FC Mathematics Examination – Suggested answers 2000/2001

A1

Given $x-2=\sqrt{a+4}$, when x =3, then a equals

A. 2

B. –2

C. 3

D. -3

A2.

$a^4 - y^4 =$

A. $(a+y)(a-y)(a^2-y^2)$

B. $(a-y)(a+y)(a^2+y^2)$

C. $(a+2y)(a-2y)(a^2-y^2)$

D. $(a^2+y^2)(a^2-2ay-y^2)$

A3

$\sin 215°$

A. $\cos 35°$

B. $\sin 35°$

C. $-\sin 35°$

D. $-\cos 35°$

A4.

Given $\cos x = -0.5$, then x equals

A. $30°$

B. $60°$

C. $180°$

D. $240°$

A5

The 7th term of the arithmetic sequence is –8, -2 ,4 …….is

A. 28

B. 27

C. 26

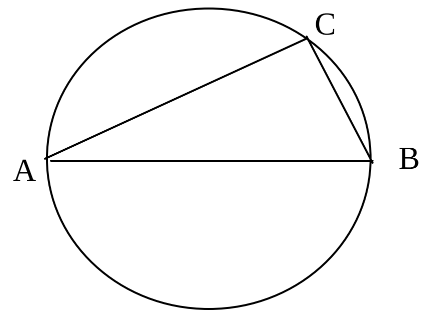
D. 25

A6.

AB is a diameter of the circle ABC. Given $\angle CAB$ equals 35°

$\angle ABC$ equals

A. 45°
B. 55°
C. 65°
D. 90°



A7.

If the y-intercept of the line $4x+3y+2=\lambda$ is equal to 1, then $\lambda =$

A. 2
B. 3
C. 4
D. 5

A8.

The equation $2x^2 + kx + 18 = 0$ has two identical roots. k equals

A. -12
B. –6
C. ±12
D. ±6

A9.

Tea X costs $50 per 100g and tea Y costs $40 per 100g. A new brand of tea is produced by mixing X and Y in the ratio 2:3 by weight. How much would 100g of the new brand of tea cost?

A. $44
B. $45
C. $46
D. $48

A10.

The mode of the set of numbers {2,3,7,5,5,1,5,8,8,6,5,7,19,2,4} is

A. 2
B. 5
C. 7
D. 8

A1

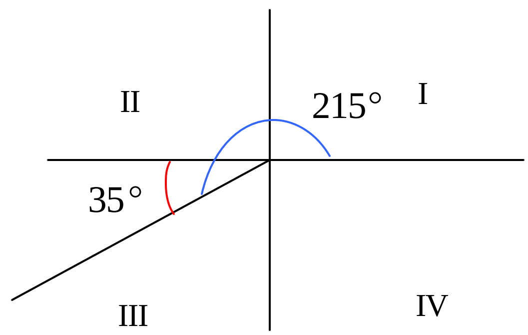
$When\ x=3, \quad 3-2=\sqrt{a+4} \Rightarrow \quad \sqrt{a+4}=1 \quad \Rightarrow a=1-4=-3$

A2.

$a^4-y^4= \quad (a^2)^2-(y^2)^2 \quad =(a^2+y^2)(a^2-y^2) \quad =(a^2+y^2)(a+y)(a-y)$

A3

$\sin 215° \quad =-\sin 35°$

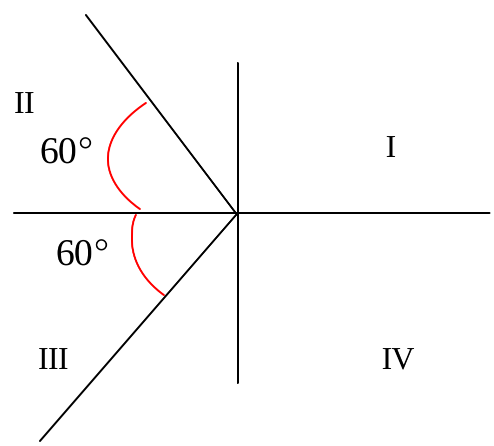


A4.

