HONG KONG INSTITUTE OF VOCATIONAL EDUCATION
FOUNDATION COURSES EXAMINATION

Course : Foundation Certificate/BCTT **Course Code** : **03601T/1, 05601T/1, 05601B/1, 03602**

Stream : Business/Technical

Module Title : Mathematics

Module Code : CMV6103

Session : 2001/2002 **Date** : 30 May 2002

Time Allowed : 2 hours **Time** : 7:00 to 9:00 pm

This examination paper has 7 pages (excluding this covering page)

Instructions to Candidates

(a) Answer **ALL** questions in section A (10 multiple choice questions). Each question carries 2 marks. (20 marks)

(b) Answer **ALL** questions in section B (7 short questions). Each question carries 5 marks. (35 marks)

(c) Answer **ANY 3** questions out of 5 in section C (long questions). Each question carries 15 marks. (45 marks)

(d) Multiple choice **answer sheet** is provided for section A. Answers of section B and C must be written in the answer book provided.

(e) Approved calculators may be used.

Section B Short Questions (35 marks)

Answer ALL questions in this section. Each question carries 5 marks.

B1. Solve the equation $2\sin^2 x + 3\sin x - 2 = 0$ for $0° < x < 360°$

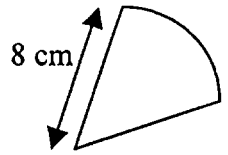
B2. Solve the inequality $3x^2 + x + 1 > 3$.

B3. A wire 20 cm long is bent into the form of a sector of radius 8 cm.

(a) Find the arc length of the sector.

(b) Find the sector angle in radians.

(c) Calculate the area of the sector.

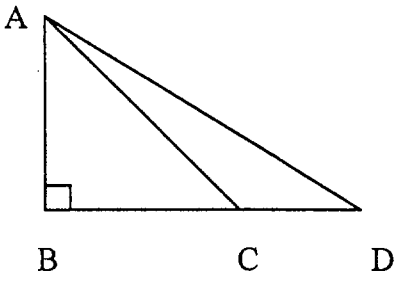


B4. In the figure below, AB represents a light house of height 25 m. C and D are 2 boats with angles of elevation $40°$ and $30°$ respectively.

(a) Find the distance BC.

(b) Find the distance between the 2 boats.

[All answers correct to 1 decimal place]

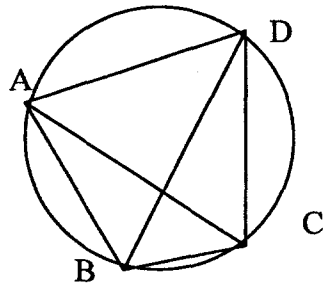


B5. The price of tea X is \$240 per kg and the price of tea Y is \$300 per kg. Tea A is a mixture of X and Y by weight in the ratio 2 to 1.

(a) What is the price of 1 kg of tea A?

(b) A new type of tea is made by mixing tea X to tea A in the ratio 1 to 3 by weight. What is the price of 1 kg of the new type of tea?

B6. ABCD are points on the circumference of a circle. Given $\angle BAC=27^\circ$ and $\angle ADB=46^\circ$. Find (a) $\angle BDC$ and (b) $\angle ABC$.



B7. There are 5 red balls, 6 white balls and 7 green balls in a box.

(a) A ball is randomly drawn from the box. Find the probability that the ball is red.

(b) What is the probability that at least 1 red ball is drawn in 2 successive random selections? No replacement is allowed.

Section C Conventional Questions (45 marks)

Answer ANY THREE questions in this section. Each question carries 15 marks.

C1. The roots of the equation $x^2 + (2p+1)x + p+3 = 0$ are α and β.

(a) Write down the value of $\alpha + \beta$ in terms of p. (1 marks)

(b) Write down the value of $\alpha\beta$ in terms of p. (1 marks)

(c) Write down the value of $\alpha^2 + \beta^2$ in terms of p. (3 marks)

(d) Find the values of p if the equation has identical roots. (4 marks)

(e) Find the range of values of p if the roots of the equation are real. (3 marks)

(f) Solve the equation when p = -3. (3 marks)

C2. The cost of a sushi lunch box is partly constant and partly inversely proportional to the number of sushi lunch boxes ordered. It is known that the unit cost of 1000 sushi lunch boxes is $33 and the unit cost of producing 4000 sushi lunch boxes is $27.

