Evaluate

a. For $x > 0$, prove that $\ln x \leq x - 1$ where the equality holds if and only if $x = 1$.

b. Prove that $\ln \frac{r}{r-1} < \frac{1}{r-1}$ for $r > 1$. Hence deduce that

$$\ln n < \sum_{k=1}^{n-1} \frac{1}{k}$$

for $n = 2, 3, 4, \ldots$.

(7 marks)