HONG KONG INSTITUTE OF VOCATIONAL EDUCATION
FOUNDATION COURSES SUPPLEMENTARY EXAMINATION

Course : Foundation Certificate/BCTT **Course Code** : **03601T/1, 05601T/1, 05601B/1, 03602**

Stream : Business/Technical

Module Title : Mathematics

Module Code : CMV6103

Session : 2001/2002 **Date** : 6 August 2002

Time Allowed : 2 hours **Time** : 7:00 to 9:00 pm

This examination paper has 7 pages (excluding this covering page)

Instructions to Candidates

(a) Answer **ALL** questions in section A (10 multiple choice questions). Each question carries 2 marks. (20 marks)

(b) Answer **ALL** questions in section B (7 short questions). Each question carries 5 marks. (35 marks)

(c) Answer **ANY 3** questions out of 5 in section C (long questions). Each question carries 15 marks. (45 marks)

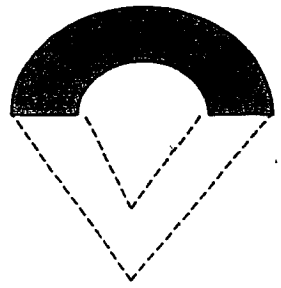
(d) Multiple choice **answer sheet** is provided for section A. Answers of section B and C must be written in the answer book provided.

(e) Approved calculators may be used.

Section B Short Questions (35 marks)

Answer ALL questions in this section. Each question carries 5 marks.

B1. A bridge is constructed in the form of a circular arc of a sector with sector angle 45° as shown in below. The radius of the inner sector is 12 m and the radius of the outer sector is 13 m.



(a) Find the arc length of the outer sector in terms of π.

(b) Find the area highlighted in terms of π.

B2. Solve the inequality $\frac{5x-1}{3} > \frac{x-3}{2}$.

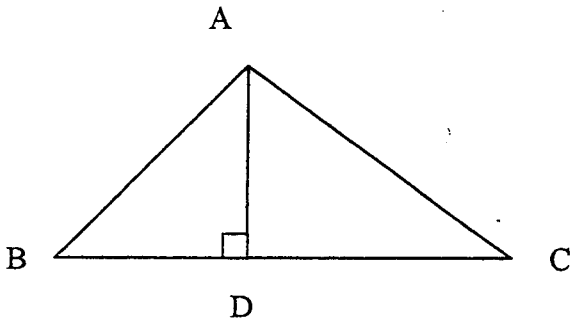
B3. The roots of the equation $x^2 - 2x + 5 = 0$ are α and β.

(a) Wrtie down the value of $\alpha + \beta$ and $\alpha\ \beta$.

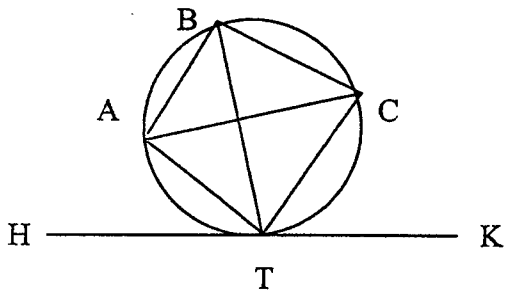
(b) Find the value of $\alpha^2 + \beta^2$.

B4. Solve the equation $2(\sin x - 1) + 3 = 0$ for $0° < x < 360°$.

B5. In $\triangle ABC$, AB= 10cm, $\angle ABC = 40°$ and $\angle BAC = 82°$. AD is a perpendicular drawn from A to BC. Find the length of (a) AD and (b) AC.
[Answers correct to 2 decimal places]



B6. HTK is a tangent to the circle. ABCT are points on the circumference of the circle. Given TA = TC and $\angle CTK = 56°$, find (a) $\angle ABC$ and (b) $\angle CAT$.



B7. A is the point (1, 0) and B is the point (4, -6).
(a) Find the equation of a line joining A and B.
(b) Find the coordinates of a point K dividing AB internally in the ratio 2:1.

