Unit 14 : Mensuration

Objectives

Students should be able to

- identify common plane figures and solids
- find the area of common plane figures
- understand the properties of similar figures
- find the volume of common solids
- to solve miscellaneous problems involving daily applications of mensurations

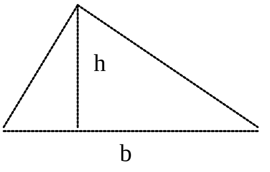
Mensuration

1. Common plane figures

1.a Triangle

Area $= \frac{1}{2}$ **base** x **height**

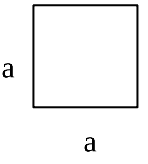
$= \frac{1}{2}$ **b h**



1.b Square

Area = (**length of a side**)^2

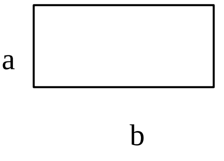
= $\mathbf{a^2}$



1.c Rectangle

Area =**length** x **breadth**

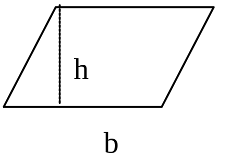
= a b



1.d Parallelogram

Area =**base** x **height**

= b h

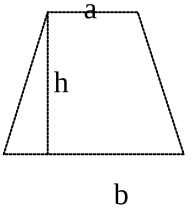


1.e Trapezium

Area

$= \frac{1}{2}$ (sum of lengths of the parallel sides) x height

$= \frac{h}{2}$ (a+b)

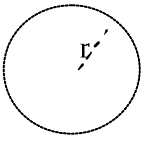


1.f Circle

Circumference = $2\pi r$

Area =πr^2

where r = radius

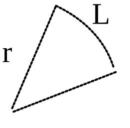


1.g Sector

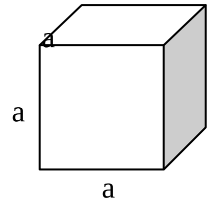
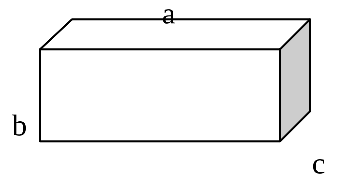
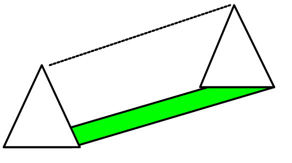
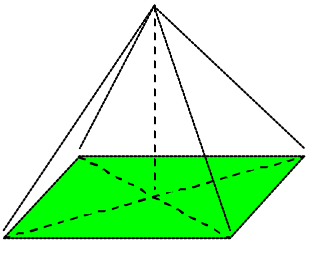
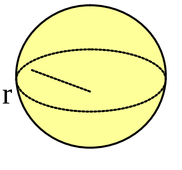
Area $A = \frac{1}{2} r^2 q$

Arc length L = $r\theta$

Example: a chinese fan



2. Solids

2.a Cube **Surface area** = $6a^2$ **Volume** = a^3 *Example*: dice	
2.b Rectangular cuboid **Volume** = a b c **Surface area** =4ab + 2bc *Example*: mahjong	
2.c Prism **Volume** = (Regular cross-sectional area)x Length **Surface area** =area of 3 rectangular faces + area of 2 triangular faces *Example*: chocolate bar	
2.d Pyramid on rectangular base **Surface area** =area of 4 slant faces + area of rectangular base **Volume** = $\frac{1}{3}$ base area x height *Example*: pyramids in Egypt	
2.e Sphere **Surface Area** = $4\pi r^2$ **Volume** = $\frac{4}{3}\pi r^3$ *Example*: basket balls	

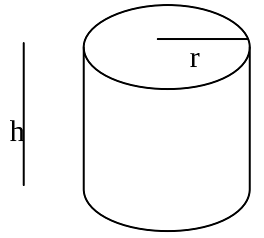
2.f. Cylinder

Surface Area $=2\pi r^2+2\pi rh$

Volume $=\pi r^2 h$

where r = radius and h = height

Example: can



2.g Cone

Surface Area $=\pi r^2+\pi rL$

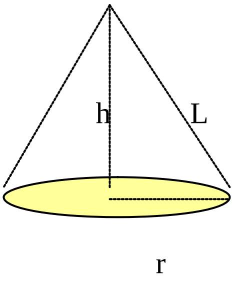
Volume $=\frac{1}{3}$ base area x height

$=\frac{1}{3}\pi r^2 h$

where r = radius and h = height

$L=\sqrt{r^2+h^2}$

Example: conic paper cup

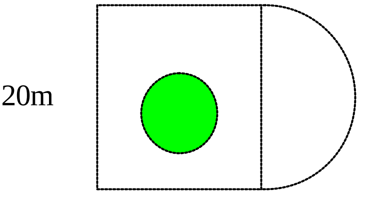


Example 1

A plot of land is in the shape of a semi-circle joining a square as shown. The length of one side of the square is 20 m.

