Suggested answers for FDC revision

1 May 2003

CMV6111 FD/FC Foundation Mathematics Revision

Section A Multiple Choice questions (20 marks)

Answer All questions. Each question carries 2 marks.

A 1

If $x = \dfrac{1+2b}{1-2b}$, then b =

$x(1-2b) = 1+2b$

$x - 2bx = 1+2b$

$2b(1+x) = x-1$

$\therefore \quad b = \dfrac{x-1}{2(1+x)} = \dfrac{1}{2}\left(\dfrac{x-1}{x+1}\right)$

OR

When $x = 2$,

$2 = \dfrac{1+2b}{1-2b}$

$2 - 4b = 1+2b$

$6b = 1$

$\therefore b = \dfrac{1}{6}$

Test all the answers from A to D, put x =2

A. $\dfrac{1+2x}{1-2x}$

$b = \dfrac{1+2(2)}{1-2(2)} = \dfrac{5}{-3}$

B. $\dfrac{1}{2}\left(\dfrac{1+x}{1-x}\right)$

$b = \dfrac{1}{2}\left(\dfrac{1+2}{1-2}\right) = \dfrac{3}{-2}$

C. $\frac{1}{2}\left(\frac{x-1}{x+1}\right)$

$b=\frac{1}{2}\left(\frac{2-1}{2+1}\right)=\frac{1}{6}$ √

D. $\frac{x-1}{2}$

$b=\frac{2-1}{2}=\frac{1}{2}$

$\frac{1}{2}\left(\frac{x-1}{x+1}\right)$

$\frac{x-1}{2}$

A2.

The equation $2x^2 + x - 1 = 0$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = \frac{1}{2}\ or\ -1$ Ans: A

2 -1
1 +1

OR

$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-1)}}{2(2)}=\frac{-1\pm\sqrt{9}}{4}$

$=\frac{-1\pm3}{4}=\frac{1}{2}\ or\ -1$

A3

$\frac{2x+1}{-3}>4$

$2x+1<4(-3)$

$2x+1<-12$

$x<-\frac{13}{2}$

$x<-6.5$

Ans: A

A4.

$x^2 - xy + x - y = x(x-y) + (x-y) = (x-y)(x+1) = (x+1)(x-y)$

Ans: D

OR

Put $x = 3$, $y = 2$

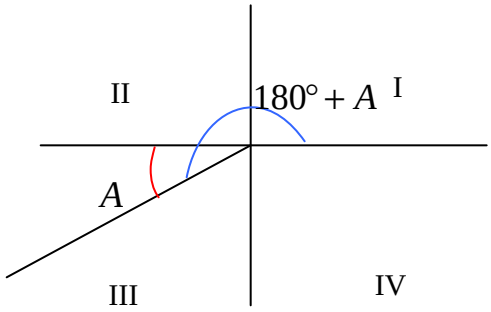
$x^2 - xy + x - y = 3^2 - 3(2) + 3 - 2 = 9 - 6 + 1 = 4$

A. (x + 1)(x + y) = (3 + 1)(3 + 2) = 20
B. (x - 1)(x - y) = (3 - 1)(3 - 2) = 2
C. (x - 1)(x + y) = (3 - 1)(3 + 2) = 10
D. (x + 1)(x - y) = (3 + 1)(3 - 2) = 4 √

A5.

$\sin(180° + A) = -\sin A$

Ans: C



OR

Put A = 30°

$\sin(180° + A) =$

$\sin(180^\circ + 30^\circ) = \sin(210^\circ) = -\frac{1}{2}$

A. $\cos A = \cos 30^\circ = \frac{\sqrt{3}}{2}$

B. $\sin A = \sin 30^\circ = \frac{1}{2}$

C. $-\sin A = -\sin 30^\circ = -\frac{1}{2}$ √

D. $-\cos A = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

A6.
An arithmetic sequence (A.P.) (n + 3)
When n = 10 ,

General term $T(n) = (n+3) = (10+3) = 13$

Ans: D

A7.
Two fair dice are thrown. The probability that the outcome is an **even number** on both dice
Outcome of even number = {2,4,6}

P(an even number on both dice) = $\frac{3}{6} \times \frac{3}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Ans: B

A8.
The line 3y = x + 4 passes through the point
Put *x* and *y* to the equation,

A. (-1,1) $3(1) = (-1) + 4 = 3$ √

B. (1,-1) $3(-1) \neq (1) + 4 = 5$

C. (1,1) $3(1) \neq (1) + 4 = 5$

D. (-1,-1) $3(-1) \neq (-1) + 4 = 3$

A9. Which component comprises 25% of the total?