$\cos x=-0.5$

$if\ \cos y=0.5, \quad y=60°\ or\ 300°$

$when\ \cos x=-0.5 \quad \Rightarrow x=120°\ or\ 240°$



A5

The arithmetic sequence is –8, -2 ,4 ……..

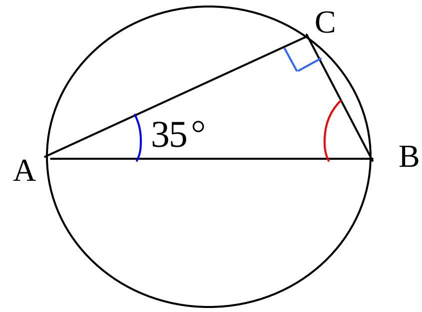
$T(n)=a+(n-1)d$

$\because a=-8, \quad d=-2-(-8)=6$

$\therefore T(7)=-8+(7-1)6=-8+36 \ =28$

A6.

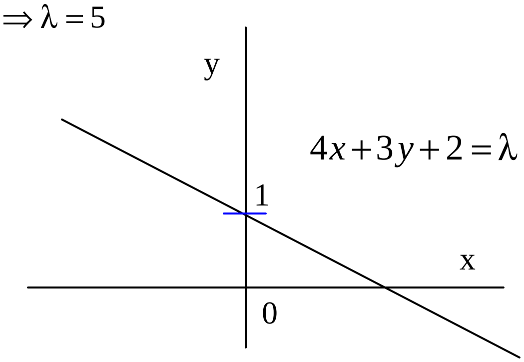
$\angle ABC = 90° - 35° = 55°$



A7.

The line $4x + 3y + 2 = \lambda$

when $x = 0$, y-*intercept* $= 1$, $\therefore 4(0) + 3(1) + 2 = \lambda \quad \Rightarrow \lambda = 5$



A8.

$2x^2 + kx + 18 = 0$

The equation has two identical roots $\therefore \Delta = b^2 - 4ac = 0$

$k^2 - 4(2)(18) = 0 \quad \Rightarrow k^2 = 144 \quad \Rightarrow k = \pm\sqrt{144} \quad \Rightarrow k = \pm 12$

A9.

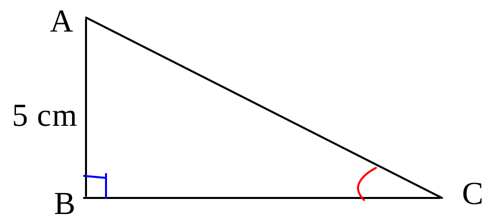
The new tea cost of 100g $= (100 \times \frac{2}{2+3}) \times \frac{50}{100} + (100 \times \frac{3}{2+3}) \times \frac{40}{100} \quad = 20 + 24 \quad = \$44$

A10.

{2,3,7,5,5,1,5,8,8,6,5,7,19,2,4}

The mode is 5.

B1.



$Given\, cosC = \frac{1}{3},$

$(a) \quad \angle ACB = \cos^{-1}\frac{1}{3} = 70.5° \; (correct\ to\ 1\ decimal\ place)$

$(b) The\ length\ of\ AC:$

$Consider\ \Delta ABC, \quad sin\angle ACB = \frac{5}{AC} \quad \Rightarrow AC = \frac{5}{\sin 70.529°} = 5.30\, cm\; (correct\ to\ 2\ decimal\ place)$

B2.