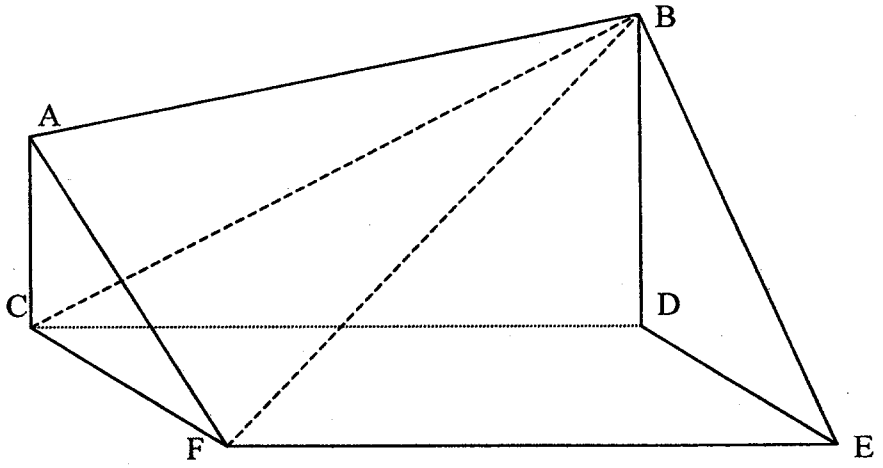
(a) Find the relation between the unit cost and the number of sushi lunch boxes ordered. (7 marks)

(b) Calculate the cost of a sushi lunch box when 500 sushi lunch box are ordered. (3 marks)

(c) Find the number of sushi lunch boxes ordered when the unit cost is $30. (3 marks)

(d) It is known that the formula found in (a) is only valid when the number of lunch boxes does not exceed 8000. What is the lowest cost possible ? (2 marks)

C3. AB is a straight road constructing along a slope on a horizontal rectangular site CDEF. AB is vertically above CD. $\angle$ACF and $\angle$BDE are right angles. It is known that EF = 100 m, AC = 30 m, BD = 50 m and CF= 40 m.



(a) Calculate the length of the road AB . (2 marks)

(b) A cable car ropeway is planned to be constructed from F to B. Find the length of the ropeway. (2 marks)

(c) Find the angle of inclination of the ropeway. (4 marks)

(d) A drainage system is to be constructed along the plane CBF. Calculate $\angle$CBF and the area of the plane CBF. (7 marks)
[All answers correct to 1 decimal place]

C4. The diameter of rings of successive models are in G.P. The diameter of a ring of the first model is 150 mm and the diameter of a ring of the second model is 120 mm.

1st model 2nd model 3rd model

(a) Find the common ratio of the G.P. (2 marks)

(b) Find the **radius** of a ring of the third model. (3 marks)

(c) Find the sum of **circumference** of rings of the first 10 models to the nearest mm. (6 marks)

(d) If infinitely many models are produced, find the total sum of **circumference** of rings of all the models in terms of π. (4 marks)

C5. The frequency distribution of the weights of 100 students is as follows:

weight/kg	Frequency
35 to 39	6
40 to 44	20
45 to 49	34
50 to 54	25
55 to 59	15

(a) Draw a histogram for the distribution of weight. (3 marks)

(b) Complete the table below and construct a cumulative frequency polygon for the distribution.

(6 marks)

weight up to /kg	cumulative frequency
34.5	
39.5	
44.5	
49.5	
54.5	
59.5	

(c) Find median of the distribution from the cumulative frequency polygon.

(2 marks)

(d) Overweight students would be selected for special physical exercise program. If 30% of the students are to be selected, what is the critical weight ?

(2 marks)

(e) How many students weigh less than 42 kg? (2 marks)

*** END OF PAPER ***

CMV6103 FC Mathematics Examination 01/02

Section A Multiple Choice questions (20 marks)

Answer All questions. Each question carries 2 marks.

A1. If $x = \frac{1+2b}{1-2b}$, then b =

A. $\frac{1+2x}{1-2x}$

B. $\frac{1}{2}\left(\frac{1+x}{1-x}\right)$

C. $\frac{1}{2}\left(\frac{x-1}{x+1}\right)$

D. $\frac{x-1}{2}$

A2. The graph of $y = x^2 + x - 6$

A. cuts the x-axis at 2 different points
B. cuts the x-axis at one and only one points
C. does not cut the x-axis
D. represents a circle.