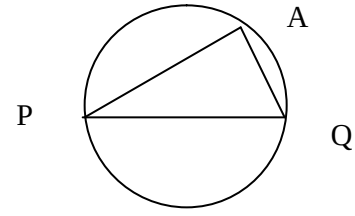
25% = $\frac{1}{4} = \frac{90°}{360°}$

Ans: A

A10.
PQ is a diameter of the circle. A is a point on the circumference. ∠APQ would be

$\angle PAQ = 90°$
therefore, ∠APQ
must be less than 90°

Ans: A



Section B Short Questions (40 marks)

Answer ALL questions in this section. Each question carries 5 marks.

B1

$2^x = 3$

$x\log 2 = \log 3$

$x = \dfrac{\log 3}{\log 2} = 1.585$

B2

$3x^2 + x + 1 > 3$

$3x^2 + x - 2 > 0$

$(3x - 2)(x + 1) > 0$

$\therefore \quad x < -1 \text{ or } x > \dfrac{2}{3}$

3 -2
1 +1

B3

$2\sin^2 x + 3\sin x - 2 = 0 \quad$ for $0^o < x < 360^o$.

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \dfrac{1}{2}$ *or* -2 (*rejected*) $\quad \because -1 \le \sin x \le 1$

$\therefore \quad x = 30° \text{ or } 150°$

2 -1
1 +2

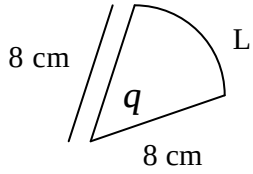
B4

A wire 20 cm long is bent into the form of a sector of radius 8 cm.

a) Find the arc length of the sector.

Let L be the arc length of the sector

L = 20 – 8 – 8 = 4 cm



b) Find the sector angle in radians.

$L = rq$

$4 = 8q$

$q = \frac{4}{8} = \frac{1}{2} rad$

c) Calculate the area of the sector.

$A = \frac{1}{2} r^2 q = \frac{1}{2} rL = \frac{1}{2} \cdot 8 \cdot 4 = 16 cm^2$

B5

In the figure below, ABC is a triangle such that AB = 20m, **AC** = 30 m and $\angle BAC = 45^o$. Find the length BC.

[Answer correct to 1 decimal place]

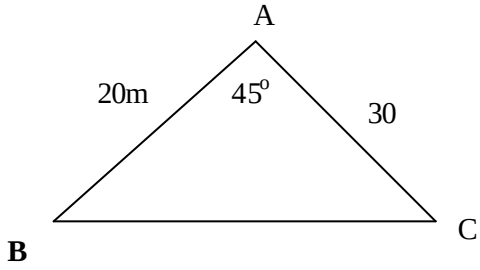
Using cosine formula,

$BC^2 = 20^2 + 30^2 - 2(20)(30)\cos 45°$

$= 400 + 900 - 848.53 = 451.47$

$\therefore \quad BC = \sqrt{451.47} = 21.248 \approx 21.2m$

(correct to 1 decimal place)



B6

The price of tea X is \$240 per kg and the price of tea Y is \$300 per kg. Tea Z is a mixture of X and Y by **weight** in the ratio **2 to 1**.

a) What is the price of 1 kg of tea Z?
tea X : tea Y = 2 : 1
For 1 kg of tea Z, the price should be

$$(1\times\frac{2}{2+1})\times\$240+(1\times\frac{1}{2+1})\times\$300$$
$$=\frac{2}{3}\times\$240+\frac{1}{3}\times\$300$$
$$=\$260$$