$The\ equation\ of\ the\ circle\ is:$

$(x-1)^2 + [y-(-3)]^2 = 2^2$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 4$

$\therefore \quad x^2 + y^2 - 2x + 6y + 6 = 0$

B3

$-7 < 2x^2 - 15x$

$2x^2 - 15x > -7$

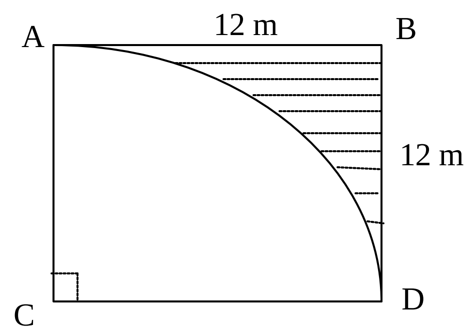
$2x^2 - 15x + 7 > 0$

$2x$	-1
$1x$	-7

$(2x-1)(x-7) > 0$

$\therefore x < \frac{1}{2} \quad or \quad x > 7$

B4.



$The\ area\ of\ the\ shaded\ region:$

$= 12^2 - \frac{1}{2}(12)^2(\frac{\pi}{2}) \quad = 144(1-\frac{\pi}{4}) \quad = 30.9\ m$

$(correct\ to\ 1\ decimal\ place)$

B5.

Given: $x : (y - 1) = 3 : 4$

(a) $\dfrac{x}{(y-1)} = \dfrac{3}{4} \quad \Rightarrow x = \dfrac{3}{4}(y-1)$

(b) if $2x + y = 21 \quad \Rightarrow y = 21 - 2x \cdots\cdots (1)$

Sub (1) to (a), $\quad x = \dfrac{3}{4}(21 - 2x - 1)$

$4x = 60 - 6x \quad \Rightarrow x = 6, \quad y = 21 - 2(6) = 9$

B6.

(a) $P(\text{the red book is drawn}) = \dfrac{6}{2+4+6} = \dfrac{6}{12} = \dfrac{1}{2}$

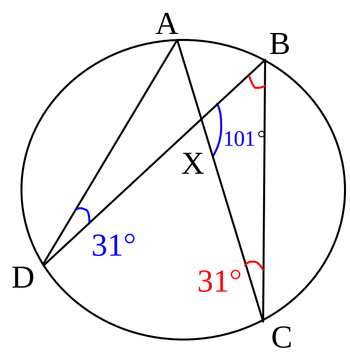
(b) $P(\text{the first book is blue and the second book is green is drawn without replacement}) = \dfrac{2}{12} \times \dfrac{4}{11} = \dfrac{2}{33}$

B7.

$\angle ADX = \angle ACB = \angle XCB = 31^\circ \quad (\angle \text{ in the same segment})$

Consider the ΔBXC, $\quad \angle CBX = 180^\circ - \angle BXC - \angle XCB$

$= 180^\circ - 101^\circ - 31^\circ = 48^\circ$



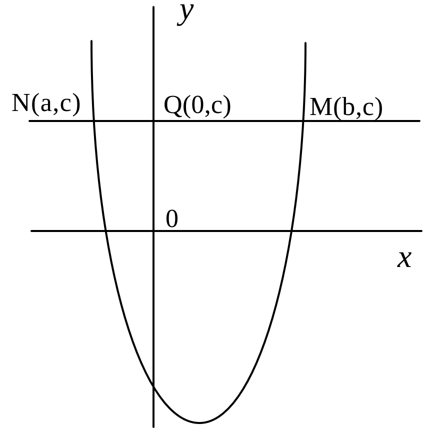
C1.

(a) $Given: \quad y = x^2 - 4x + 3 = c \quad when\ x = a\ or\ b$

$\therefore \quad a\ and\ b\ are\ the\ roots\ of\ the\ equation$

$x^2 - 4x + (3 - c) = 0$

$sum\ of\ root = a + b = \frac{-(-4)}{1} = 4$



(b) $product\ of\ roots = ab = \frac{(3-c)}{1} = (3-c)$

(c) $Given: \quad a = b$

$From\ (a), \quad a + a = 4 \quad \Rightarrow a = 2$

$From\ (b), \quad a \cdot a = 3 - c = 2 \times 2 = 4$

$\therefore c = 3 - 4 = -1$

(d) $Given: \quad MQ = 2NQ$

$MQ = b - 0 = b$

$NQ = 0 - a = -a \quad (\because a\ is\ negative)$

$\therefore \quad b = -2a$

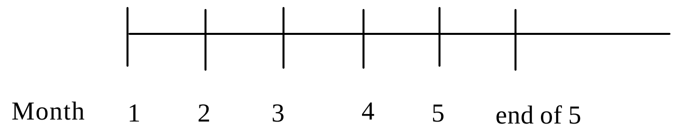
$From\ (a), \quad a + (-2a) = 4 \quad \Rightarrow a = -4$