A3. When $\frac{2x+1}{-3} > 4$, then

A $x < -6.5$
B $x < -5.5$
C $x > -6.5$
D $x > -5.5$

A4. $x^2 - xy + x - y =$

A $(x + 1)(x + y)$
B $(x - 1)(x - y)$
C $(x - 1)(x + y)$
D $(x + 1)(x - y)$

A5. $\sin(180° + A) =$

A $\cos A$
B $\sin A$
C $-\sin A$

D -cos A

A6. The nth term of an arithmetic sequence is (n + 3). The sum of the first 10 terms equals

A 13
B 47
C 85
D 170

A7. The number of roots for the equation $\sin^2 x + \sin x - 2 = 0$ in the range $0 < x < 2p$

A 0
B 1
C 2
D 3

A8. Two fair dice are thrown. The probability that the outcome is an even number on both dice is

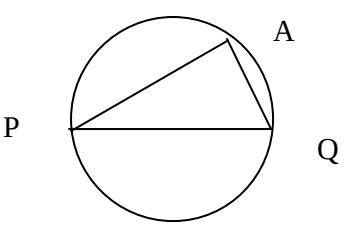
A $\frac{1}{2}$
B $\frac{1}{4}$
C $\frac{5}{36}$
D $\frac{7}{36}$

A9. Which of the following line is perpendicular to the line 3y = x + 4-y ?

A x-4y+1=0
B x+4y+1=0
C 4x-y+1=0
D 4x+y+1=0

A10. In the figure below, PQ is a diameter of the circle. A is a point on the circumference. Given PQ=10cm and $\angle APQ=20°$, then AQ

A 9.40cm
B 6.42cm
C 3.64cm
D 3.42cm



Suggested answers for FC Exam

3 May 2003

CMV6103 FC Mathematics Examination 01/02

Section A Multiple Choice questions (20 marks)

Answer All questions. Each question carries 2 marks.

A 1

If $x = \dfrac{1+2b}{1-2b}$, then b =

$x(1-2b) = 1+2b$

$x - 2bx = 1+2b$

$2b(1+x) = x-1$

$\therefore \quad b = \dfrac{x-1}{2(1+x)} = \dfrac{1}{2}\left(\dfrac{x-1}{x+1}\right)$

OR

When $x = 2$,

$2 = \dfrac{1+2b}{1-2b}$

$2 - 4b = 1+2b$

$6b = 1$

$\therefore b = \dfrac{1}{6}$

Test all the answers from A to D, put x =2

A. $\dfrac{1+2x}{1-2x}$

$b = \dfrac{1+2(2)}{1-2(2)} = \dfrac{5}{-3}$

B. $\dfrac{1}{2}\left(\dfrac{1+x}{1-x}\right)$

$b = \dfrac{1}{2}\left(\dfrac{1+2}{1-2}\right) = \dfrac{3}{-2}$

C. $\frac{1}{2}\left(\frac{x-1}{x+1}\right)$

$b=\frac{1}{2}\left(\frac{2-1}{2+1}\right)=\frac{1}{6}$ √

D. $\frac{x-1}{2}$

$b=\frac{2-1}{2}=\frac{1}{2}$

$\frac{1}{2}\left(\frac{x-1}{x+1}\right)$

A2

$y=x^2+x-6$

$\Delta=1^2-4(1)(-6)=1+24=25>0$

Therefore, there are two roots and they locate the x-axis at 2 different points

Ans: A

A3

$\frac{2x+1}{-3}>4$

$2x+1<4(-3)$

$2x+1<-12$

$x<-\frac{13}{2}$

$x<-6.5$

Ans: A

A4.

$x^2 - xy + x - y = x(x-y) + (x-y) = (x-y)(x+1) = (x+1)(x-y)$

Ans: D

OR

Put $x = 3$, $y = 2$

$x^2 - xy + x - y = 3^2 - 3(2) + 3 - 2 = 9 - 6 + 1 = 4$

A. $(x + 1)(x + y) = (3 + 1)(3 + 2) = 20$
B. $(x - 1)(x - y) = (3 - 1)(3 - 2) = 2$
C. $(x - 1)(x + y) = (3 - 1)(3 + 2) = 10$
D. $(x + 1)(x - y) = (3 + 1)(3 - 2) = 4$ √

A5

$\sin(180° + A) = -\sin A$

Ans: C

OR

Put $A = 30°$

$\sin(180° + A) =$

$\sin(180^\circ + 30^\circ) = \sin(210^\circ) = -\frac{1}{2}$

A. $\cos A = \cos 30^\circ = \frac{\sqrt{3}}{2}$
B. $\sin A = \sin 30^\circ = \frac{1}{2}$
C. $-\sin A = -\sin 30^\circ = -\frac{1}{2}$ √
D. $-\cos A = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

A6.