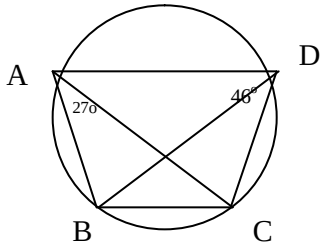
b) A new type of tea is made by adding 1 kg of tea X is added to 3 kg of tea Z. What is the price of 1 kg of the new type of tea?
Cost of 4 kg of the new type of tea

$$=1\times\$240+3\times\$260$$
$$=\$1020$$

The price of 1 kg of the new type of tea $=\$\frac{1020}{4}=\$255$

B7

ABCD are points on the circumference of a circle. Given $\angle BAC=27^o$ and $\angle ADB=46^o$. Find (a) $\angle BDC$ and (b) $\angle ABC$.

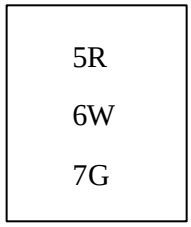


a) $\angle BDC=\angle BAC=27^o$

b) $\angle ABC = 180^o - \angle ADB - \angle BDC$
$= 180^o - 46^o - 27^o = 107^o$

B8

There are 5 red balls, 6 white balls and 7 green balls in a box.
A ball is randomly drawn from the box. a) Find the probability that the ball is red
.
b) What is the probability that at least 1 red ball is drawn in 2 successive random selections assume no replacement is allowed?



a) P(the ball is red) $= \frac{5}{5+6+7} = \frac{5}{18}$

b) P(at least 1 red ball is drawn in 2 selections) = 1- P(no red ball is drawn)

$= 1 - \frac{13}{18} \times \frac{12}{17} = \frac{25}{51} = 0.49$

OR

P(at least 1 red ball is drawn in 2 selections) $= \frac{5}{18} \times \frac{4}{17} + \frac{13}{18} \times \frac{5}{17} + \frac{5}{18} \times \frac{4}{17} = \frac{25}{51} = 0.49$

Pick 1 red ball | Pick 1 red ball | Pick 2 red balls

Section C Structure Questions (40 marks)

Answer ANY FOUR questions in this section. Each question carries 10 marks.

C1 A function is given by $y = 2x^3 - 9x^2 + 12x - 3$.

(a) Copy and complete the table below in your answer book. (4 marks)

x	-1	0	0.5	1	1.5	2	2.5	3
y								

(b) Plot a graph of y aginst x for $-1 \le x \le 3$. (3 marks)

(c) Solve the equation $2x^3 - 9x^2 + 12x - 5 = 0$. (3 marks)
[Answer correct to 2 decimal places]

a) A function is given by $y = 2x^3 - 9x^2 + 12x - 3$.
Copy and complete the table below in your answer book. (4 marks)

x	-1	0	0.5	1	1.5	2	2.5	3
y	**-26**	**-3**	**1**	**2**	**1.5**	**1**	**2**	**6**

Program can be set to calculate the required y values.

MODE EXP P1 ENT 1 Min x^y $3 \times 2 - 9 \times$ MR x^2 $+12 \times$ MR $- 3 =$ MODE .

