

## Tutorial 11 : Reduction principle and simple trigonometric equations

1. Evaluate the following without using tables or calculators.

(a) 
$$\frac{\sin 225^\circ \cos 315^\circ - \tan 120^\circ}{\sin(-60^\circ) \tan 210^\circ}$$

(b) 
$$5 \sin 217^\circ \cos 233^\circ + 5 \sin 127^\circ \cos 323^\circ$$

2. Simplify 
$$\frac{\cos(\frac{3p}{2} + A)}{\tan(A - p)} + \frac{1}{\tan(\frac{p}{2} - A)} - \cos(A - 2p)$$

3. How many roots are there in the equation  $\sin x = 0.5$  where  $0^\circ \leq x \leq 360^\circ$  ?

4. Find the values of  $\theta$  from  $0^\circ$  to  $360^\circ$  inclusive satisfying the equation  $5 \sin^2 q - 2 \sin q \cos q - 3 \cos^2 q = 0$

5. Solve the equation  $4 \sin A = 3 \tan A$  for  $0^\circ \leq A \leq 360^\circ$ .

6. Solve  $\cos(3q - 45^\circ) = \frac{1}{2}$  where  $0^\circ \leq q \leq 360^\circ$ .

## Solutions to tutorial 13

$$\begin{aligned}
 1. \quad (a) \quad & \frac{\sin 225^\circ \cos 315^\circ - \tan 120^\circ}{\sin(-60^\circ) \tan 210^\circ} \\
 &= \frac{\sin(180^\circ + 45^\circ) \cos(360^\circ - 45^\circ) - \tan(180^\circ - 60^\circ)}{-\sin 60^\circ \tan(180^\circ + 30^\circ)} \\
 &= \frac{(-\sin 45^\circ) \cos 45^\circ - (-\tan 60^\circ)}{-\sin 60^\circ \tan 30^\circ} \\
 &= \frac{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \sqrt{3}}{-\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}}} = 1 - 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & 5 \sin 217^\circ \cos 233^\circ + 5 \sin 127^\circ \cos 323^\circ \\
 &= 5 \sin(180^\circ + 37^\circ) \cos(270^\circ - 37^\circ) + 5 \sin(90^\circ + 37^\circ) \cos(360^\circ - 37^\circ) \\
 &= 5(-\sin 37^\circ)(-\sin 37^\circ) + 5 \cos 37^\circ \cos 37^\circ \\
 &= 5(\sin^2 37^\circ + \cos^2 37^\circ) = 5
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{\cos(\frac{3\pi}{2} + A)}{\tan(A - \pi)} + \frac{1}{\tan(\frac{\pi}{2} - A)} - \cos(A - 2\pi) \\
 &= \frac{\sin A}{-\tan(\pi - A)} + \frac{1}{\frac{1}{\tan A}} - \cos(2\pi - A) \\
 &= \frac{\sin A}{-(-\tan A)} + \tan A - \cos A \\
 &= \frac{\sin A}{\frac{\sin A}{\cos A}} + \tan A - \cos A \\
 &= \cos A + \tan A - \cos A = \tan A
 \end{aligned}$$

3. There are 2 roots. Check the graph of  $\sin x$ .

$$\begin{aligned}
 4. \quad & 5 \sin^2 q - 2 \sin q \cos q - 3 \cos^2 q = 0 \\
 & (5 \sin q + 3 \cos q)(\sin q - \cos q) = 0 \\
 & \therefore \tan q = -\frac{3}{5} \quad \text{or} \quad \tan q = 1
 \end{aligned}$$

$$q = 180^\circ - 30.96^\circ, 360^\circ - 30.96^\circ; \quad 45^\circ, 180^\circ + 45^\circ$$

$$\therefore \theta = 149.04^\circ, 329.04^\circ; 45^\circ, \quad 225^\circ$$

5.  $4\sin A = 3\tan A$

$$4\sin A - 3\frac{\sin A}{\cos A} = 0$$

$$\sin A\left(4 - \frac{3}{\cos A}\right) = 0$$

$$\sin A = 0 \text{ or } \cos A = \frac{3}{4}$$

$$A = 0^\circ, 180^\circ, 360^\circ, 41.41^\circ, 360^\circ - 41.41^\circ$$

$$A = 0^\circ, 180^\circ, 360^\circ, 41.41^\circ, 318.59^\circ$$

6. Since  $0^\circ \leq \theta \leq 360^\circ$ ,  $0^\circ \leq 3\theta \leq 1080^\circ$

$$\cos(3q - 45^\circ) = \frac{1}{2}$$

$$3q - 45^\circ = 60^\circ, 360^\circ - 60^\circ, 360^\circ + 60^\circ, 720^\circ - 60^\circ, 720^\circ + 60^\circ, 1080^\circ - 60^\circ$$

$$q = 35^\circ, 115^\circ, 155^\circ, 235^\circ, 275^\circ, 355^\circ$$