There was a question asked on problem 5 in the chapter on Binomial Theorem.

$$f(r) = C_0^n C_r^n + C_1^n C_{r+1}^n + \cdots + C_{n-r}^n C_n^n$$

1. Show that $C_r^n = C_{n-r}^n$.
2. By considering the expansion of $(1+x)^{2n}$, show that

$$f(r) = \frac{(2n)!}{(n+r)!(n-r)!}$$

3. Show

$$C_0^n f(0) + C_1^n f(1) + \cdots + C_n^n f(n) = \frac{(3n)!}{n!(2n)!}$$

First part is quite easy!

For the second part, note that the right side is the coefficient of x^{n+r} in the binomial expansion of $(1+x)^2$.

Using the first part, you can see that $f(r)$ is exactly the coefficient of x^{n+r} in the binomial expansion of $(1+x)(1+x)$.

For the third part, note that the right side is the coefficient of x^n (or x^{2n}) in the binomial expansion of $(1+x)^3$.

You should then consider the binomial expansion of $(1+x)^n(1+x)^{2n}$. Some ordering may have to be reversed.