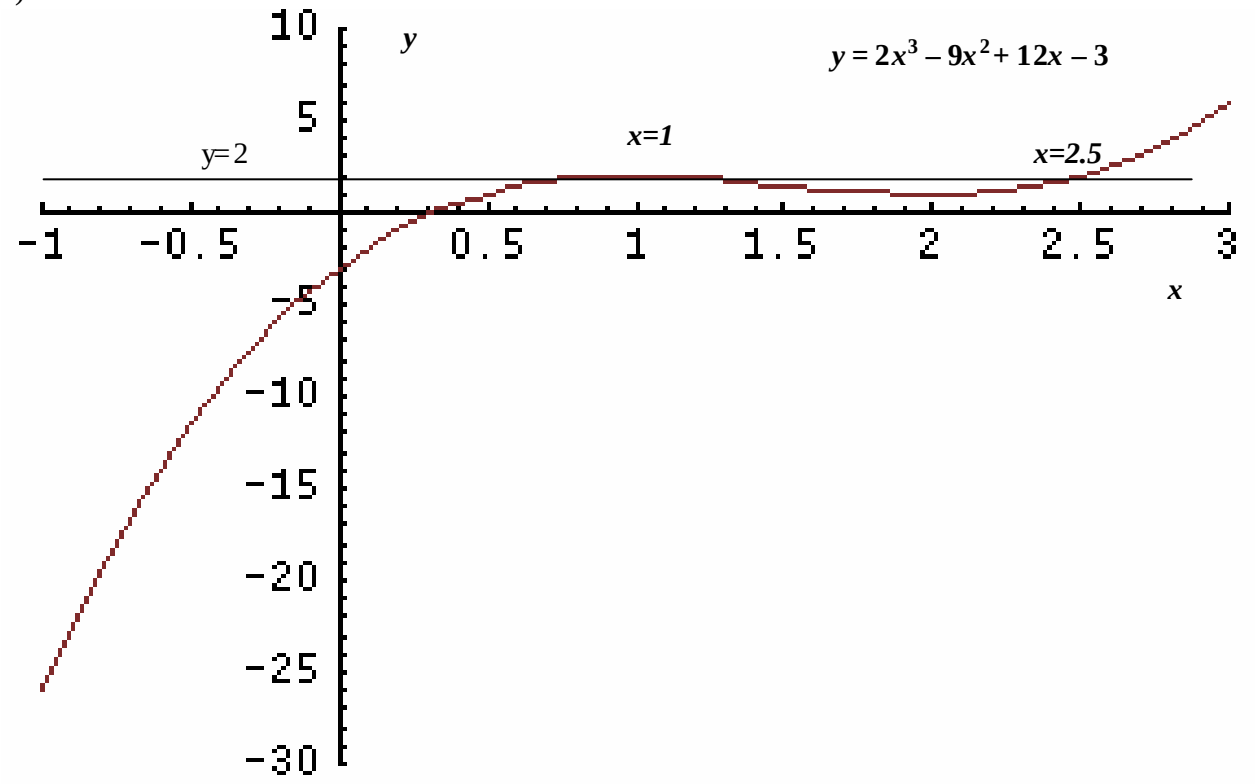
The output should be 2. (Check when $x = 1$, $y = 2$)

Execute the program by pressing **P1 -1 ENT**, the output should be -26.

When $x = 3$, $y = 6$
Any calculator can output the result by pressing

3 Min x^y $3 \times 2 - 9 \times$ MR x^2 $+12 \times$ MR $- 3 =$

b)



c)

Solve the equation $2x^3 - 9x^2 + 12x - 5 = 0$. (3 marks)
[Answer correct to 2 decimal places]

$2x^3 - 9x^2 + 12x - 5 = 0$

$2x^3 - 9x^2 + 12x - 5 + 2 = 0 + 2 = y$

$2x^3 - 9x^2 + 12x - 3 = y = 2$

Therefore plot a horizontal line $y=2$ on the graph,
x = 1.00 or 2.50

C2 The consumption of fuel per km of a hover boat is **<u>partly constant</u>** and **<u>partly varies as the square of the speed</u>** of the boat. When the speed is 40 km per hour, the fuel consumption is 9.2 litre per km. When the speed is 50 km per hour, the fuel consumption is 13.7 litre per km.

(a) Find an **equation** connecting the fuel consumption and the speed of the boat. (6 marks)

(b) If the fuel consumption is 6.32 litre per km, calculate the speed of the boat. (2 marks)

(c) What is the fuel consumption in litre per km when the speed of the boat is 60 km per hour ? (2 marks)

a) Let c be the consumption of fuel per km, s be the speed of km per hour and k_1, k_2 be two constants.

The equation connecting the fuel consumption and the speed of the boat. is :

$c = k_1 + k_2 s^2$

When s = 40 , c = 9.2

Therefore , $9.2 = k_1 + k_2 (40)^2$

$9.2 = k_1 + 1600\, k_2 \cdots\cdots (1)$

When s = 50 , c = 13.7

Therefore , $13.7 = k_1 + k_2 (50)^2$

$13.7 = k_1 + 2500\, k_2 \cdots\cdots (2)$

(2) – (1),

$\therefore 900\, k_2 = 4.5$

$k_2 = 0.005 = 5 \times 10^{-3}$

$13.7 = k_1 + 2500\, k_2$

$-\quad (9.2 = k_1 + 1600\, k_2)$

……

$4.5 = 900\, k_2$

Sub k_2 to (1),

$9.2 = k_1 + 1600(0.005)$

$\therefore k_1 = 1.2$

Therefore the equation is $c = 1.2 + 0.005 s^2$

b) Putting, c=6.32 to the equation,

$6.32 = 1.2 + 0.005 s^2$

$s^2 = \dfrac{6.32 - 1.2}{0.005} = 1024$

$s = \sqrt{1024} = 32$

The speed of the boat is 32 km per hour.

