FC Supplementary Examination I 2000/01

Section B Short Questions

B1.

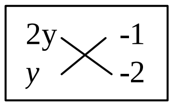
$2\cos^2 x - 5\cos x + 2 = 0 \qquad 0° < x < 360°$

$Let\ y = \cos x, \quad \Rightarrow \quad 2y^2 - 5y + 2 = 0$

$(2y-1)(y-2) = (2\cos x - 1)(\cos x - 2) = 0$

$\therefore \quad \cos x = \frac{1}{2} \quad or \quad 2\ (rejected)$

$x = 60°\ or\ 300°$



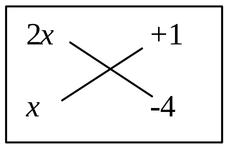
B2.

$2x^2 - 7x > 4$

$2x^2 - 7x - 4 > 0$

$(2x+1)(x-4) > 0$

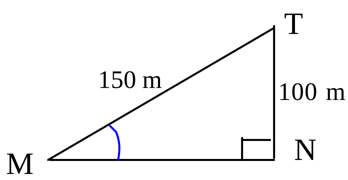
$\therefore \ x < -\frac{1}{2} \quad or \quad x > 4$

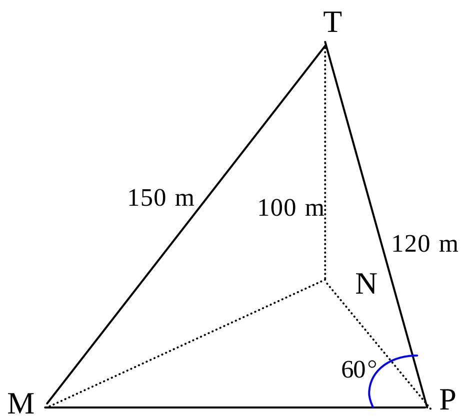


B3.

$The\ new\ mixture\ cost\ of\ 100g \quad = \quad (20)\times\frac{30}{100} + (80)\times\frac{20}{100} \quad = 6 + 16 \quad = \$22$

B4.





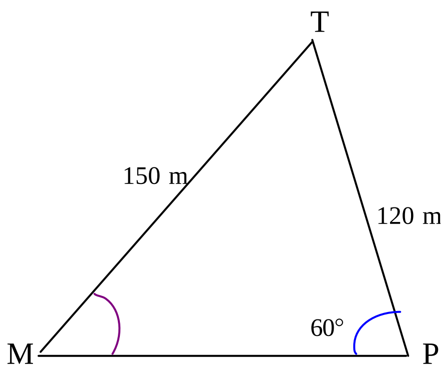
$(a) \quad \sin \angle TMN = \frac{100}{150} \quad \Rightarrow \angle TMN = \sin^{-1}\frac{100}{150} = 41.8°$

$(Correct\ to\ 1\ decimal\ place)$

(b) Consider ΔTMP, by using sine formula,

$$\frac{\sin\angle TMP}{120}=\frac{\sin 60}{150} \Rightarrow \sin\angle TMP = 120\times\frac{\sin 60}{150}$$

$$\Rightarrow \angle TMP = 43.9^\circ \quad \text{(Correct to 1 decimal place)}$$

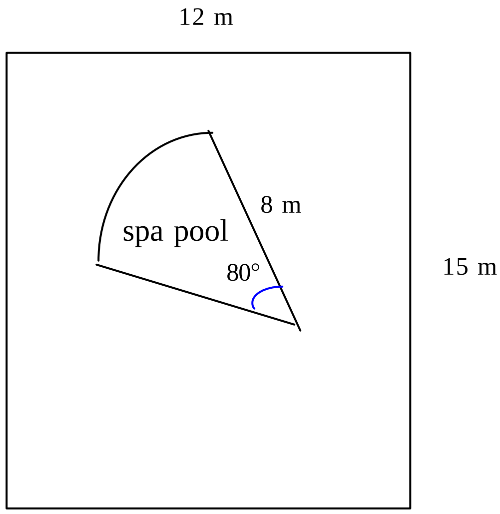


B5.

