \infty} g(t) f'(t) = 0$.

(i) Use the Mean Value Theorem to show that

$$f(n) + f'(n)(t - n) \leq f(t) \leq f(n) + f'(n + 1)(t - n)$$

for all $t \in [n, n + 1]$, where n is a positive integer greater than a.

(3 marks)