$From\ (b), \quad a \cdot -2a = 3 - c = -4 \times -2(-4) = -32$

$\therefore c = 3 + 32 = 35$

C2.

This is a question about geometric progression.



(a) i) *The length of the longest bar* $= aR^4$

ii) *The total length of all the bars* $= \dfrac{a(R^5 - 1)}{R - 1}$

(b) i) *The accrued amount (total deposit plus any interest earned) at the end of 2 months* =

$5000(1+.01)^2 + 5000(1+.01) =$ *\$10,101 (correct to the nearest dollar)*

ii) *The accrued amount of Joey at the end of five consecutive months* $= 5000(1+.01)^5 + 5000(1+.01)^4 +$

$5000(1+.01)^3 + 5000(1+.01)^2 + 5000(1+.01)^1 = 5000\left[1.01^5 + 1.01^4 + 1.01^3 + 1.01^2 + 1.01\right]$

$= 5000\dfrac{\left[1.01(1.01^5 - 1)\right]}{1.01 - 1} = \$25{,}760.$ *(correct to the nearest dollar)*

C3.

(a) By distant formula,

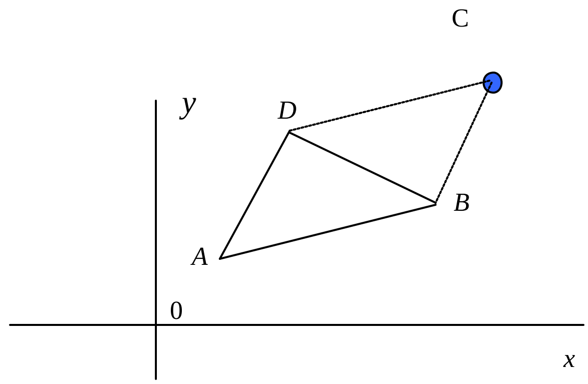
$AD = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{10}$

$AB = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{17}$

$BD = \sqrt{(5-2)^2 + (2-4)^2} = \sqrt{13}$

$\because$ $AD \neq AB \neq BD$, there is no equal pair in the triangle

$\therefore$ ΔABD is not an isosceles triangle



(b) The slope of line $AB = \frac{2-1}{5-1} = \frac{1}{4}$

The equation of the line AB :

$\frac{y-1}{x-1} = \frac{1}{4} \Rightarrow 4y - 4 = x - 1 \quad \therefore 4y - x - 3 = 0$

(c) The slope of line $AD = \frac{4-1}{2-1} = 3$

The equation of the line AD :

$\frac{y-1}{x-1} = 3 \Rightarrow y - 1 = 3x - 3 \quad \therefore y - 3x + 2 = 0$

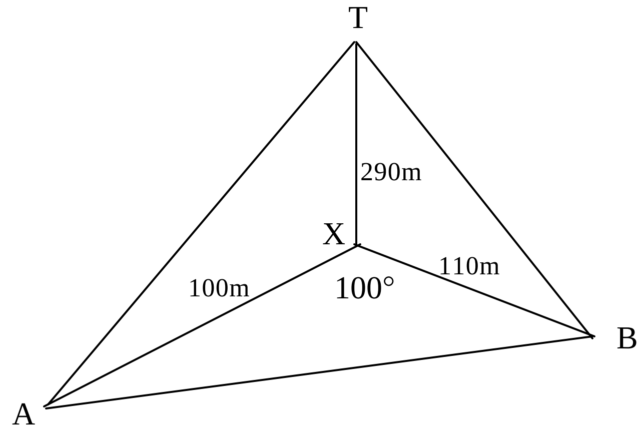
(d) The slope of line $DC = \frac{5-4}{6-2} = \frac{1}{4}$ = The slope of line AB (From a)

