## Tutorial 20 : Applications of standard deviation

## Problems

1. The Hong Kong unemployment rate in the year of $4 / 2001-5 / 2002$ was as following:

| $4 / 2001$ | 4.5 |
| :--- | :--- |
| $5 / 2001$ | 4.5 |
| $6 / 2001$ | 4.5 |
| $7 / 2001$ | 4.7 |
| $8 / 2001$ | 4.9 |
| $9 / 2001$ | 5.3 |
| $10 / 2001$ | 5.5 |
| $11 / 2001$ | 5.8 |
| $12 / 2001$ | 6.1 |
| $1 / 2002$ | 6.7 |
| $2 / 2002$ | 6.8 |
| $3 / 2002$ | 7.0 |
| $4 / 2002$ | 7.1 |
| $5 / 2002$ | 7.4 |

Calculate the standard deviation of unemployment rate:
a) For $4 / 2001-12 / 2001$
b) For $1 / 2002-5 / 2002$
c) For all 14 months.
2. The following table shows the distribution of heights of 50 students:

| Height (cm) | Frequency |
| :---: | :---: |
| $160-164$ | 8 |
| $165-169$ | 12 |
| $170-174$ | 14 |
| $175-179$ | 7 |
| $180-184$ | 6 |
| $185-189$ | 3 |

Find the range and standard deviation of heights.
3. Find the mean and standard deviation of the 5 numbers in term of x :
$x-5, x-3, x-2, x+1, x+4$.
4. The mean of the five numbers $6,9,2, x, y$ is 5 and the standard deviation is $\sqrt{6}$. Find the values of $x$ and $y$.
5. Find Peter's standard scores in English and Chinese

| Subject | Peter's mark | class mean | standard deviation |
| :--- | :--- | :--- | :--- |
| English | 70 | 55 | 10 |
| Chinese | 66 | 50 | 8 |

6. Keeping the standard score unchanged, find the adjusted marked for the original mark 55.

|  | mean | standard deviation |
| :--- | :--- | :--- |
| original | 40 | 10 |
| adjusted | 50 | 12 |

7. Keeping the standard score unchanged, find the mean $x$ of the adjusted marks.

|  | Peter's mark | mean | standard deviation |
| :--- | :--- | :--- | :--- |
| original | 38 | 42 | 8 |
| adjusted | 44 | $x$ | 12 |

8. Find the mean $m$ and standard deviation $\sigma$ of marks. It is given that:

| raw mark | standard score |
| :--- | :--- |
| 72 | -0.6 |
| 90 | 1.2 |

9. In a Mathematics examination, the marks obtained by 15000 students are normally distributed with a mean of 52 and a standard deviation of 16 . The percentages of marks lying within 1 and 2 standard deviations from the mean are $68 \%$ and $96 \%$ respectively.
(a) Find the number of students who score less than 68.
(b) If the top $2 \%$ of students are awarded a distinction, what is the minimum mark a student must get in order to receive a distinction?
10. The weights of 1000 students are normally distributed with mean 68 kg and standard deviation 3 kg . If $68 \%$ of the students lie within one standard deviation of the mean and $96 \%$ lie within 2 standard deviations of the mean, find
(a) number of students who are heavier than 74 kg .
(b) number of students whose weight lie between 62 kg and 71 kg .

## Activity

Below is an extract from the web site : http://www.mste.uiuc.edu/hill/dstat/dstat.html

Statistics are all around us. In fact it would be difficult to go through a full week without using statistics.

Imagine watching a football game where no one kept score. The action itself might provide enough excitement to hold your attention for a while, but think of all the drama that would be lost if winning and losing weren't at issue.

Imagine going to the grocery store and trying to find the best buy on a box of doggie treats for your dog, Fluffy. Without statistics this task would come down to simple guess work. You could never know for sure if that worthless mutt were getting the best (cheapest) treats for your dollar.

Without statistics we couldn't plan our budgets, pay our taxes, enjoy games to their fullest, evaluate classroom performance... Are you beginning to get the picture? We need statistics.