(a) Sector angle $\theta = \frac{80}{180}\cdot\pi$

The area of the spa pool $=\frac{1}{2}r^2\theta = \frac{1}{2}\cdot 8^2\cdot\frac{80}{180}\cdot\pi = 44.7m^2$

(Correct to 1 decimal place)



(b) The area of the pavement surrounding the pool

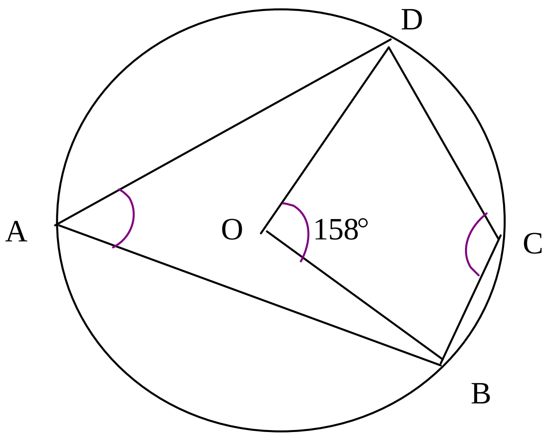
$= 12\times 15 - 44.680 = 135.3m^2$

B6.

(a) The probability that the flower is pink $=\frac{15}{10+25+15+10}=\frac{15}{60}=\frac{1}{4}$

(b) The probability that both flowers are red $=\frac{25}{10+25+15+10}\times\frac{24}{10+24+15+10}=\frac{25}{60}\times\frac{24}{59}=\frac{10}{59}$

B7.



$\angle BAD = \frac{158^\circ}{2} = 79^\circ$

($\angle$ *at centre twice* $\angle$ *at* O^{ce})

$\angle BCD = 180^\circ - \angle BAD = 180^\circ - 79^\circ = 101^\circ$

($\therefore$ *opp.*$\angle$*.cyclic quad*)

Section C

C1.

$2x^2 - (k+1)x + (k-1) = 0$ $\quad \alpha$ *and* β *are roots*

(*a*) $\alpha + \beta = \frac{-[-(k+1)]}{2} = \frac{k+1}{2}$

(*b*) $\alpha\beta = \frac{k-1}{2}$

(*c*) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\frac{k+1}{2})^2 - 2\cdot\frac{k-1}{2} = \frac{k^2+2k+1-4k+4}{4}$

$= \frac{k^2-2k+5}{4}$

(*d*) $\Delta = [-(k+1)]^2 - 4(2)(k-1) = 0$

$(k+1)^2 - 8(k-1) = 0 \Rightarrow \quad k^2+2k+1-8k+8 = k^2-6k+9 = (k-3)^2 = 0$

$\therefore k = 3$

(*e*) *From (d)*, $\Delta = (k-3)^2 >= 0$

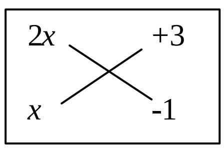
$\therefore$ *The roots of the equation are always real for all real values of* k

(*f*) *when k = -2,* $2x^2 - (k+1)x + (k-1) = 0$

$2x^2 - (-2+1)x + (-2-1) = 2x^2 + x - 3 = 0$

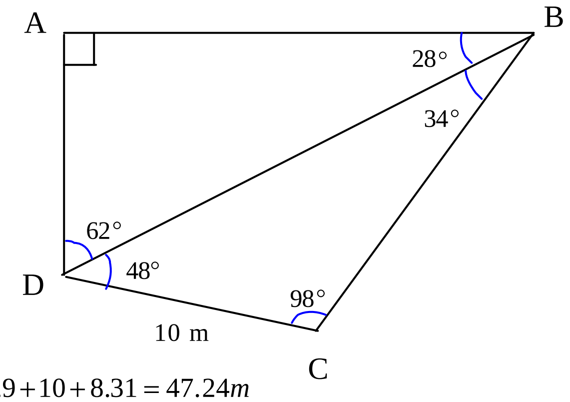
$(2x+3)(x-1) = 0$

$\therefore \quad x = -\frac{3}{2}$ *or* 1



C2.