An arithmetic sequence (A.P.) (n + 3)

When n = 10 ,

General term $T(n) = (n+3) = (10+3) = 13$

Ans: D

A7

$\sin^2 x + \sin x - 2 = 0$ for $0 < x < 2\pi$.

$(\sin x - 1)(\sin x + 2) = 0$

$\sin x = 1 \ \ or \ -2 \ (rejected) \quad \because -1 \le \sin x \le 1$

$\therefore \quad x = 90°$

$or \quad x = \frac{90}{180} \times p$

$= \frac{p}{2}$

Ans: B

A8.

Two fair dice are thrown. The probability that the outcome is an **even number** on both dice

Outcome of even number = {2,4,6}

P(an even number on both dice) = $\frac{3}{6} \times \frac{3}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Ans: B

A9.

Which of the following line is perpendicular to the line 3y = x + 4-y ?

3y = x + 4-y

4y = x + 4

Slope $m_1 = \frac{1}{4}$

Assume the slope which is perpendicular to the line 4y = x + 4 is m_2

$m_1 \cdot m_2 = -1$

$\frac{1}{4} m_2 = -1$

$m_2 = -4$

Check the slope of the lines below:

A. x-4y+1=0 Slope $m_2 = \frac{1}{4}$

B. x+4y+1=0 Slope $m_2 = -\frac{1}{4}$

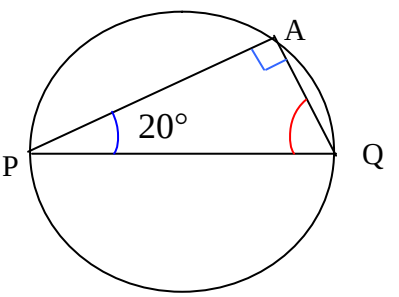
C. 4x-y+1=0 Slope $m_2 = 4$

D. 4x+y+1=0 Slope $m_2 = -4$ √

Ans: D

A10

Given PQ=10cm and ∠APQ=20°, then AQ



∠PAQ=90°

$\sin 20° = \frac{AQ}{PQ}$

$AQ = PQ \sin 20° = 10 \sin 20° = 10 \times 0.342 = 3.42\ cm$

Ans: D

Section B Short Questions

B1

$2\sin^2 x + 3\sin x - 2 = 0$ for $0° < x < 360°$.

2 -1
1 +2

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ or -2 (*rejected*) $\because -1 \le \sin x \le 1$

$\therefore \quad x = 30° \text{ or } 150°$

B2

$3x^2 + x + 1 > 3$

2 -2
1 +1

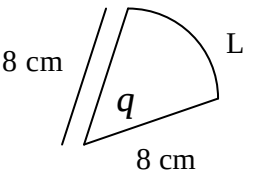
$3x^2 + x - 2 > 0$

$(3x - 2)(x + 1) > 0$

$\therefore \quad x < -1 \text{ or } x > \frac{2}{3}$

B3

A wire 20 cm long is bent into the form of a sector of radius 8 cm.



a) Find the arc length of the sector.

Let L be the arc length of the sector

L = 20 – 8 – 8 = 4 cm

b) Find the sector angle in radians.

$L = rq$

$4 = 8q$

$q = \frac{4}{8} = \frac{1}{2} rad$

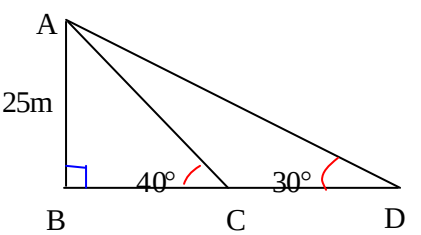
c) Calculate the area of the sector.

