Limit of Sequence

Time allowed: One hour

To be administered on Monday, 19 February, 2001.

Please use single-line composition paper.

1. If $\{a_n\}$ is a sequence, what is the meaning of $\lim_{n\to\infty} a_n = L$?
2. Let $u_1 = \sqrt{3}$, $u_{n+1} = \sqrt{3 + u_n}$ for $n > 1$. Prove that $\lim_{n\to\infty} u_n$ exists and evaluate the limit.
3. Consider the sequence $\{u_n\}$ in which $u_1 = 0$, $u_{n+1} = 2n - u_n$ for $n > 1$. Using mathematical induction, or otherwise, show that

$$2u_n = 2n - 1 + (-1)^n$$

for $n \geq 1$. Hence find $\lim_{n\to\infty} \frac{u_n}{n}$.

4. (a) Let $\{a_n\}$ be a sequence. If $\lim_{n\to\infty} a_n = l$, prove that

$$\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots + a_n}{n} = l.$$

(b) Using (a), show that

$$\lim_{n\to\infty} \frac{1}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) = 0.$$

The solutions will be posted to the website tonight.