Let's take a look at the most basic form of statistics, known as descriptive statistics. This branch of statistics lays the foundation for all statistical knowledge (pretty important, huh?), but it is not something that you should learn in order that you can use it in the distant future. Descriptive statistics can be used NOW, in English class, in physics class, in history, at the football stadium, in the grocery store. You probably already know more about these statistics than you think.

If you like, you may skip to one of the following topics:
Mode
Median
Mean
Central Tendency
Variation
Range
Variance
Standard Deviation
Log on to the web site and work on each of the above topics.

## Solution to Tutorial 20

1. a) s.d. $=0.61$
b) s.d. $=0.27$
c) s.d. $=1.07$
2. range $=189.5-159.5=30$
class marks $162,167, \ldots, 187$
From calculator, standard deviation $=7.14$
3. mean $=x+(-5-3-2+1+4) / 5$

$$
=x-1
$$

$$
\begin{aligned}
\text { s.d. } & =\sqrt{\frac{(-4)^{2}+(-2)^{2}+(-1)^{2}+(2)^{2}+(5)^{2}}{5}} \\
& =3.16
\end{aligned}
$$

4. $6+9+2+x+y=5(5)$

$$
\begin{equation*}
\text { So, } y=8-x \text {. } \tag{1}
\end{equation*}
$$

$$
\sqrt{\frac{(1)^{2}+(4)^{2}+(-3)^{2}+(x-5)^{2}+(y-5)^{2}}{5}}=\sqrt{6}
$$

$$
\begin{aligned}
& 26+(x-5)^{2}+(y-5)^{2}=5(6) \\
& \text { So, } x^{2}+y^{2}=34 \quad-----(2)
\end{aligned}
$$

Put (1) into (2)

$$
\begin{aligned}
& x^{2}+(8-x)^{2}=34 \\
& x^{2}-8 x+15=0 \\
& (x-3)(x-5)=0
\end{aligned}
$$

$$
x=3 \text { or } 5
$$

$$
\text { Answer: } x=3, y=5 \text { or } x=5, y=3 \text {. }
$$

5. Standard score for English

$$
\begin{aligned}
& =(70-55) / 10 \\
& =1.5
\end{aligned}
$$

Standard score for Chinese

$$
\begin{aligned}
& =(66-50) / 8 \\
& =2
\end{aligned}
$$

6. Let $x$ be the adjusted mark

$$
\begin{aligned}
& (x-50) / 12=(55-40) / 10 \\
& (x-50)=18 \\
& x=68
\end{aligned}
$$

7. Let $x$ be the mean of the adjusted marks,

$$
\begin{aligned}
& (44-x) / 12=(38-42) / 8 \\
& 44-x=-6 \\
& x=50 .
\end{aligned}
$$

8. $(72-m) / s=-0.6$
$(90-m) / \mathrm{s}=1.2$
$m-0.6 s=72$
$m+1.2 s=90$
$1.8 s=18$
$s=10$
$\mathrm{m}=72+0.6(10)=78$
9. (a) standard score $=(68-52) / 16=1$

There are $34 \%$ of marks between standard scores 0 to 1 .
There are $50 \%$ of marks less than standard score 0 .
No. of students $=15000(34 \%+50 \%)=12600$
(b) There are $2 \%$ of marks above standard score 2 .

Let $x$ be the minimum mark to get distinction,
$(x-52) / 16=2$
$x=52+2(16)=84$
10. (a) standard score $=(74-68) / 3=2$

There are $2 \%$ of weights more than standard score 2 .
No. of students $=1000(2 \%)=20$.
(b) standard score $=(62-68) / 3=-2$
standard score $=(71-68) / 3=1$
There are $48 \%$ of weights between standard scores -2 to 0 .
There are $34 \%$ of weights between standard scores 0 to 1 .
No. of students $=1000(48 \%+34 \%)=820$.
Activity
Please refer to the web site http://www.mste.uiuc.edu/hill/dstat/dstat.html