$A = \frac{1}{2} r^2 q = \frac{1}{2} rL = \frac{1}{2} \cdot 8 \cdot 4 = 16 cm^2$

B4

a) Find the distance BC

Consider ΔABC



$\tan 40° = \frac{25}{BC}$

$BC = \frac{25}{\tan 40°} = 29.8m$ (correct 1 decimal place)

b) Find the distance between the 2 boats

Consider ΔABD

$\tan 30° = \frac{25}{BD}$

$BD = \frac{25}{\tan 30°} = 43.3$ (correct 1 decimal place)

The distance between the 2 boats = CD = BD – BC

$= 43.3 - 29.8 = 13.5m$ (correct 1 decimal place)

B5

The price of tea X is \$240 per kg and the price of tea Y is \$300 per kg. Tea A is a mixture of X and Y by **weight** in the ratio **2 to 1**.

a) What is the price of 1 kg of tea A?
tea X : tea Y = 2 : 1
For 1 kg of tea A, the price should be

$$(1\times\frac{2}{2+1})\times\$240+(1\times\frac{1}{2+1})\times\$300$$
$$=\frac{2}{3}\times\$240+\frac{1}{3}\times\$300$$
$$=\$260$$

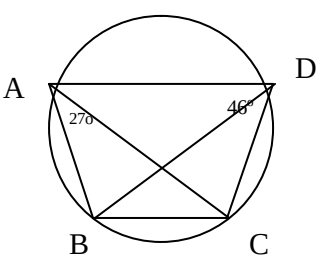
b) A new type of tea is made by mixing tea X to tea A in the ratio **1 to 3** by **weight**. What is the price of 1 kg of the new type of tea?

The price of 1 kg of the new type of tea

$$=(1\times\frac{1}{1+3})\times\$240+(1\times\frac{3}{1+3})\times\$260$$
$$=\frac{1}{4}\times\$240+\frac{3}{4}\times\$260$$
$$=\$255$$

B6

ABCD are points on the circumference of a circle. Given $\angle BAC=27^o$ and $\angle ADB=46^o$. Find (a) $\angle BDC$ and (b) $\angle ABC$.

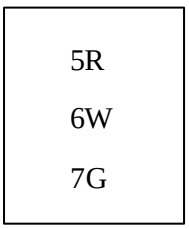


a) $\angle BDC=\angle BAC=27^o$

b) $\angle ABC = 180^o - \angle ADB - \angle BDC$
$= 180^o - 46^o - 27^o = 107^o$

B8

There are 5 red balls, 6 white balls and 7 green balls in a box.
A ball is randomly drawn from the box. a) Find the probability that the ball is red
.
b) What is the probability that at least 1 red ball is drawn in 2 successive random selections assume no replacement is allowed?



a) P(the ball is red) $= \frac{5}{5+6+7} = \frac{5}{18}$

b) P(at least 1 red ball is drawn in 2 selections) = 1- P(no red ball is drawn)
$= 1 - \frac{13}{18} \times \frac{12}{17} = \frac{25}{51} = 0.49$

OR

P(at least 1 red ball is drawn in 2 selections) $= \frac{5}{18} \times \frac{4}{17} + \frac{13}{18} \times \frac{5}{17} + \frac{5}{18} \times \frac{4}{17} = \frac{25}{51} = 0.49$

Pick 1 red ball | Pick 1 red ball | Pick 2 red balls

C2

a) Let c be the cost of a sushi lunch box, n be number of sushi lunch boxes ordered and k_1, k_2 be two constants.

The equation connecting the unit cost and the number of sushi lunch boxed ordered is:

$$c = k_1 + \frac{k_2}{n}$$

When $n = 1000$, $c = 33$

Therefore , $33 = k_1 + \frac{k_2}{1000} \cdots\cdots (1)$

When $n = 4000$, $c = 27$

Therefore , $27 = k_1 + \frac{k_2}{4000} \cdots\cdots (2)$

$(1) - (2)$,

$$\therefore k_2 (\frac{1}{1000} - \frac{1}{4000}) = 6$$

$$k_2 = \frac{6}{(\frac{1}{1000} - \frac{1}{4000})} = 8000$$

$$k_2 = 8000$$

$$33 = k_1 + \frac{k_2}{1000}$$

$$- \quad (27 = k_1 + \frac{k_2}{4000})$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$6 = k_2 (\frac{1}{1000} - \frac{1}{4000})$$

