Unit 17 : Definition of Probability

Learning Objectives

The students should be able to :

- List and define the terms used in probability
- Carry out simple operations on events
- Define the probability of an event
- State the basic properties of probability
- Solve simple problems in probability

Definition of Probability

Introduction

The probability P that an event E that may happen is given by

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} \qquad \text{(F1)}$$

In order to use this formula, we need to be able to **count the number of outcomes correctly**.

Counting

1.1 Counting by listing

We use the {} to enclose the outcomes. For example, the possible outcomes of tossing a coin are {H, T}; the possible outcomes of tossing a die are {1, 2, 3, 4, 5, 6}.

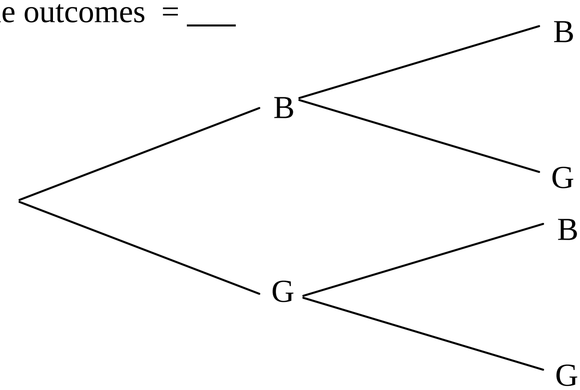
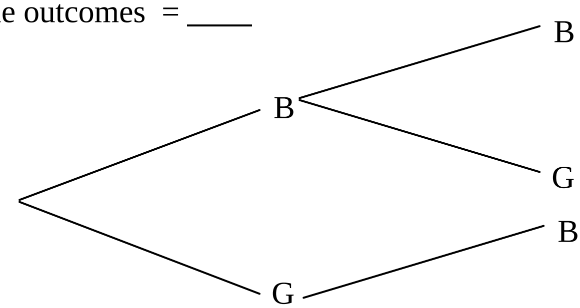
Example 1	Consider tossing of a fair coin twice and we want to have one head only. a) Find the number of possible outcomes. b) Find the number of favourable outcomes. c) Find the probability of getting one head only.
Solution	a) possible outcomes = {HH, ____, TH, ____} number of possible outcomes = __. b) favourable outcomes = {____, ____} number of favourable outcomes = ____. c) P(only one head) $= \frac{\_\_}{4} = \frac{\_\_}{\_\_}$ #

Example 2	Consider tossing of a fair coin and a die, a) Find the number of possible outcomes. b) Find the probability of getting a head and an even number.
Solution	a) possible outcomes = {1__, __H, 3__, __H, 5H, 6H, __T, 2__, __T, 4__, 5T, ___}number of possible outcomes = ___ b) favourable outcomes = {___, ___, ___} number of favourable outcomes = ___. c) P(one head and an even number) $= \frac{\_\_}{12} = \frac{\_\_\_}{\_\_\_}$ #

Example 3	When two dice are rolled, find the number of possible outcomes.
Solution	S = {(1,1), ……………, (1,6), (2,1), ……………(2,6), …………………………, (6,1), ……………, (6,6)}. Number of possible outcomes = __.

1.2 Counting by using Tree diagram

- The number of leaves is the number of outcomes

Example 4	A family has two children. Find the number of possible outcomes of their genders.
Solution	number of possible outcomes = ___ 
Example 5	A certain family has two children and it is known that one of them is a boy. Find the number of possible outcomes of the their genders.
Solution	number of possible outcomes = ____ 

1.3 Counting by calculation

The methods of listing and tree diagram always work. But it is troublesome to do the full listing. For simple cases, we may calculate the number of outcomes directly.

Number of outcomes = product of number of possible outcomes of each individual variable (F2)

Example 6	Calculate the number of possible outcomes of the following cases: a) tossing two coins b) tossing a coin and a die c) tossing two dice d) tossing a die twice e) tossing a coin three times
Solution	a) number of possible outcomes = 2(__) = __ b) number of possible outcomes = 2(__) = __ c) number of possible outcomes = __ (__) = __ d) number of possible outcomes = __ (__) = __ e) number of possible outcomes = __ (2)(__) = __

- We cannot use F2 to solve Example 5. That is why we always need to have the ability to list the wanted outcomes directly.

