Unit11: Reduction principle and Simple trigonometric equations

Objectives

Students should be able to

- state the reduction principle
- use the reduction principle to find the trigonometric ratios of angles related to special angles such as $30°$, $45°$ and $60°$
- solve simple trigonometric equations

Reduction principle and simple trigonometric equations

1. Reduction Principle

1.1 Reference Angle

The reference angle α for an angle θ is the positive acute angle formed by the terminal side of θ and the x-axis.

Given $\sin\alpha = 4/5$, $\cos\alpha = 3/5$ and $\tan\alpha = 4/3$, you can easily fill in the table below.

Angles	Quadrant	$\sin\theta$	$\sin\alpha$	$\cos\theta$	$\cos\alpha$	$\tan\theta$	$\tan\alpha$
$\theta = \alpha$	I	$\frac{4}{5}$	$\frac{4}{5}$		$\frac{3}{5}$	$\frac{4}{3}$	$\frac{4}{3}$
$\theta = (180°-\alpha)$	II	$\frac{4}{5}$		$-\frac{3}{5}$	$\frac{3}{5}$	$-\frac{4}{3}$	
$\theta = (180°+\alpha)$	III		$\frac{4}{5}$		$\frac{3}{5}$		$\frac{4}{3}$
$\theta = (360°-\alpha)$	IV	$-\frac{4}{5}$	$\frac{4}{5}$	$\frac{3}{5}$		$-\frac{4}{3}$	$\frac{4}{3}$

The trigonometric ratios of an angle θ and its reference angle α have the same numerical value,

i.e. $\sin q = \pm \sin a$

$\cos q = \pm \cos a$

$\tan q = \pm \tan a$

where the choice of sign (+ or -) depends on the quadrant in which θ lies.

1.2 The Reduction Principle

In the first quadrant, $\theta = \alpha$, we have

$\sin q = \sin a$

$\cos q = \cos a$

$\tan q = \tan a$

In the second quadrant, $\theta = (180°-\alpha)$, we have

$\sin q = \sin(180° - a) = \sin a$

$\cos q = \cos(180° - a) = -\cos a$

$\tan q = \tan(180° - a) = -\tan a$

In the third quadrant, $\theta = (180°+\alpha)$, we have

$$\sin\theta = \sin(180° + \alpha) = -\sin\alpha$$
$$\cos\theta = \cos(180° + \alpha) = -\cos\alpha$$
$$\tan\theta = \tan(180° + \alpha) = \tan\alpha$$

In the fourth quadrant, $\theta = (360°-\alpha)$, we have

$$\sin\theta = \sin(360° - \alpha) = -\sin\alpha$$
$$\cos\theta = \cos(360° - \alpha) = \cos\alpha$$
$$\tan\theta = \tan(360° - \alpha) = -\tan\alpha$$

N.B. It is also interesting to note that

$\sin(90°+\theta) = \cos\theta$; $\quad\sin(-\theta) = -\sin\theta$

$\cos(90°+\theta) = -\sin\theta$; $\quad\cos(-\theta) = \cos\theta$

$\tan(90°+\theta) = -\cot\theta$; $\quad\tan(-\theta) = -\tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

1.3 Trigonometric ratios of some special angles

The trigonometric ratios of some special angles are listed below for easy reference. These could be found by using Pythagoras' theorem.

Ratio\θ	0^o	$30^o(\frac{p}{6})$	$45^o(\frac{p}{4})$	$60^o(\frac{p}{3})$	$90^o(\frac{p}{2})$
$\sin\theta$	0	$\frac{1}{2}$		$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1		$\frac{\sqrt{2}}{2}$		0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$		$\sqrt{3}$	8

Example 1

Find the values of the following without using a calculator.

(i) $\cos 150°$ (ii) $\tan(-\frac{p}{3})$

Solution

(i) $\cos 150° = \cos(180^o - 30^o)$

$= \cos\_\_\_^o$

$=$

(ii) $\tan(-\frac{p}{3}) = \tan(-60^o) = -\tan\_\_\_^o =$

Example 2

$Simplify \frac{\sin(90° - q)}{\sin(90° + q)}$

Solution

$\frac{\sin(90° - q)}{\sin(90° + q)} = \frac{\cos}{\cos} =$

Example 3

$Simplify \frac{\sin(180° - q)}{\cos(180° + q)}$

Solution

$\frac{\sin(180° - q)}{\cos(180° + q)} = \frac{\sin q}{\_\_\cos q} =$

2. Simple Trigonometric Equations

We shall demonstrate some methods in solving simple equations involving trigonometric ratios of angle.

Example 4 Solve the equation

$2\sin q - 1 = 0,\ 0° \le q \le 360°$

Solution $\sin q =$, q = ___°, ____°

Example 5

Solve the equation $4\cos^2 q - 3 = 0,\ 0 \le q \le 2p$

Solution

Factorizing, $(\cos\theta + \frac{\sqrt{3}}{2})(\cos\theta - \frac{\sqrt{3}}{2}) = 0$ or simply $\cos q = \pm$____

when $(\cos\theta + \frac{\sqrt{3}}{2}) = 0$,

$q = \frac{5p}{\_\_}, \frac{\_\_p}{6}$

when $(\cos\theta - \frac{\sqrt{3}}{2}) = 0$,

$q = \frac{\_\_}{6}, \frac{\_\_p}{6}$

Example 6 Solve the equation

$5\sin q - 2\cos^2 q - 1 = 0,\ 0° \le q \le 360°$

Solution

Since $\sin^2 q + \cos^2 q = 1$,

the equation becomes

$5\sin q - 2(1 - \sin^2 q) - 1 = 0$

$(2\sin q - 1)(\sin q + \_\_\_) = 0$

gives $\sin q = 1/2$

whence q = ___°, ___°

[Note: $(\sin q + 3) = 0$ has ____ solution.]