(a) *Consider* ΔBCD, *by using sine formula,*



$$\frac{BD}{\sin 98^\circ} = \frac{10}{\sin 34^\circ} \Rightarrow BD = \sin 98^\circ \times \frac{10}{\sin 34^\circ} = 17.71m$$

(*Correct to 2 decimal places*)

(b) $$\frac{BC}{\sin 48^\circ} = \frac{10}{\sin 34^\circ} \Rightarrow BC = \sin 48^\circ \times \frac{10}{\sin 34^\circ} = 13.29m$$

(*Correct to 2 decimal places*)

(c) *Consider* ΔABD,

$AD = \cos 62^\circ \cdot BD = \cos 62^\circ \times 17.71 = 8.31m$

$AB = \sin 62^\circ \cdot BD = \sin 62^\circ \times 17.71 = 15.64m$

The perimeter of $ABCD = AB + BC + CD + AD = 15.64 + 13.29 + 10 + 8.31 = 47.24m$

(d) *The cost of fencing* $ABCD = \$10 \times 47.24 = \$472$

(*Correct to the nearest dollar*)

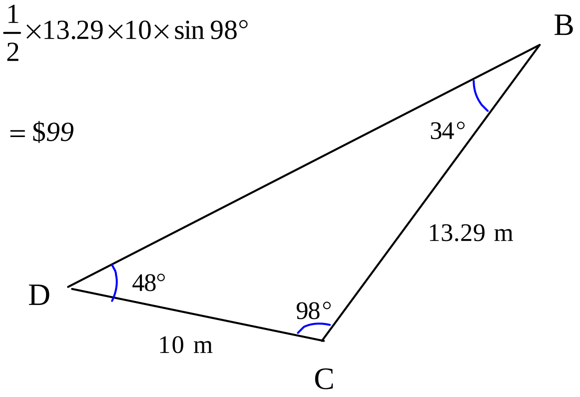
(e) *Consider* ΔBCD,

The area of the triangular plot $BCD = \frac{1}{2} BC \cdot CD \sin \angle BCD = \frac{1}{2} \times 13.29 \times 10 \times \sin 98^\circ$

$= 65.80 \approx 66m^2$ (*Correct to the nearest* m^2)

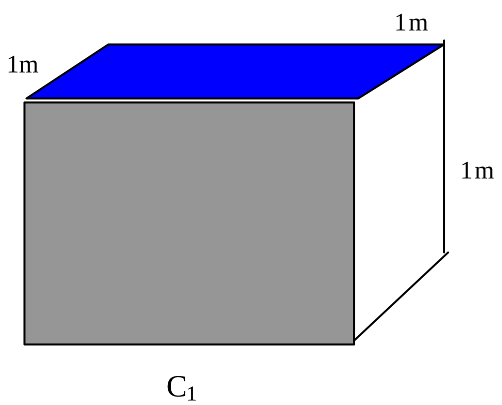
(f) *The cost of mowing the triangular plot* $BCD = \$1.50 \times 66 = \$99$

(*Correct to the nearest dollar*)

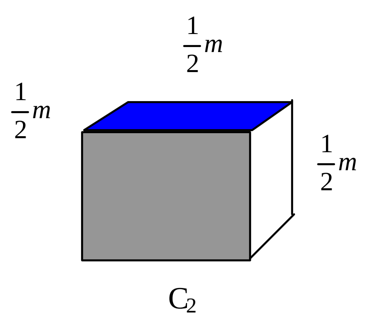


C3.