c) Putting, s=60 to the equation,

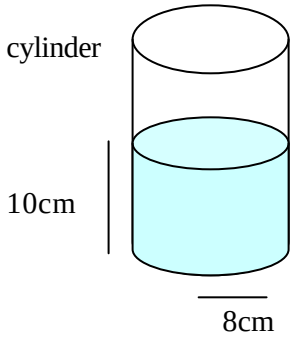
$c = 1.2 + 0.005(60)^2$

$c = 19.2$

The fuel consumption is 19.2 litre per km

C3

A cylinder has a radius 8 cm is filled with water up to a level of 10 cm.



a) Calculate the cross-sectional area of the cylinder. (3 marks)

A sphere of radius 4 cm is dropped into the cylinder and sinks to the bottom.

b) Find the volume of the sphere. (3 marks)

c) Find the rise in water level in the cylinder. (4 marks)

[All answers correct to 1 decimal place]

a) The cross-sectional area of the cylinder

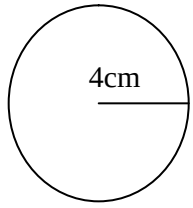
$A = r^2 p = (8)^2 p = 64p = 210.1 cm^2$

(correct to 1 decimal place)

b) The volume of the sphere

$V = \frac{4}{3} r^3 p = \frac{4}{3}(4)^3 p = 268.1 cm^3$

(correct to 1 decimal place)



c) The volume of the sphere equals the increase of water volume in the cylinder

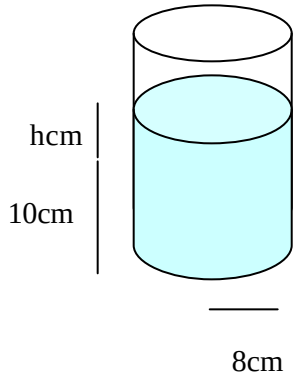
$268.1 = h.A$

$A = 201.1$ from part a)

$h(201.1) = 268.1$

$h = \frac{268.1}{201.1} = 1.3 cm$

(correct to 1 decimal place)



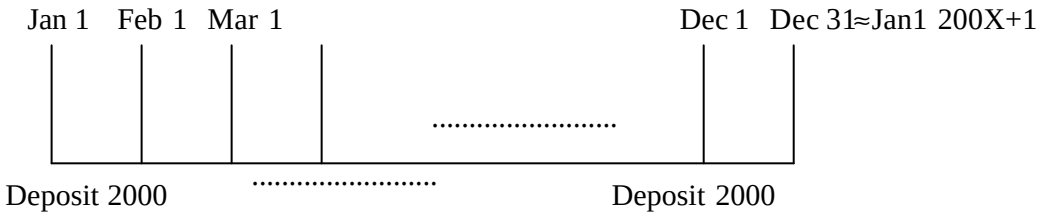
C4

Alex deposits $2000 in a bank on 1 Jan 200X. The investment is compounded monthly at a rate 1.2% per year.

a) Calculate the interest earned on 31 Dec 200X. (4 marks)

b) Alex agrees with the bank to deposit $2000 on the first day of each month for the whole year of 200X. Calculate the total interest earned on 31 Dec 200X.
(6 marks)

a)



Alex had earned 12 months of interest since 1 Jan 200X.
The principle with interest on 31 Dec 200X become

$$= 2000(1+\frac{1.2\%}{12})^{12} = 2000(1+\frac{0.012}{12})^{12} = \$2024.13$$

The interest earned on 31 Dec 200X $= 2024.13 - 2000 = \$24.13$

b)