The slope of line $BC = \frac{5-2}{6-5} = 3$ = The slope of line AD (From b)

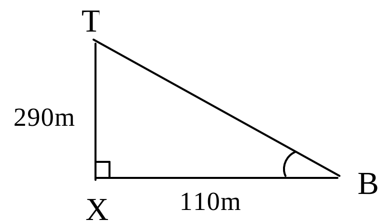
$\therefore$ $DC \,//\, AD$ and $BC \,//\, AD$

Hence ABCD forms a parallelogram.

C4.



(a)

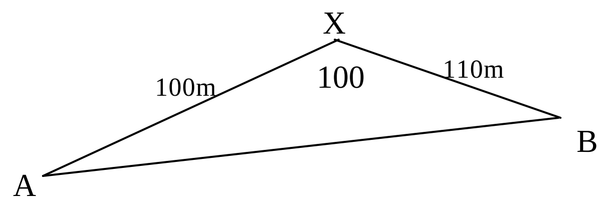


(a) *Consider* ΔTXB,

$$\tan\angle TBX = \frac{290}{110} \Rightarrow \angle TBX = \tan^{-1}\left(\frac{290}{110}\right) = 69.23°$$

(b) *The minimum length of the cable* $= TX + AX = 290 + 100 = 390m$

(c) *Consider* ΔABX *, by cosine formula,*



$$AB^2 = AX^2 + BX^2 - 2AX \cdot BX \cos\angle AXB$$

$$= 100^2 + 110^2 - 2(100)(110)\cos 100° = 25,920$$

$$\therefore \quad AB = \sqrt{25920} = 161.0\ m$$

(d) *The area of* $\Delta AXB = \frac{1}{2}(AX)(BX)\sin\angle AXB = \frac{1}{2}(100)(110)\sin\angle 100° = 5,416.44\ m$

(*correct to 2 decimal places*)

(e) *The cost of building the garden* =

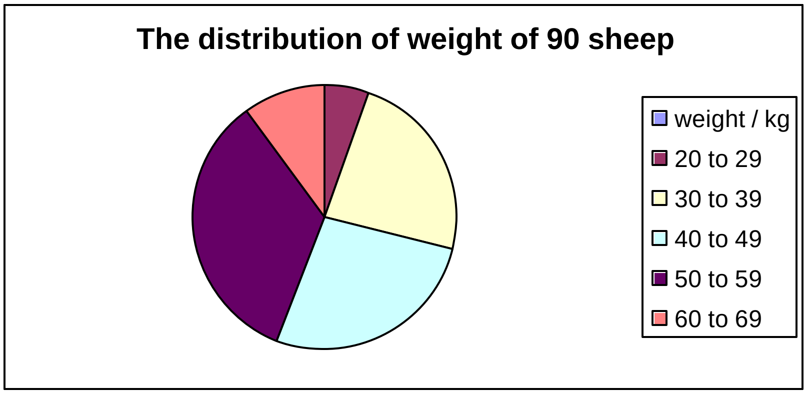
The area of $\Delta AXB \times$ *the cost of building* $= 5,416.44 \times 12 = \$64,997$

(*correct to the nearest dollar*)