Sub k_2 to (1),

$$33 = k_1 + \frac{8000}{1000}$$

$$\therefore k_1 = 33 - 8 = 25$$

Therefore the equation is $c = 25 + \frac{8000}{n}$

b) Putting, n=500 to the equation,

$$c = 25 + \frac{8000}{500} = \$41$$

The cost is \$41

c) Putting, c=30 to the equation,

$$30 = 25 + \frac{8000}{n}$$

$$\frac{8000}{n} = 30 - 25 = 5$$

$$\therefore n = \frac{8000}{5} = 1600$$

The number of sushi lunch boxes is 1600.

d) Putting, n=8000 to the equation,

$$c = 25 + \frac{8000}{8000}$$

$$\therefore c = 25 + 1 = 26$$

The lowest cost is \$26

C4.

a) The common ratio of G.P. $= \frac{120}{150} = \frac{4}{5} = 0.8$

b) The radius of a ring of the third model $= \frac{1}{2}(\frac{4}{5} \times 120) = 48\,\text{mm}$

$$S(n) = \frac{a(1-R^n)}{1-R}$$

c) $a = 150, R = 0.8, n = 10$

$$S(10) = \frac{150(1-0.8^{10})}{1-0.8} = 669.47$$

The sum of circumference of rings of the first 10 models $= 669.47p = 2103.2 \approx 2103$ mm (correct to nearest mm)