Section C Long Questions (45 marks)

Answer ANY THREE questions in this section. Each question carries 15 marks.

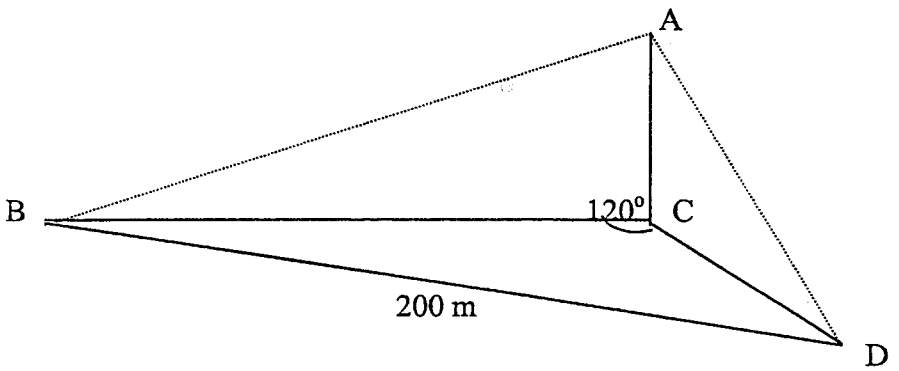
C1. The cost of flying a sport aircraft is partly constant and partly proportional to the square of the weight of the load. The cost of flying is \$1005 per hour when the load is 250 kg. The cost of flying is \$1280 per hour when the load is 300 kg.

(a) Let y be the cost in dollars and x be the weight of the load in kg, find an equation relating x and y. (7 marks)

(b) Calculate the cost of flying per hour when the load is 120 kg. (4 marks)

(c) It is known that the maximum load that the aircraft can carry is 550 kg. Find the cost of flying the aircraft for 2.5 hours. (4 marks)

C2. AC is a vertical tower erecting from a horizontal triangular base BCD. $\angle ACB$ and $\angle ACD$ are right angles. It is known that BD = 200 m, CD = 80 m, $\angle BCD = 120^\circ$ and the angle of elevation of A from D is 38°.
[Answers all correct to 1 decimal place]



(a) Calculate the height of the tower AC. (2 marks)

(b) Find the length of BC. (6 marks)

(c) Find the angle of elevation of A from B. (3 marks)

(d) $\triangle BCD$ is planned to be a garden. Find the area of $\triangle BCD$ to the nearest m^2 and the cost of construction if it costs \$150 per m^2 for the garden. (4 marks)

C3. Alex deposits $2000 in a bank on 1 Jan 2002. The investment is compounded monthly at a rate 1.2% per year.

(a) Calculate the interest earned on 31 Dec 2002. (4 marks)

(b) Alex agrees with the bank to deposit $2000 on the first day of each month for the whole year of 2002. Calculate the total interest earned on 31 Dec 2002.
[Hint: find the sum of the G.P. $a + a(1+r) + a(1+r)^2 + a(1+r)^3 + \ldots + a(1+r)^{n-1}$] (6 marks)

(c) Betty invests her money in the following way. She deposits $6000 on 1 Jan 2002, $6000 on 1 Apr 2002, $6000 on 1 Jul 2002 and $6000 on 1 Oct 2002. The amount of money is compounded quarterly (3 months) at a rate 1.2% per annum. Calculate the total interest earned by Betty on 31 Dec 2002. (5 marks)

C4. The profit of producing a certain computer part is given by the function $f(x) = x^3 - 3x^2 - 10x + 24$ where x is the number of years elapsed.