Month	Jan 1	Feb 1	Mar 1	Apr 1		Dec 1
No. of months passed	12	11	10	9		1
The principle with interest	$2000(1+\frac{0.012}{12})^{12}$	$2000(1+\frac{0.012}{12})^{11}$				$2000(1+\frac{0.012}{12})^{1}$

The sum of principle with interest started from Jan1 200X to 31 Dec 200X become

$$2000(1+\frac{0.012}{12})^{12} + 2000(1+\frac{0.012}{12})^{11} + 2000(1+\frac{0.012}{12})^{10} + \cdots\cdots + 2000(1+\frac{0.012}{12})^{2} + 2000(1+\frac{0.012}{12})^{1}$$

$$= 2000(1+\frac{0.012}{12})^{1} + 2000(1+\frac{0.012}{12})^{2} + \cdots\cdots + 2000(1+\frac{0.012}{12})^{11} + 2000(1+\frac{0.012}{12})^{12}$$

It is a G.P.,

$$a = 2000(1+\frac{0.012}{12})^1, R = \frac{2000(1+\frac{0.012}{12})^2}{2000(1+\frac{0.012}{12})^1} = (1+\frac{0.012}{12}), n = 12$$

$$S(n) = \frac{a(R^n - 1)}{R - 1}$$

$$S(12) = \frac{2000(1+\frac{0.012}{12})[(1+\frac{0.012}{12})^{12} - 1]}{(1+\frac{0.012}{12}) - 1} = \frac{2002(1.001^{12} - 1)}{1.001 - 1} = \$24156.57$$

Alex had deposited $2000 \times 12 = \$24000$

The total interest earned on 31 Dec 200X.

$= 24156.57 - 24000 = \$156.57$

C5

A(3,4), B(0,1) and C(4,1) form a triangle on the x – y plane.

a) If D is the mid-point of BC, find the coordinates of D. (2 marks)

b) Find the distance between A and B. (3 marks)

c) Find the perpendicular distance between A and the line BC. (1 mark)
[Hint: sketch the points A, B and C.]

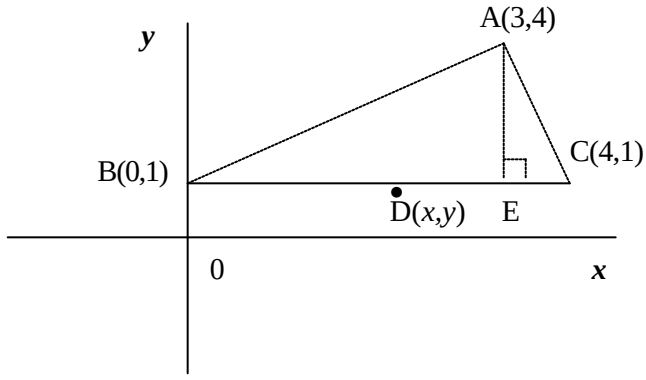
d) Find the equation of the line passing through the points A and B.
(3 marks)

a)

$$x = \frac{0+4}{2} = 2$$

$$y = \frac{1+1}{2} = 1$$

The coordinates of D is (2,1)



b) The distance between A and B

$$= \sqrt{(3-0)^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} = \sqrt{9 \cdot 2}$$

$$= \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}\ units$$

c) The perpendicular distance between A and the line BC is AE

$$= 4 - 3 = 1\ unit$$

d) The equation of the line passing through the points A and B is

$$\frac{y-1}{x-0} = \frac{4-1}{3-0} = \frac{3}{3} = 1$$

$$\therefore \frac{y-1}{x} = 1$$

$$y - 1 = x$$

$$y = x + 1$$

C6

The frequency distribution of the lengths of 100 butterflies (to the nearest mm) is as follows:

length/mm	Frequency
15 to 19	15
20 to 24	25
25 to 29	35
30 to 34	15
35 to 39	10

a) What is the probability that a butterfly randomly chosen from the above sample has a length between 14.5 mm and 19.5 mm? (3 marks)

b) Complete the table below and construct a cumulative frequency polygon for the distribution. (6 marks)

c) Read from your cumulative frequency polygon the median of the distribution. (1 mark)

