Tutorial 1 Indices and logarithms

1. Carry out each of the following operation. Write all answers with positive exponents and simplify where possible.

   (a) $\dfrac{7^{-3}}{7^{-5}}$ (b) $\dfrac{3^5 \cdot 2^2}{3^8}$

   (c) $\dfrac{4^6 \cdot 3^4}{4^5 \cdot 3^3}$ (d) $\left(\dfrac{2}{5}\right)^{-2}$

2. Evaluate the following :

   (a) $49^{-\frac{1}{2}}$ (b) $8^{-\frac{1}{3}}$

3. In the compound interest formula $A = P\,(1 + r)^n$,

   (a) if $P = 100{,}000$, $r = 8\ \%$, $n = 5$, find the value of A,

   (b) if $A = 21{,}003.42$, $P = 10{,}000$, $n = 5$, find the value of r ,

   (c) if $A = 80957.13$, $P = 35000$, $r = 15\%$, find the value of n.

4. The loudness, measured in decibels, is defined by the function

   $$b = 10 \log\left(\frac{I}{I_0}\right),$$

   where P is the intensity of the sound and I_0 is the minimum intensity detectable. How many times greater is the intensity of $\mathbf{b_1} = 105$ dB (factory) than the intensity of $\mathbf{b_2} = 80$ dB (busy street) ?

Solution

1.(a) $\frac{7^{-3}}{7^{-5}} = 7^{-3-(-5)} = 7^{-3+5} = 7^2 = 49$

(b) $\frac{3^5 \cdot 2^2}{3^8} = \left(\frac{3^5}{3^8}\right) 2^2 = 3^{5-8} \cdot 2^2 = 3^{-3} \cdot 2^2 = \frac{2^3}{3^3} = \frac{4}{27}$

(c) $\frac{4^6 \cdot 3^4}{4^5 \cdot 3^3} = 4^{6-5} \cdot 3^{4-3} = 4^1 \cdot 3^1 = 4 \cdot 3 = 12$

(d) $\left(\frac{2}{5}\right)^{-2} = \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{1}{\frac{4}{25}} = \frac{25}{4}$

2. (a) $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$

b) $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

3. (a) $A = P(1 + r)^n$

$A = 100{,}000\,(1 + 8\%)^5$

$= 100{,}000\,(1 + 0.08)^5$

$= 100{,}000\,(1.08)^5$

$= 146{,}932.81$

(b) $A = P(1 + r)^n$

$21{,}003.42 = (10{,}000)(1 + r)^5$

$\frac{21{,}003.42}{10{,}000} = (1 + r)^5$

$(1 + r)^5 = 2.100342$

$\therefore\ 1 + r = \sqrt[5]{2.100342}$

$= (2.100342)^{1/5}$

$1 + r = 1.16$

$\therefore\ r = 1.16 - 1$

$= 0.16$

or

$= 16\%$

(c) $A = P(1 + r)^n$

$$80957.13 = 35000\,(1 + 15\%)^n$$

$$\frac{80957.13}{35000} = (1.15)^n$$

$$\log\frac{80957.13}{35000} = n\log 1.15$$

$$0.36419 = n(0.060698)$$

$$\therefore n = 6$$

4. Given $b_1 = 105$ dB

$$\therefore \quad 105 = 10\log\frac{I_1}{I_0}$$

$$10.5 = \log\frac{I_1}{I_0}$$

$$10^{10.5} = I_1/I_0$$

$$I_1 = 10^{10.5}\, I_0 \ldots\ldots \bullet$$

Similary, $b_2 = 80$ dB

$$\therefore \quad 80 = 10\log\frac{I_2}{I_0}$$

$$8 = \log\frac{I_2}{I_0}$$

$$10^8 = \frac{I_2}{I_0}$$

$$I_2 = 10^8\, I_0 \ldots\ldots ,$$

• ÷ , :

$$\frac{I_1}{I_2} = \frac{10^{10.5} I_0}{10^8 I_0}$$

$$= 10^{2..5}$$

$$= 316$$

The sound intensity in the factory is 316 times greater than the busy street.