$$S(\infty) = \frac{a}{1-R}$$

d) $a = 150, R = 0.8$

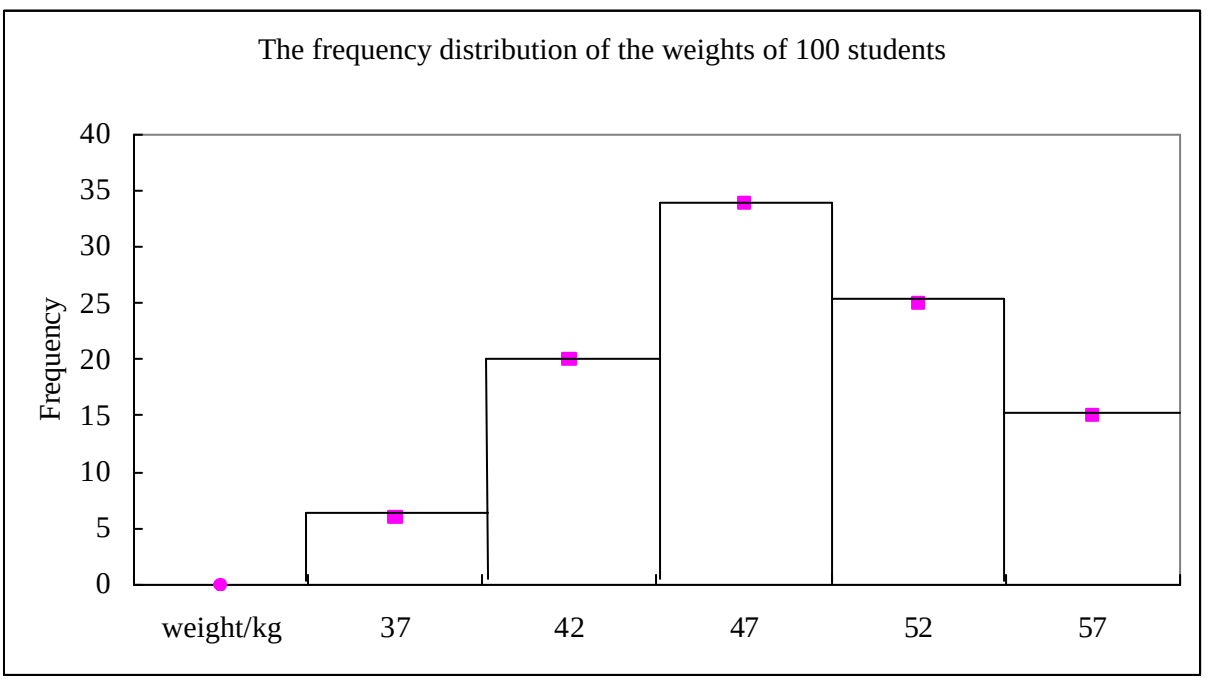
$$S(\infty) = \frac{150}{1-0.8} = 750$$

The total sum of circumference of rings of the all the models = $750p$ mm

C5.
The frequency distribution of the weights of 100 students

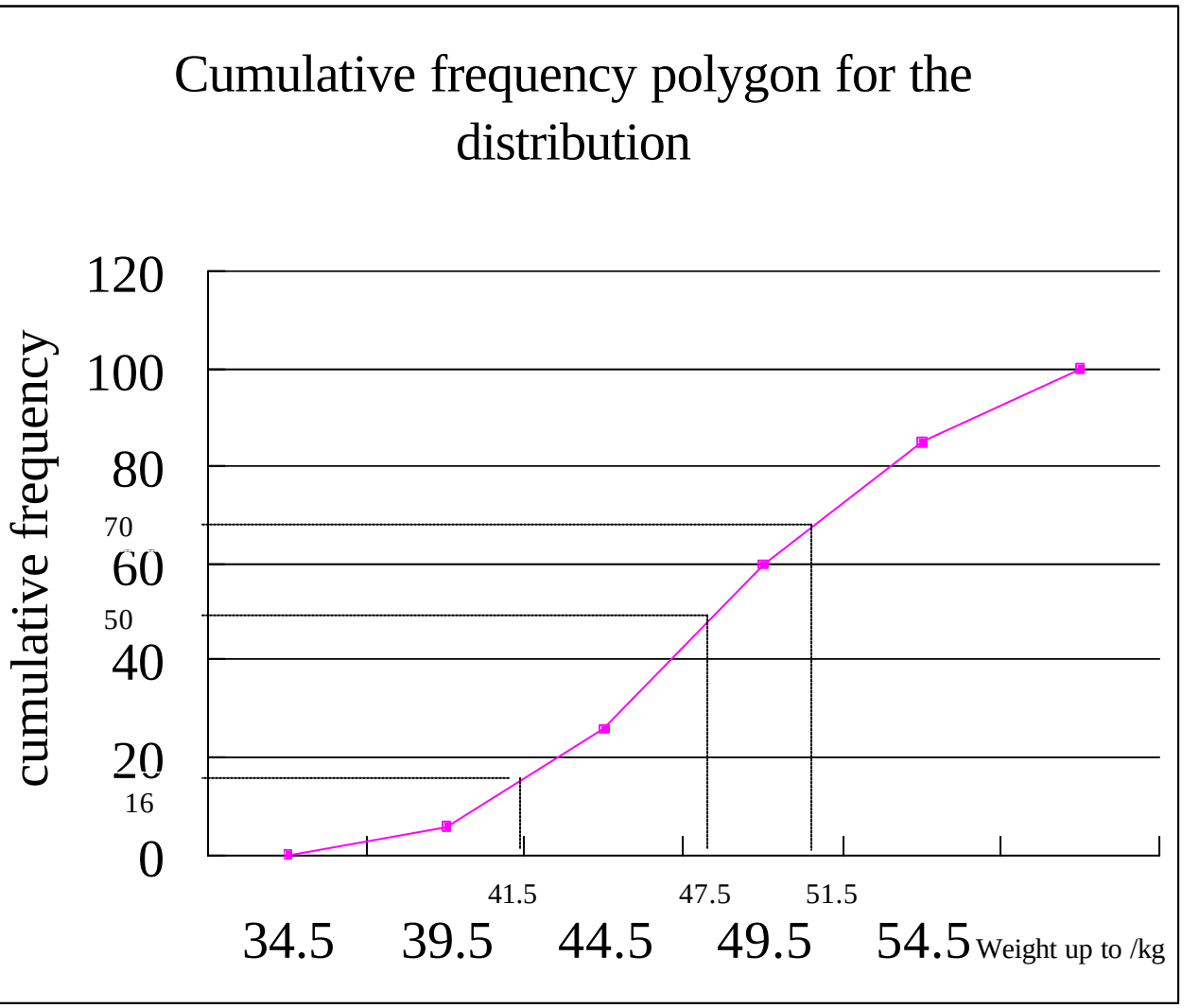
a)

weight/kg	**Class mark**	Frequency
35 to 39	**37**	6
40 to 44	**42**	20
45 to 49	**47**	34
50 to 54	**52**	25
55 to 59	**57**	15



The histogram for the distribution of weight

b)



c) The median is 48.

d) 30% of the students are selected. The number should be $0.3 \times 100 = 30$
70 students are not selected. The critical weight should be 52 kg..

e) 41.5 should be marked. From the graph, 16 students who weigh less than 42 kg.