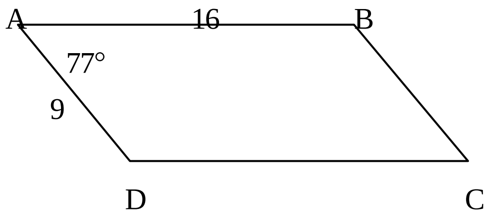
Tutorial 12: The Sine formula and the cosine formula

1. Solve the triangle PQR with p = 32, q = 49, $\angle P = 36°$.
   [There are two possible sets of solutions.]

2. In ΔXYZ, y = 7, z = 11 and $\angle Z = 48°$. Find $\angle Y$.

3. In a parallelogram ABCD, AB = 16, AD = 9, $\angle BAD = 77°$. Find the lengths of the diagonals.



4. In ΔPQR, p : q : r = 8 : 17 : 13. Find all the angles in the triangle.

Solution

1. By Sine Formula,

$$\frac{\sin Q}{49} = \frac{\sin 36°}{32},$$

$$\sin Q = \frac{49 \sin 36°}{32} = 0.9000$$

$$\therefore \angle Q = 64.16° \text{ or } 180° - 64.16°$$

$$= 64.16° \text{ or } 115.84°$$

When $\angle Q = 64.16°$,

$\angle R = 180° - 36° - 64.16°$ [180^o- ? P- ? Q]

$= 79.84°$

$$r = \frac{32 \sin 79.84°}{\sin 36°}$$

$= 53.59$

When $\angle Q = 115.84°$,

$\angle R = 180° - 36° - 115.84°$

$= 28.16°$

$$r = \frac{32 \sin 28.16°}{\sin 36°}$$

$= 25.69$

$\therefore \angle Q = 64.16°, \angle R = 79.84°, r = 53.59$ or

$\angle Q = 115.84°, \angle R = 28.16°, r = 25.69$.

2. $$\frac{\sin Y}{7} = \frac{\sin 48°}{11}$$

$$\sin Y = \frac{7 \sin 48°}{11}$$

$= 0.4729$

$\therefore \angle Y = 28.22°$ or $180° - 28.22°$

$=$ 28.22° or 151.78° (rejected)

$= 28.22°$

In this case ∠Y cannot be 151.78° because the angles sum of a triangle cannot exceed 180°.

3. In ΔABD, apply cosine rule

$BD^2 = 9^2 + 16^2 - 2 \times 9 \times 16 \cos 77°$

= 272.21

BD = 16.50

Similarly, in ΔABC,

$BC = 9, \angle ABC = 180° - 77° = 103°$

$\therefore AC^2 = 9^2 + 16^2 - 2 \times 9 \times 16 \cos 103°$

= 401.79

AC = 20.04

4. Let p = 8k, q = 17k, r = 13k.

$\angle$Q = the angle opposite to the longest side

By Cosine Formula,

$$\cos Q = \frac{(8k)^2 + (13k)^2 - (17k)^2}{2(8k)(13k)}$$

= -0.26923

$\therefore \angle Q = 105.62°$

By Sine Formula,

$$\frac{\sin P}{8k} = \frac{\sin 105.62°}{17k}$$

$$\sin P = \frac{8 \sin 105.62°}{17}$$

$\therefore \angle P = 26.95°$

? R =180°- ? P- ? Q

$\angle R = 180° - 26.95° - 105.62°$

$= 47.43°$

$\therefore \angle P = 26.95°, \angle Q = 105.62°, \angle R = 47.43°$