Homework Assignment for 3-D Coordinate Geometry

(1) Find the equation of the plane which passes through $(1,3,-4)$ with normal vector $2i+2j+k$.
(2) Transform the equation of the plane $x-2y+2z+3=0$ in normal form and hence write down the direction cosines and its distance from the origin to the plane.
(3) Find the equation of the plane through $(1,0,-2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$.
(4) Find the equation of the plane passing through $P(0,-1,0)$ and $Q(0,0,1)$ and making an angle $120°$ with the plane $\pi: y-z-2=0$.
(5) Find the equation of the plane passing through the points $P(3,1,-1)$ and $Q(2,-1,3)$ and being parallel to the x-axis.
(6) Find the equation of the plane containing the point $(1,1,0)$ with vectors $u=-2i+3j+5k$ and $v=-3i-2j+k$ lying on it.
(7) Find the equation of the plane which passes through $P_1(1,1,1)$, $P_2(2,-3,0)$ and $P_3(5,0,6)$.
(8) Find the equation of the plane which passes through the intersection of the planes $\pi_1: 6x+4y+3z+5=0$ and $\pi_2: 2x+y+z-2=0$ and also 3 units away from the origin.
(9) Find the points of intersection of the line joining the points $P_1(2,13,13)$ and $P_2(5,-14,-11)$ and the sphere $\pi: x^2+y^2+z^2=50$.
(10) Find the direction ratio of the line

$$L: \begin{cases} 3x+2y-z-1=0 \\ 2x-y+2z-3=0 \end{cases}.$$

(11) Transform the equation of the line

$$L: \begin{cases} 3x-y-z+6=0 \\ 2x-3y+2z-5=0 \end{cases}$$

into symmetric form.
(12) Find the equation of the line passing through the point $P(-4,5,3)$ and being perpendicular to the lines $\frac{x-2}{3}=\frac{y-5}{2}=\frac{z-5}{5}$ and $\frac{x+1}{6}=\frac{y-2}{-4}=\frac{z+1}{3}$.
(13) Show that the straight line $L: \begin{cases} x=z+4 \\ y=2z+3 \end{cases}$ lie wholly on the plane $2x+3y-8z=17$.
(14) Find the foot of perpendicular from the origin to the line $\frac{x+9}{5}=y-1=\frac{z-10}{-4}$.
(15) Find the length of the perpendicular from $(3,9,-1)$ to the line

$$\frac{x+8}{-8}=y-31=\frac{z-13}{5}.$$

(16) Find the perpendicular distance from the point $P(3,1,2)$ to the line $\frac{x-8}{3}=\frac{y-1}{2}=\frac{z-3}{2}$.
(17) Given a plane $3x+y-2z-1=0$ and a line $\begin{cases} 3x-2y+7=0 \\ y+3z=1 \end{cases}$. Find the angle between them.

(18) Find the equation of the plane passing through the point $P(3,6,-12)$ and being parallel to the two lines $L_1 : \begin{cases} x+3y-3=0 \\ 3y+z-5=0 \end{cases}$ and $L_2 : \begin{cases} 2x-z=10 \\ y=3 \end{cases}$.

(19) Find the equation of the plane π containing the line $L_1 : \begin{cases} x+2z=4 \\ y-z=8 \end{cases}$ and being parallel to the line $L_2 : \frac{x-3}{2} = \frac{y+4}{3} = \frac{z-7}{4}$.

(20) Find the equation of the plane π containing the point $(0,3,-4)$ and the line $L : \frac{x-2}{2} = \frac{y+3}{-2} = z-1$.

(21) Find the equation of the plane containing two intersecting lines $L_1 : \frac{x-2}{3} = \frac{y+1}{4} = \frac{z}{-2}$ and $L_2 : \frac{x-2}{-1} = \frac{y+1}{3} = \frac{z}{2}$.

(22) Find the equation of the plane containing two parallel lines $L_1 : \frac{x+1}{3} = \frac{y-2}{2} = z$ and $L_2 : \frac{x-3}{3} = \frac{y+4}{2} = z-1$.

(23) (a) Find the equation of the plane which contains the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and is perpendicular to the plane $ax+by+cz+d=0$.
(b) Find the equation in symmetric form for the orthogonal projection of the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z+1}{3}$ on the plane $x+y+z=1$.

(24) Given the lines $L_1 : 15x = 5y = 3z$ and $L_2 : \frac{x+2}{3} = \frac{y+1}{4} = \frac{z}{5}$. If the line L_2 is the orthogonal projection of L_1 on the plane π, find the equation of this plane.

(25) Find the equation in general form of the projection of the line $L_1 : \frac{x+1}{3} = \frac{y-2}{2} = \frac{z-3}{-1}$ on the plane $\pi_1 : x+y+2z=4$. Find the projection of the point $(-1,2,3)$ on this plane.

(26) (a) Find the coordinates of the mirror image of $A(4,0,3)$ with respect to $\pi : x-2y+2z-1=0$.
(b) Find the equations of the mirror image of $L : \frac{x-4}{3} = \frac{y}{-1} = \frac{z-3}{2}$ with respect to π.

(27) Show that the lines $L_1 : \frac{x-3}{2} = \frac{y-2}{-5} = \frac{z-1}{3}$ and $L_2 : \frac{x-1}{-4} = \frac{y+2}{1} = \frac{z-6}{2}$ are coplanar and find the point of intersection. Also find the plane containing L_1 and L_2.

(28) Show that the lines $\frac{x+2}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ and $\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z-2}{4}$ do not intersect and the perpendicular distance between them is $\frac{11}{\sqrt{6}}$.

(29) Two lines are given by $r_1 = 2i+3j-5k+\lambda(3i+2j-4k)$ and $r_2 = 3i-j+4k+\lambda'(5i-2j-4k)$ where λ and λ' are parameters. Find the shortest distance between the lines and the position vectors of the points of closest approach.

(30) Show that the shortest distance between the lines $x = \frac{y-8}{3} = \frac{z-5}{7}$ and $x-8 = \frac{y-1}{-2} = \frac{z-11}{7}$ is $5\sqrt{2}$ and find the equation of the line along which it lies.