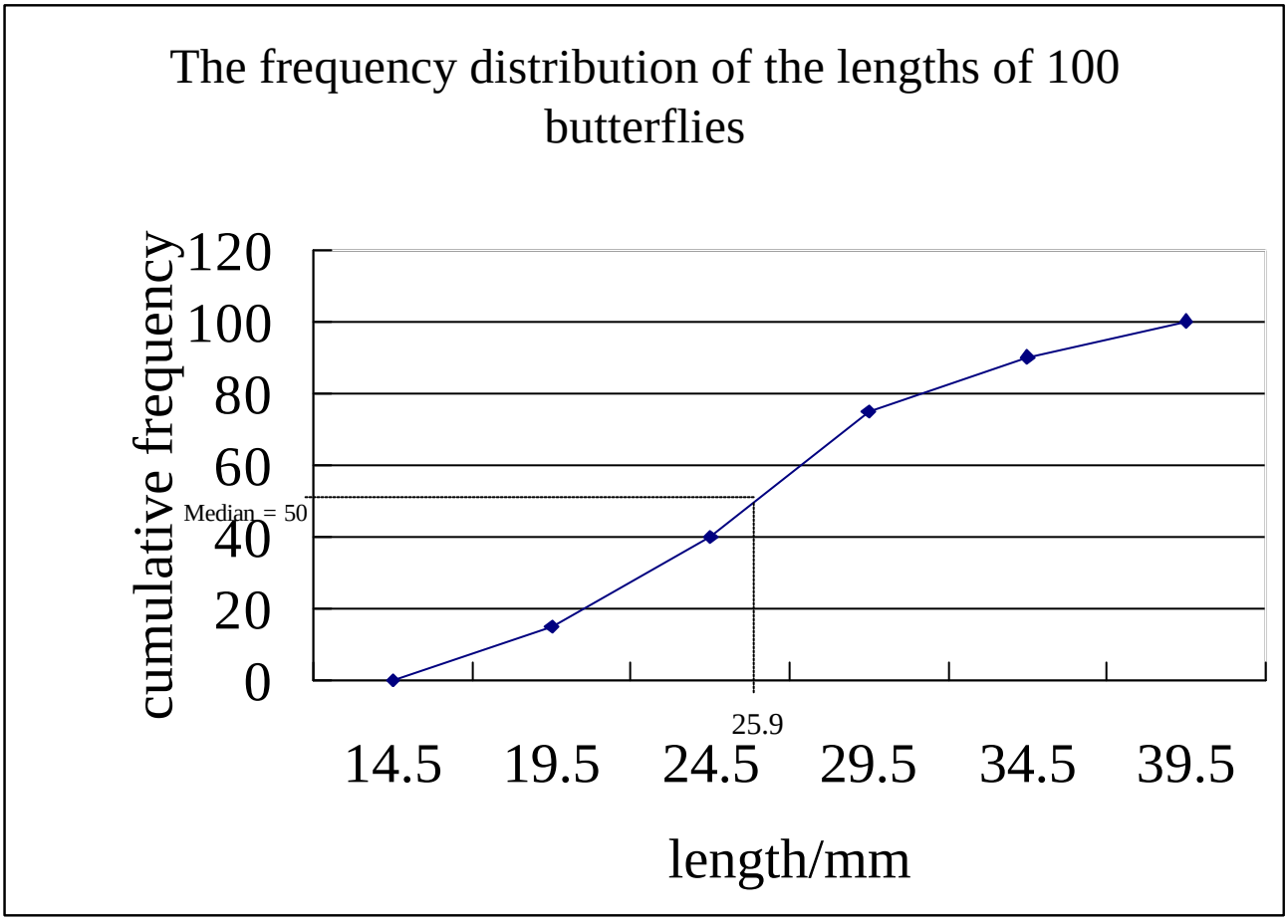
a) If a sample has length between 14.5 mm and 19.5 mm, class 15 – 19 will be chosen.

P(class 15 – 19 will be chosen) $= \frac{15}{100} = \frac{3}{20} = 0.15$

b)

Length up to /mm	cumulative frequency
14.5	**0**
19.5	**15**
24.5	**40**
29.5	**75**
34.5	**90**
39.5	**100**

b)



c) The median of the distribution = 25.9*mm*