(a) Calculate the profit when x equals 5. (2 marks)

(b) Find f(0) and f(1). What does f(1) represent? (3 marks)

(c) Show that $(x + 3)$ is a factor of $f(x)$. (3 marks)

(d) Factorize $f(x)$ completely. (4 marks)

(e) Solve the equation $x^3 - 3x^2 - 10x + 24 = 0$. (3 marks)

C5. The frequency distribution of life-span (in hours) of 200 light bulbs is as follows:

life-span/hours	Frequency
0 and less than 200	15
200 and less than 400	40
400 and less than 600	75
600 and less than 800	40
800 and less than 1000	30

(a) Draw a histogram for the distribution of life-span of light bulbs. (2 marks)

(b) By completing the table below, construct a cumulative frequency polygon for the distribution. (6 marks)

life-span up to/hr	cumulative frequency
0	
200	
400	
600	
800	
1000	

(c) Read from your cumulative frequency polygon the median of the distribution. (2 marks)

(d) It was decided to select light bulbs with life-span shorter than 500 hours to be cast away. How many light bulbs are to be cast away? (2 marks)

(e) A light bulb is randomly selected. What is the probability that the bulb selected is of life-span longer than 700 hours ? (3 marks)

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FC Supplementary Examination 2001/02 Suggested answers

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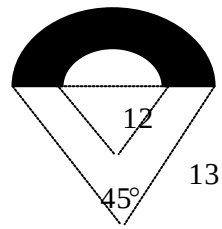
Section B Short Questions (35 marks)

Answer ALL questions in this section. Each question carries 5 marks.

B1



The arc length of the outer sector

a)

$$L = rq = 13(\frac{45}{180}p) = \frac{13}{4}p \text{ m}$$

b) The area highlighted =

$$\frac{1}{2}r_1^2 J - \frac{1}{2}r_1^2 \sin J - (\frac{1}{2}r_2^2 J - \frac{1}{2}r_2^2 \sin J)$$

$$= [\frac{1}{2}13^2(\frac{p}{4}) - \frac{1}{2}13^2 \sin 45°]$$

$$- [\frac{1}{2}12^2(\frac{p}{4}) - \frac{1}{2}12^2 \sin 45°] = \frac{25p}{8} - \frac{25\sqrt{2}}{4}$$

$$= \frac{25}{8}(p - 2\sqrt{2})m^2$$

B2

$$\frac{5x - 1}{3} > \frac{x - 3}{2}$$

$$2(5x - 1) > 3(x - 3)$$

$$10x - 2 > 3x - 9$$

$$7x > -7$$

$$\therefore \quad x > -1$$

B4

$2(\sin x - 1) + 3 = 0$ for $0^o < x < 360^o$.

$2\sin x - 2 + 3 = 0$

$2\sin x + 1 = 0$

$$\sin x = -\frac{1}{2}$$

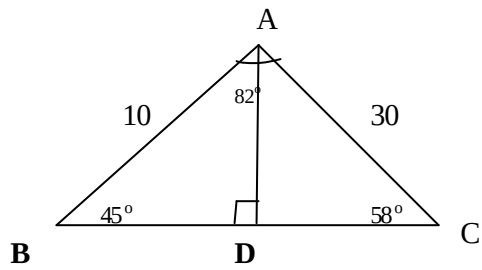
$$\therefore \quad x = -30° \text{ or } 210° (180° + 30°) \text{ or } 330° (360° - 30°)$$

B5

Consider ΔABD,

a) $\sin 40° = \dfrac{AD}{10} \Rightarrow AD = 10 \sin 40° = 6.43cm$

(correct to 2 decimal place)



b) $\angle ACB = 180° - 82° - 40° = 58°$

Consider ΔACD,

$\sin 58° = \dfrac{AD}{AC} = \dfrac{6.43}{AC} \Rightarrow AC = \dfrac{6.43}{\sin 58°} = 7.58cm$

(correct to 2 decimal place)

OR

Consider ΔABC,

using sine formula,

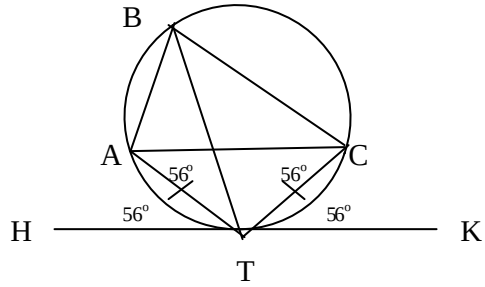
$\dfrac{AC}{\sin 40°} = \dfrac{10}{\sin 58°}$

