Unit 2 Basic Technique of Solving Problems

Learning Objectives

The students should be able to:

- Solve linear equations
- Solve equations of the form $ax^n = b$
- Transform a given equation into the above forms if necessary.
- Solve simple problems involving equations of the above two forms.

1. Introduction

In solving mathematical problems, usually we need to solve equations. In this unit we are going to revise the basic technique of solving two types of simple equations: linear equations and equations of the form $ax^n = b$. Then, we shall demonstrate the basic technique of solving problems.

2. Solving of Linear equations

Steps:

1. Remove the brackets by expansion.
2. Group the unknown terms to one side of the equation and the constant terms to the other side.
3. Simplify the LHS and RHS of the equation into the form $ax = b$.
4. $x = \frac{b}{a}$, value found by mental calculation or by calculator

E.g. 1	Solve $2x+5=4x-7$	Explanation
Solution	$2x+5=4x-7$ $2x-4x=-7-5$ $-2x=-12$ $x=6_{\#}$	No brackets, no need to do step 1 Step 2 Step 3 Step 4

E.g. 2	Solve $3(x+2)-4(x-7)=5$	Explanation
Solution	$3(x+2)-4(x-7)=5$ $3x+$ ___ $-4x+$ ___ $=5$ $3x-4x=5-$ ___ $-$ ___ $-x=-$ ____ $x=$ ___ #	 Step 1 Step 2 Step 3 Step 4

Once we are familiar with the method, steps 1 and 2 may be combined.

E.g. 3	Solve $3x-2(x+1)=5(x+2)$	
Solution	$3x-2(x+1)=5(x+2)$ $3x-$ ___ $x-5x=10+2$ $-4x=$ ___ $x=-$ ___ #	Step 1 & 2 Step 3 Step 4

E.g. 4	Solve $\frac{h+7}{2}-\frac{1}{3}=\frac{1}{2}-\frac{h+9}{9}$	
Solution	$\frac{h}{2}+\frac{h}{9}=\frac{1}{2}-1-\frac{7}{2}+\frac{1}{3}$ $\frac{11}{\_\_\_}h=\frac{-11}{\_\_\_}$ $h=-$ ___ #	Step 1 & 2 Step 3 Step 4

3. Solving of equation of the form $ax^n = b$.

- $x = \pm\left(\frac{b}{a}\right)^{\frac{1}{n}}$ if n is even, two solutions
- $x = \left(\frac{b}{a}\right)^{\frac{1}{n}}$ if n is not even

E.g. 5	Solve $x^2 = 4$	Explanation
Solution	$x^2 = 4$ $x = \pm(\_\_\_)^{\frac{1}{2}}$ $= \_\_\_ \text{ or } -\_\_\_$ #	$n = 2$, even
E.g. 6	Solve $x^3 = 8$	
Solution	$x^3 = 8$ $x = 8^{\frac{1}{3}}$ $= \_\_\_$ #	$n = 3$, not even
E.g. 7	Solve $2\sqrt[3]{x} = 8$	
Solution	$2\sqrt[3]{x} = 8$ $2x^{\frac{1}{3}} = 8$ $x = \left(\frac{8}{\_\_\_}\right)^3$ $= \_\_\_$ #	$n = \frac{1}{3}$, not even
E.g. 8	$v^2 - u^2 = 2as$. Find v if $u = 10$, $a = 10$ and $s = 15$.	
Solution	$v^2 - u^2 = 2as$ $v^2 - \_\_\_^2 = 2(10)(\_\_\_)$ $v^2 = 2(10)(\_\_\_) + \_\_\_^2$ $v = \pm(\_\_\_\_)^{\frac{1}{2}}$ $= \_\_\_ \text{ or } -\_\_\_$ #	

4. Transformation of the equations

- If the unknown is at the denominator of an equation. We may multiply the equation by the denominator.
  The resulting equation may be solved by the above technique.

E.g. 9	$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. Find v if u = 30 and f = 10	
Solution	$\frac{1}{\_\_\_}+\frac{1}{v}=\frac{1}{\_\_\_}$ $\frac{v}{30}+1=\frac{v}{\_\_\_}$ $\frac{v}{30}-\frac{v}{\_\_\_}=-1$ $\left(-\frac{1}{\_\_\_}\right)v=-1$ $= \_\_\_ _{\#}$	
E.g. 10	Solve $\frac{1}{x-3}+7=5-\frac{2}{x-3}$	
Solution	$1+\_\_\_\_(x-3)=5(x-3)-\_\_\_$ $\_\_\_\ x-5x=-15-2-1+21$ $2x=\_\_\_$ $x=\_\_\_\_ _{\#}$	

- If $(x - a)^n$ exists, treating $(x - a)$ as a variable is another method.

E.g. 11	Solve $\frac{6}{(x-3)^3}+15=7-\frac{2}{(x-3)^3}$	
Solution	Let $y = x - 3$ (1) $\frac{6}{y^3}+15=7-\frac{2}{y^3}$ $6+\_\_\_\ y^3=\_\_\_\ y^3-2$ $\_\_\ y^3=-8$ $y^3=-\_\_\_$ $y=-\_\_\_$ By (1) $x - 3 = -\_\_\_$ $x = \_\_\_ _{\#}$	