Unit 13: Trigonometry – Problems in 2 dimensions

Learning Objectives

Students should be able to

- use compass bearing and true bearing to measure directions
- define the 'angle of elevation' and the 'angle of depression'
- use trigonometric formulas in solving two-dimensional problems
- formulate and solve practical two-dimensional problems

Trigonometry – Problems in 2 dimensions

1. Basic Terms

1.1 Bearing

Bearing is used to indicate the direction of an object from a given point.

(a) *Compass bearing* : the angle measured either form the north or from the south.
i.e. Nx^oE or Nx^oW or Sx^oE or Sx^oW where $0^o<x<90^o$

(b) *True bearing* : the angle measured clockwisely from the north direction.

Example 1

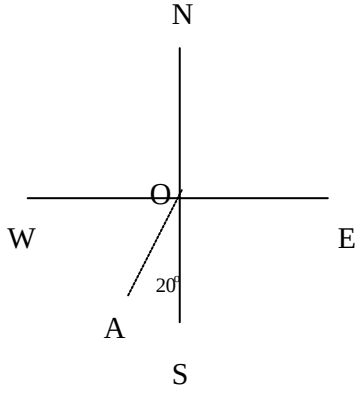


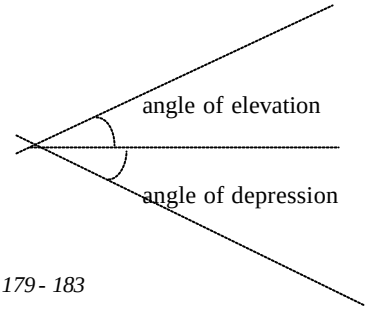
Figure 1

The compass bearing of A from O is S ^{o}W while
the true bearing of A from O is ^{o}.

1.2 Angle of Elevation and Angle of Depression

The angles of elevation and depression are the angles between the line of sight and the horizontal.

Figure 2



Reference : Maths for HK4B P.179 - 183

2. Applications to Problems in Two Dimensions

2.1 Solving problems in 2 dimensions involving right-angled triangles

For a right-angled triangle,

$a^2 + b^2 = c^2$

$\sin x = b/c$

$\cos x = a/c$

$\tan x = b/a$

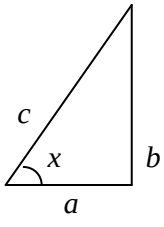


Figure 3

Example 2

The angle of elevation of a balloon from a man on the ground is 41^o and it is 200m from the man.

(a) What is the height of the balloon above the ground ?

(b) If the balloon rises vertically 100 m further, what is the angle of elevation of the balloon from the man ?

[Answer correct to 1 decimal place]

Solution:

(a) $\sin 41^o = AN/AM$

$AN = AM \sin 41^o = 200 \sin 41^o$

$= 131.2$ m

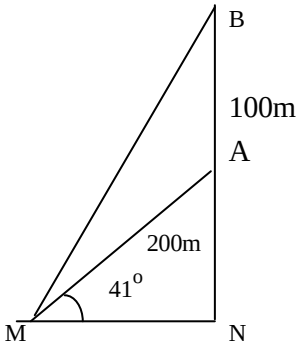


Figure 4

(b) $\cos 41^o =$ /AM

MN = AM cos 41^o = ____cos 41^o

=_____ m

$\tan \angle BMN = BN/MN = (100 +$ _____)/_____

$\angle BMN =$ _____^{o}

2.2 Solving problems in 2 dimensions using the sine or the cosine formula

The Sine Formula: In any ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

The Cosine Formula: In any ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$

Example 3

A bridge 11 km long connects city T in the south and city K in the north. The true bearing of a ship S is 120^o from T. The distance between the ship S and T is 15 km. Find the distance between the ship S and K.

[Answer correct to 1 decimal place]

Solution

An appropriate drawing is necessary in solving such a problem.

By the cosine rule,

$KS^2 = 11^2 +$ ____^{2} - 2 x 11 x _____ cos____^{o}

= _____

KS = _____ km

[Answer correct to 1 decimal place]

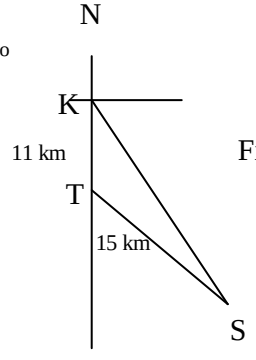


Figure 6

Example 4

A student tried to measure the height of Yashima hill. The angle of elevation of the top of the hill was found to be 23^o from a point A and 44^o from a point B. The distance between A and B was 300 m. Find the height of Yashima hill to the nearest m.

Solution

From figure 7,

$\angle AYB = 44^o - \_\_\_\_^o$

$= \_\_\_\_\_^o$

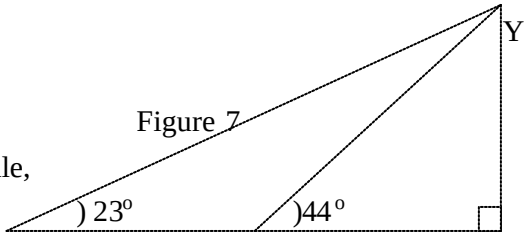


Figure 7

BY can be found from the sine rule,

$BY/\sin 23^o = \_\_\_\_/\sin \_\_\_^o$

BY =_____ m

(to the nearest m)

Example 5

In figure 8, a ship leaves a harbour A and sails at 22 km/h for 3 hours on a course of N75° E to another harbour B. It then sails at 18 km/h for 7 hours on a course of S25° E to harbour C. Find

(a) the distance between A and C

(b) the compass bearing of A from C.

[Answers correct to 1 decimal place]

Solution:

(a) Distance AB = speed x time = ____ x 3 =____ km

BC =___ x 7 =___ km

$\angle ABC = 75^o + \_\_\_^o = \_\_\_^o$

$AC^2 = AB^2 + BC^2 - 2AB.BC \cos ? ABC$

$= 66^2 + \_\_\_^2 - 2 \times 66 \times \_\_\_\_ \cos \_\_\_^o$

AC = _____ km

(b) By $\sin C/66 = \sin \_\_\_^o/152.1$

$C = \_\_\_\_^o$

The compass bearing of A from C is N_____ ° W.

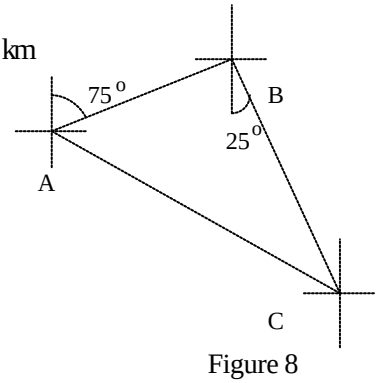


Figure 8