Tutorial 1 Indices and logarithms

1. Carry out each of the following operation. Write all answers with positive exponents and simplify where possible.

(a) $\dfrac{7^{-3}}{7^{-5}}$ (b) $\dfrac{3^5 \cdot 2^2}{3^8}$

(c) $\dfrac{4^6 \cdot 3^4}{4^5 \cdot 3^3}$ (d) $\left(\dfrac{2}{5}\right)^{-2}$

2. Evaluate the following :

(a) $49^{-\frac{1}{2}}$ (b) $8^{-\frac{1}{3}}$

3. In the compound interest formula $A = P(1 + r)^n$,
   (a) if $P = 100{,}000$, $r = 8\%$, $n = 5$, find the value of A,
   (b) if $A = 21{,}003.42$, $P = 10{,}000$, $n = 5$, find the value of r,
   (c) if $A = 80957.13$, $P = 35000$, $r = 15\%$, find the value of n.

4. The loudness, measured in decibels, is defined by the function

$$b = 10 \log\left(\frac{I}{I_0}\right),$$

where P is the intensity of the sound and I_o is the minimum intensity detectable. How many times greater is the intensity of $\mathbf{b_1}$ = 105 dB (factory) than the intensity of $\mathbf{b_2}$ = 80 dB (busy street) ?

Solution

1.(a) $\frac{7^{-3}}{7^{-5}} = 49$

(b) $\frac{3^5 \cdot 2^2}{3^8} = \frac{4}{27}$

(c) $\frac{4^6 \cdot 3^4}{4^5 \cdot 3^3} = 12$

(d) $\left(\frac{2}{5}\right)^{-2} = \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{25}{4}$

2. (a) $49^{-\frac{1}{2}} = \frac{1}{7}$

b) $8^{-\frac{1}{3}} = \frac{1}{2}$

3. (a) $A = P(1 + r)^n$

$A = 146{,}932.81$

(b) $A = P(1 + r)^n$

$r = 0.16$

or

$= 16\%$

(c) $A = P(1 + r)^n$

$n = 6$

4. Given $b_1 = 105$ dB

$b_2 = 80$ dB

$\frac{I_1}{I_2} = 316$

The sound intensity in the factory is 316 times greater than the busy street.