$AC = \sin 40° \dfrac{10}{\sin 58°} = 7.58cm$

(correct to 2 decimal place)

B6

a)



$\angle CTK = 56°$

$\angle CAT = \angle CTK = 56°$

$\because TA = TC$

$\therefore \angle ACT = \angle CAT = 56°$

$\angle ATC = 180° - \angle ACT - \angle CAT$

$= 180° - 56° - 56° = 68°$

$\angle ABC = 180° - \angle ATC = 180° - 68° = 112°$

OR

$\angle CTK = 56°$

$\angle CBT = \angle CTK = 56°$

$\because TA = TC$

$\therefore \angle ACT = \angle CAT = 56°$

$\angle ATH = \angle ACT = 56°$

$\angle ABT = \angle ATH = 56°$

$\therefore \angle ABC = \angle ABT + \angle CBT = 56° + 56° = 112°$

b)

$\angle CTK = 56°$

$\angle CAT = \angle CTK = 56°$

B7

a) The equation of the line joining A and B

$$\frac{y-0}{x-1} = \frac{0-(-6)}{1-4} = \frac{6}{-3}$$

$$\frac{y}{x-1} = -2$$

$$y = -2(x-1)$$

$$y + 2x - 2 = 0$$

b) Let (h,k) be the coordinates of the point K dividing AB internally in the ratio 2:1

$$h = \frac{1(1) + 2(4)}{2+1} = 3$$

$$y = \frac{1(0) + 2(-6)}{2+1} = -4$$

Therefore, (h,k) is (3,-4)

Section C Long Questions (45 marks)

Answer ANY **THREE** questions in this section. Each question carries 15 marks.

C1

a) Let y be the cost of flying a sport aircraft per hour in dollars, x be the weight of load in kg and k_1, k_2 be two constants.

The equation relating x and y is :

$y = k_1 + k_2 x^2$

When y = 1005, x = 250

Therefore, $1005 = k_1 + k_2(250)^2$

$1005 = k_1 + 62500\,k_2 \cdots\cdots (1)$

When y = 1280, x = 300

Therefore, $1280 = k_1 + k_2(300)^2$

$1280 = k_1 + 90000\,k_2 \cdots\cdots (2)$

(2) – (1),

$\therefore 27500\,k_2 = 275$

$k_2 = 0.01$

$1280 = k_1 + 90000\,k_2$

$-\quad (1005 = k_1 + 62500\,k_2\quad)$

$275 = 27500\,k_2$

Sub k_2 to (1),

$1005 = k_1 + 0.01(250)^2$

$1005 = k_1 + 0.01(62500)$

$\therefore k_1 = 380$

Therefore the equation is $y = 380 + 0.01x^2$

b) Putting, x=120 to the equation,

$y = 380 + 0.01x^2$

$= 380 + 0.01(120)^2$

$y = 524$

The cost of flying is \$524 per hour.

c) Putting, x=550 to the equation,

$y = 380 + 0.01x^2$

$= 380 + 0.01(550)^2$

$y = 3405$

The cost of flying is \$3405 per hour.

The cost of flying the aircraft for 2.5 hours is (3405×2.5) \$8512.5

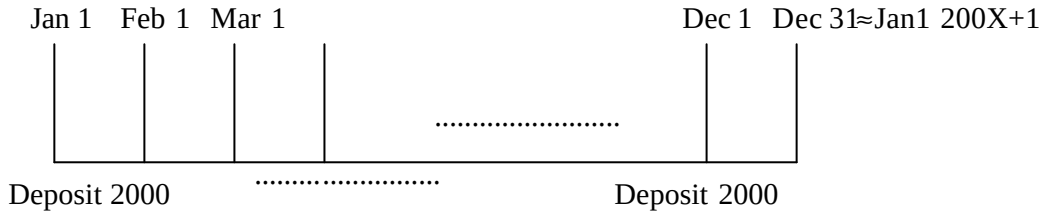
C4

Alex deposits $2000 in a bank on 1 Jan 200X. The investment is compounded monthly at a rate 1.2% per year.