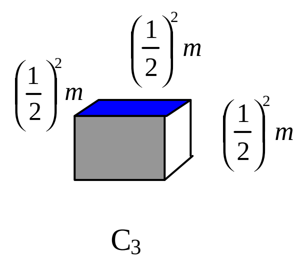
(a) *The surface area of* $C = 6 \cdot (1)^2 = 6m^2$



C_1



C_2



C_3

(b) The surface area of $C_2 = \quad 6 \cdot (\frac{1}{2})^2 = \frac{3}{2} m^2$

(c) The surface area of $C_3 = \quad 6 \cdot (\frac{1}{2})^2 (\frac{1}{2})^2 = \frac{3}{8} m^2$

The sum of surface area of C_1, C_2 and $C_3 = 6 + \frac{3}{2} + \frac{3}{8} = 7.875 m^2$

(Correct to 4 significant figures)

(d) first term   second term   third term

$6 \qquad 6 \cdot (\frac{1}{2})^2 \qquad 6 \cdot (\frac{1}{2})^2 (\frac{1}{2})^2$

$a = 6, \quad R = \frac{6 \cdot (\frac{1}{2})^2}{6} = (\frac{1}{2})^2 = \frac{1}{4}$

General term $\quad T(n) = aR^{n-1} = 6 \cdot \left(\frac{1}{4}\right)^{n-1}$

$C_7 = T(7) = 6 \cdot \left(\frac{1}{4}\right)^{7-1} = 6 \cdot \left(\frac{1}{4}\right)^{6} = 0.001465 m^2$

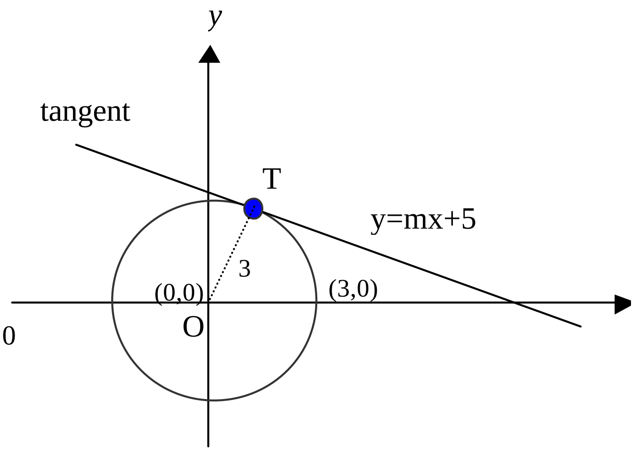
(Correct to 4 significant figures)

(e) Since $-1 < R < 1$,

the sum of surface area of all the cubes $= S(\infty) = \frac{a}{1 - R} = \frac{6}{1 - \frac{1}{4}} = \frac{6}{\frac{3}{4}} = 2 \times 4 = 8 m^2$

C4.

(a) *The equation of the circle :*



$(x-0)^2+(y-0)^2=3^2 \quad \Rightarrow x^2+y^2=9$

$\Rightarrow x^2+y^2-9=0\cdots\cdots(1)$

(b) *The equation of the tangent :* $\quad y=mx+5\cdots\cdots(2)$

$Sub\ (2)\ to\ (1), \quad x^2+(mx+5)^2-9=0$

$x^2+m^2x^2+10mx+25-9=0 \quad \Rightarrow (1+m^2)x^2+10mx+16=0$

$\Delta=0,$

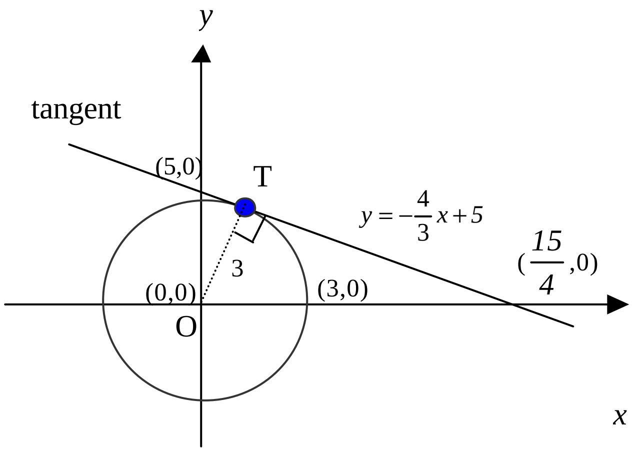
$\Delta=(10mx)^2-4\cdot(1+m^2)\cdot 16=0$

$100m^2x^2-64-64m^2=36m^2-64=0$

$m^2=\dfrac{64}{36}=\dfrac{16}{9} \quad \Rightarrow m=\pm\sqrt{\dfrac{16}{9}}=\pm\dfrac{4}{3}$

Read from the figure, the slope of the line is negative.