C5

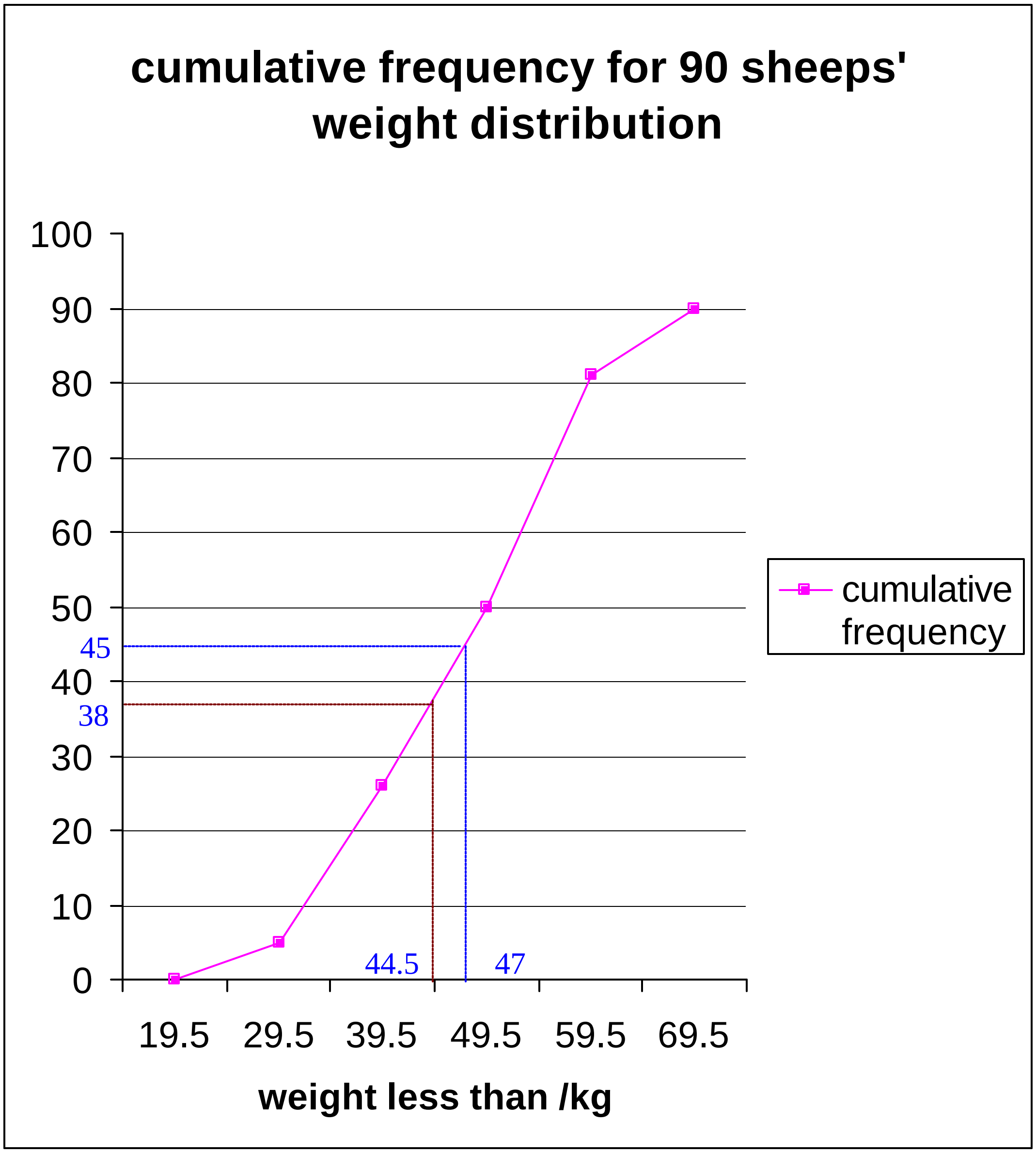
weight / kg	Frequency	relative frequency / %	sector angle / degree
20 to 29	5	5.56	20
30 to 39	21	23.33	84
40 to 49	24	26.67	96
50 to 59	31	34.44	124
60 to 69	9	10.00	36
sum	90	100	360

(a)



(b)

weight up to / kg	cumulative frequency
19.5	0
29.5	5
39.5	26
49.5	50
59.5	81
69.5	90



(c) From the graph, the median of the distribution is 47 kg.
(d) From the graph, 38 sheep would be selected