a) Calculate the interest earned on 31 Dec 200X. (4 marks)

b) Alex agrees with the bank to deposit $2000 on the first day of each month for the whole year of 200X. Calculate the total interest earned on 31 Dec 200X.
(6 marks)

a)



Alex had earned 12 months of interest since 1 Jan 200X.
The principle with interest on 31 Dec 200X become

$$= 2000(1+\frac{1.2\%}{12})^{12} = 2000(1+\frac{0.012}{12})^{12} = 2000(1.001)^{12} = \$2024.13$$

The interest earned on 31 Dec 200X $= 2024.13 - 2000 = \$24.13$

b)

Month	Jan 1	Feb 1	Mar 1	Apr 1		Dec 1
No. of months passed	12	11	10	9		1
The principle with interest	$2000(1+\frac{0.012}{12})^{12}$	$2000(1+\frac{0.012}{12})^{11}$				$2000(1+\frac{0.012}{12})^{1}$

The sum of principle with interest started from Jan1 200X to 31 Dec 200X become

$$2000(1+\frac{0.012}{12})^{12} + 2000(1+\frac{0.012}{12})^{11} + 2000(1+\frac{0.012}{12})^{10} + \cdots\cdots + 2000(1+\frac{0.012}{12})^{2} + 2000(1+\frac{0.012}{12})^{1}$$

$$= 2000(1.001)^{1} + 2000(1.001)^{2} + \cdots\cdots + 2000(1.001)^{11} + 2000(1.001)^{12}$$

b)

It is a G.P.,

$$a = 2000(1.001),\ R = \frac{2000(1.001)^2}{2000(1.001)^1} = (1.001), n = 12$$

$$S(n) = \frac{a(R^n - 1)}{R - 1}$$

$$S(12) = \frac{2000(1.001)[(1.001)^{12} - 1]}{(1.001) - 1} = \frac{2002(1.001^{12} - 1)}{1.001 - 1} = \$24156.57$$

Alex had deposited $2000 \times 12 = \$24000$

The total interest earned on 31 Dec 200X.

$= 24156.57 - 24000 = \$156.57$

c)

Month	Jan 1	Apr 1	Jul 1	Oct 1
No. of quarter passed	4	3	2	1
The principle with interest	$6000(1+\frac{1.2\%}{4})^4$	$6000(1.003)^3$	$6000(1.003)^2$	$6000(1.003)^1$

It is a G.P.,

$$a = 6000(1.003),\ R = \frac{6000(1.003)^2}{6000(1.003)^1} = 1.003, n = 4$$

$$S(n) = \frac{a(R^n - 1)}{R - 1}$$

$$S(12) = \frac{6000(1.003)[(1.003)^4 - 1]}{(1.003) - 1} = \frac{6018(1.003^4 - 1)}{1.003 - 1} = \$24180.5$$

Betty had deposited $6000 \times 4 = \$24000$

The total interest earned on 31 Dec 200X.

$= 24180.5 - 24000 = \$180.5$

C4

a) A function is given by $f(x) = x^3 - 3x^2 - 10x + 24.$

When, x = 5, $f(5) = (5)^3 - 3(5)^2 - 10(5) + 24 = 174.$

b) When, x = 0, $f(0) = (0)^3 - 3(0)^2 - 10(0) + 24 = 24.$

When, x = 1, $f(1) = (1)^3 - 3(1)^2 - 10(1) + 24 = 12.$

f(1) represents the value 12 when x=1

c) When, x = -3, $f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24 = -27 - 27 + 30 + 24 = 0.$

By factor theorem, (x+3) is a factor of f(x).