$\therefore \quad m=-\dfrac{4}{3}$



(c) *The equation of the tangent :* $\quad y=-\dfrac{4}{3}x+5$

x*-intercept : when* $y=0\ ,0=-\dfrac{4}{3}x+5 \Rightarrow x=\dfrac{15}{4}$

y*-intercept : when* $x=0\ ,y=-\dfrac{4}{3}(0)+5 \Rightarrow y=5$

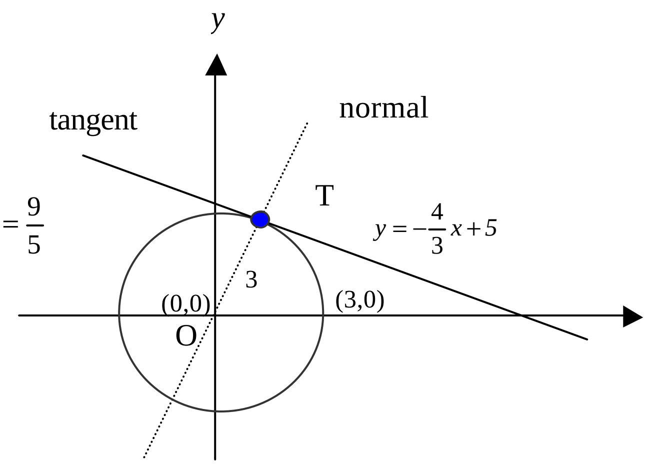
The area of the triangle $=\dfrac{1}{2}\cdot\left(\dfrac{15}{4}\cdot 5\right)=\dfrac{75}{8}unit^2$

(d) *Let* m_1 *be the slope of the normal*

$m_1\cdot -\dfrac{4}{3}=-1 \quad \Rightarrow m_1=\dfrac{3}{4}$

$\dfrac{y-0}{x-0}=\dfrac{3}{4} \quad \Rightarrow y=\dfrac{3}{4}x$

$(e) \quad y=\dfrac{3}{4}x\cdots\cdots(3)$



$Sub\ (3)\ to\ (2), \quad \dfrac{3}{4}x=-\dfrac{4}{3}x+5$

$\dfrac{3}{4}x+\dfrac{4}{3}x=5 \quad \Rightarrow \dfrac{9+16}{12}x=5 \quad \Rightarrow x=\dfrac{12}{5},\ y=\dfrac{3}{4}\cdot\dfrac{12}{5}=\dfrac{9}{5}$

$\therefore$ *The coordinate of the point T is* $\left(\dfrac{12}{5},\dfrac{9}{5}\right)$

C 5.

There are two groups of fish in the pond. The mean measures the average of two groups instead of central tendency of each group.

(a) 32, 33,40,47,47,51,53,54,58,95

$$mean = \frac{32+33+40+47+47+51+53+54+58+95}{10} = 51$$

32, 33,40,47,47,↕51,53,54,58,95

$$median = \frac{47+51}{2} = 49$$

$$mode = 47$$

(*b*) *There are two groups of fish in the pond. The mean measures the average of two groups instead of central tende*

(c) $x = \frac{1}{2}(median\ of\ Group\ A + median\ of\ Group\ B)$

Group A 51 53 54 58 95 $median = 54$

Group B 32 33 40 47 47 $median = 40$

$$\therefore x = \frac{1}{2}(54+40) = \frac{94}{2} = 47$$

(d)

mark x	$(x-\bar{x})$	$(x-\bar{x})^2$		
32	-19	361		
33	-18	324		
40	-11	121		
47	-4	16		
47	-4	16		
51	0	0		
53	2	4		
54	3	9		
58	7	49		
95	44	1936		
mean 51	sum	2836	2836/9=	315.1111
		standard derivation		17.8