(a) Find the total area of the land.

(b) A circular swimming pool of radius 5m is to be constructed in this plot and grass would be planted in all area surrounding the pool . Find the cost of plant grass given the cost for plant 1 m^2 of grass is $12.



Solution

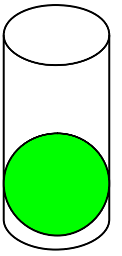
(a) Total area $=\frac{1}{2}\pi r^2+a^2$

$=\frac{1}{2}\pi$ (___)^2+____^{2} =____ m^2

(b) Area of grassland = _____ $-\pi$ (_____)^2 =_______ m^2

Cost = $12*_______ =$________(rounding up to the nearest dollar)

Example 2

A solid metal sphere is dropped inside a cylindrical glass bottle of internal radius 5 cm as shown. (a) Find the volume of the largest sphere that can fit into the bottle. (b) Find the volume of water to fill the bottle completely given the internal height of the bottle is 18 cm.	

<u>Solution</u>

a.Volume of the largest sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\ (\_\_\_)^3 =$ ______ cm^3

b.Volume of water required = internal volume of bottle – volume of sphere

$= \pi r^2 h$ – ______ = ______ – ______ = ______ cm^3

3. Similar Figures

3.1 Area ratio

If two figures have linear ratio a : b, then the area ratio would be $a^2 : b^2$.

3.2 Volume ratio

If two solids have linear ratio a : b, then the volume ratio would be $a^3 : b^3$.

Example 3

Two similar models have a linear ratio 5 to 2. The height of the larger model is 20 cm.

a. Find the height of the smaller model.

b. Find the surface area of the smaller model given the surface area of the larger model is 4000 cm^2.

c. Find the volume of the larger model given the volume of the smaller model is 8000 cm^3.

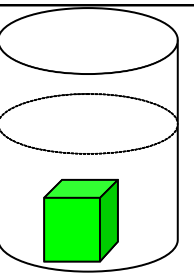
<u>Solution</u>

a. 20 : h = ___ : ___ ; height of the smaller model= ___ cm

b. $\frac{4000}{S} = \frac{\_\_\_^2}{\_\_^2}$; surface area of the smaller model = ______ cm^2

c. $\frac{V}{8000} = \frac{\_\_\_^{3}}{\_\_^{3}}$; volume of the larger model = ____________ cm^3

Example 4

A cube measures 4cm x 4cm x 4cm is dropped into a cylindrical jar of internal radius 5cm. The original water level is 5 cm. Find the new water level. (Answer correct to 2 decimal places)	

Solution

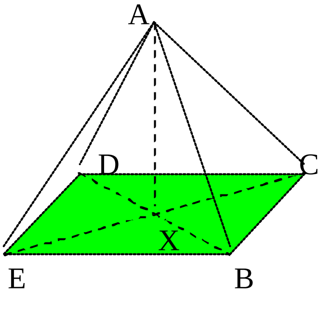
Volume of water displaced = volume of cube = _____^{3} = ______cm^3

Rise in water level = volume of water displaced/cross-sectional area of jar

=64/π (____)^2 = __________ cm

New water level = ___ + _______ = _______ = _____cm (correct to 2 decimal places)

Example 5

A pyramid was built on a rectangular base measuring 20m x 30m. The vertical height of the pyramid is 45m. (a) Find the total area of the slant surfaces.[Hint: find the length of a diagonal of the base.](b) Find the volume of the pyramid. All answers correct to the nearest integer.	

Solution

a. Length of a diagonal of the base = $\sqrt{\_\_\_^2 + \_\_\_^2}$ =________m

XB =18.03m

AB = AC = AD = AE =$\sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$ =_______m

Total area of the slant surfaces = 2ΔABC+2ΔABE

$=2[\frac{1}{2}.20(\sqrt{\_\_\_\_\_\_^2 - \_\_\_^2})]+2[\frac{1}{2}.30(\sqrt{\_\_\_\_\_^2 - \_\_\_\_^2})]$

=________+__________ = ________ = _________ m^2

b. Volume of the pyramid= $\frac{1}{3}$ base area x height

= $\frac{1}{3}$(_____x_____) x ___ = _______m^3

Web Fun Try the quick puzzle 2nd set at http://www.ex.ac.uk/cimt/