Further examples

Once we are familiar with the counting, there is no need to list out the required outcomes for simple cases.

Example 7	A dice is thrown, what is the probability of obtaining a) an odd number ? b) the number '6' ? c) a number smaller than 3 ?
Solution	a) P(odd number) = $\frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$ # b) P('6') = $\frac{\_\_}{6}$ # c) P(less than 3) = $\frac{\_\_}{6}$ #
Example 8	A family has 2 children, what is the probability that both of them are boys?
Solution	P(both are boys) = $\frac{1}{2(\_\_)} = \frac{1}{\_\_}$ #

Example 9	A family has three children. Find the probability that the first child is a boy and the second child is a girl.
Solution	Favourable outcomes = {____, BGB} P(first child is a boy and the second child is a girl) = $\frac{\_\_}{2(\_\_)(\_\_)} = \frac{1}{\_\_\_}$ #

For relatively complicated problems, it is advisable to list out the outcomes.

Example 10	A card is drawn randomly from a deck of cards. Find the probability of getting a K or the card belongs to the Heart suit with value greater than 10?
Solution	Favourable outcomes = {♠K, ♥__, ♣K, ♦__, ♥J, ♥__, ♥__} P(a K or a Heart) = $\frac{\_\_\_}{\_\_\_}$ #
Example 11	A family has 2 children, and one of them is a boy. What is the probability that both are boys?
Solution	Possible outcomes = {BB, __, __} P(the other is a boy) = P(both boys) = $\frac{\_\_}{\_\_}$ #
Example 12	A family has three children. Find the probability that the first child is a boy or the second child is a girl.
Solution	Outcome of first child is a boy = {______, BBG, _____, BGG} Outcome of second child is a Girl = {BGB, _____, GGB, _____} Favourable outcomes = {BBB, _____, BGB, _____, GGB, _______ } P(first child is a boy or the second child is a girl) = $\frac{\qquad}{2(\_\_)(\_\_)} = \frac{\_\_}{\_\_}$ #
Example 13	When three coins are tossed, find the probability of getting more heads than tails.
Solution	Favourable outcomes = {____, HHT, _____, THH}. P(more heads than tails) = $\frac{\qquad}{\_\_ \times \_\_ \times \_\_} = \frac{1}{\_\_}$ #

Finally, we introduce a very **useful** formula

$$P(E) = 1 - P(\text{not } E). \qquad \text{(F3)}$$

It means

the probability that an event may occur = 1 – the probability that the event may not occur.

F3 is useful when the direct calculation of P(not E) is easier than that of P(E).

Example 14	Three identical unbiased coins are tossed. Find the probability of getting (a) 3 tails ; (b) exactly 2 tails ; (c) at least one head ; and (d) at least one head or one tail.
Solution	a) P(3T) = $\frac{1}{\_\_ \times 2 \times \_\_} = \frac{1}{\_\_\_\_}$ # b) Favourable outcomes = {TTH, ____, ____} P(exactly 2T) = $\frac{\_\_}{\_\_}$ # c) P(at least 1H) = 1 – P(not (at least 1H)) = 1 – P(3T) = 1 - $\frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$ # d) P(at least one head or one tail) = P(any outcome) = __#

We may solve Example 12 by this formula.

Example 15	A family has three children. Find the probability that the first child is a boy or the second child is a girl.
Solution	P(first child is a boy or the second child is a girl) = 1 – P(first child is a girl and the second child is a boy) = 1 – P(GBB or ____) = 1 - $\frac{\_\_}{2(\_\_)(\_\_)} = \frac{\_\_}{\_\_}$ #

Example 16

A survey on the modes of transport for HKIVE students when they come back to school in a certain campus produced the following data:

Mode of transport	On foot	By bus	By MTR	By Train	Others
Number of students	225	130	445	165	85

If a student is randomly selected from the campus,

(a) what is the probability that he/she will travel back to school by train ?

(b) What is the probability that he/she will either travel on MTR or by train ?

(c) what is the probability that he/she will take certain type of vehicles to come back to school ?

Solution

Number of possible outcomes = 225 + 130 + ____ + 165 + ____ =

a) P(by train) = $\frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$ #

b) P(by train or by MTR) = $\frac{\_\_ + \_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$ #

c) P(by vehicle) = 1 – (not (by vehicle)) = 1 – P(on foot) = 1 - $\frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$ #