1993 Paper 2 Question 6

a. Show that if $\alpha > \beta \geq 0$, then

$$\sqrt{\frac{\alpha}{\beta+1}} > \sqrt{\frac{\beta}{\beta+1}}.$$

b. Let $u_n = \sum_{m=1}^{n} \frac{1}{2^m}\sqrt{\frac{n-m}{n-m+1}}$, $n = 1, 2, \ldots$. Use (a), or otherwise, to show that

$$u_n < u_{n+1}$$

for $n = 1, 2, \ldots$. Hence show that $\lim_{n\to\infty} u_n$ exists.

(7 marks)