d) $f(x) = x^3 - 3x^2 - 10x + 24. = (x+3)(x^2 - 6x + 8)$

$= (x+3)(x-4)(x-2)$

$$x+3 \overline{)\, x^3 - 3x^2 - 10x + 24}$$

$x^2 - 6x + 8$

$x^3 + 3x^2$

$-6x^2 - 10x$

$-6x^2 - 18x$

$8x + 24$

$8x + 24$

1 -4

1 -2

e)

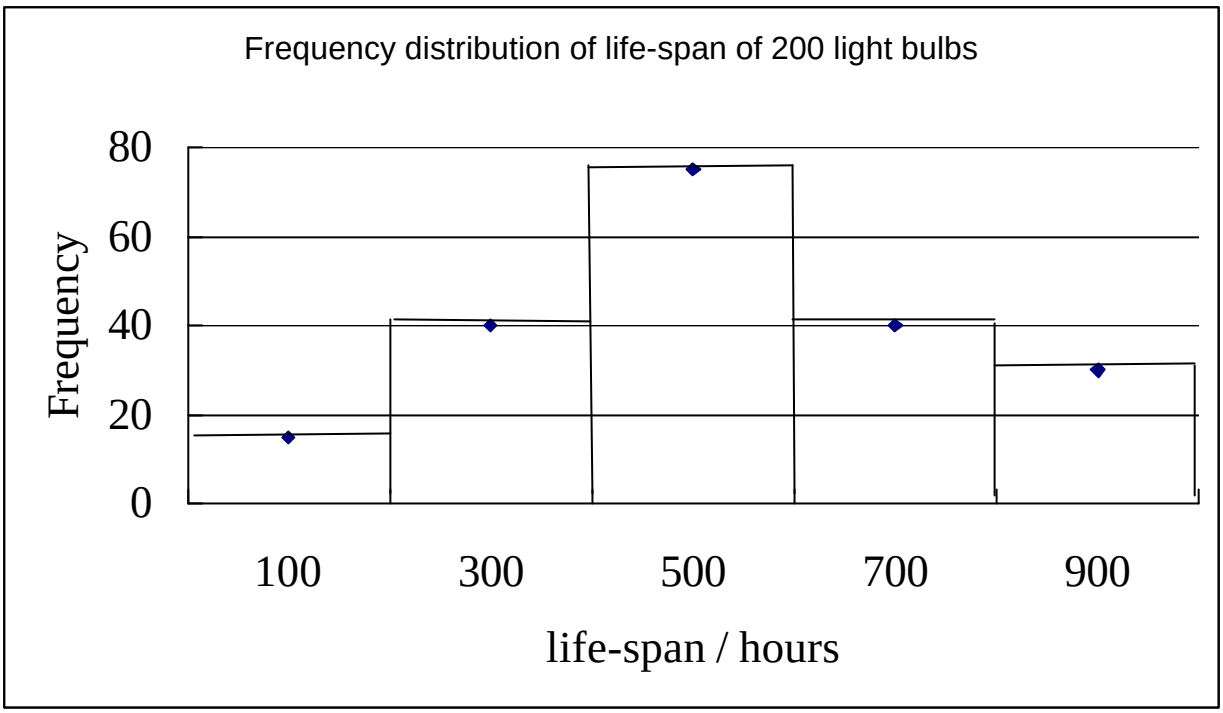
$f(x) = x^3 - 3x^2 - 10x + 24. = (x+3)(x-4)(x-2) = 0$

$\therefore x = -3, 2 \text{ or } 4$

C5

a)

life-span up to/hr	Class Mark	Frequency
0 and less than 200	**100**	15
200 and less than 400	**300**	40
400 and less than 600	**500**	75
600 and less than 800	**700**	40
800 and less than 1000	**900**	30



b)

life-span up to/hr 0	cumulative frequency **0**
200	**15**
400	**55**
600	**130**
800	**170**
1000	**200**

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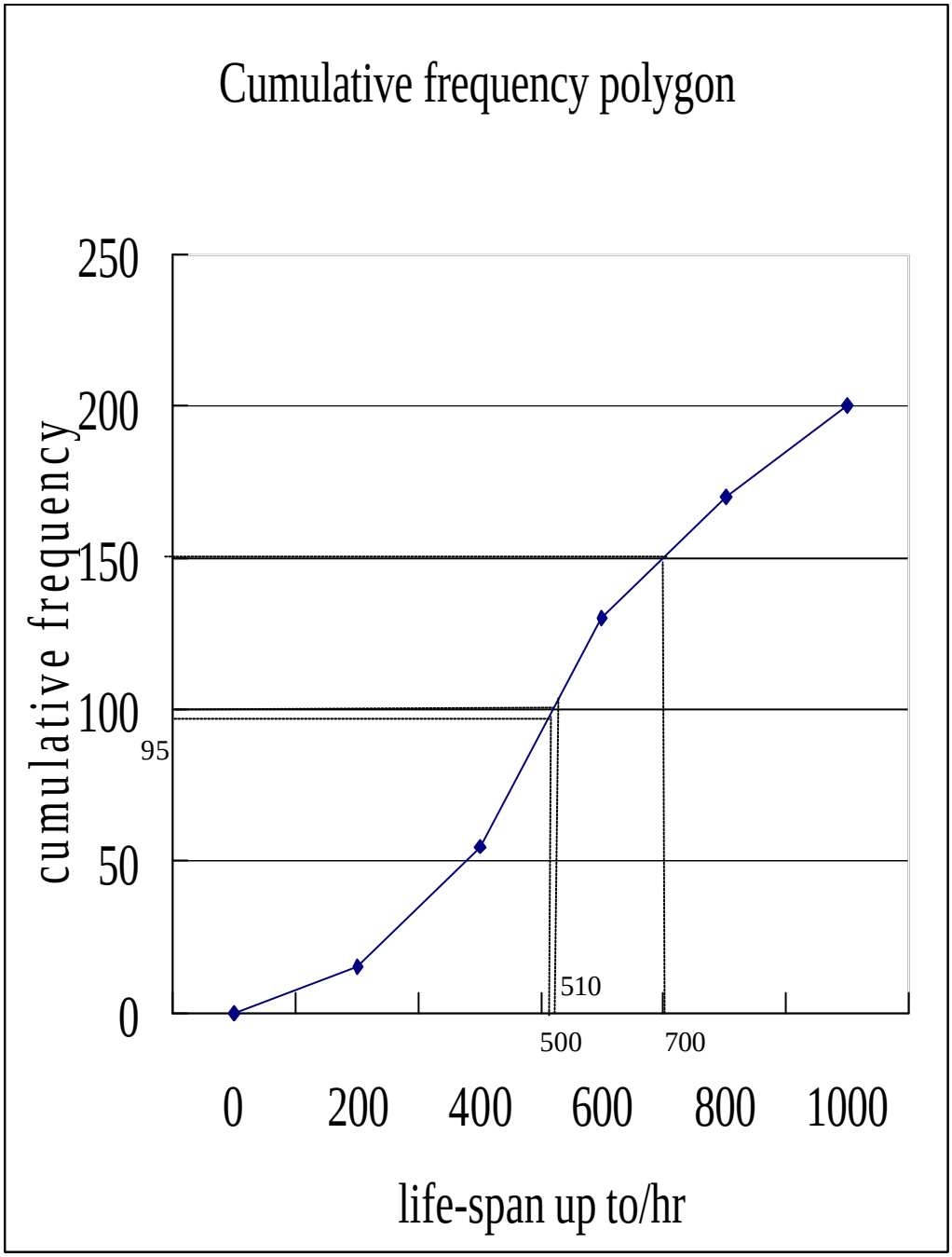

c) The median of the distribution is 510 hours.

d) 95 light bulbs.

e) P(bulb selected which has life-span longer than 700 hrs) $= \frac{200 - 150}{200} = \frac{50}{200} = 0.25$