Tutorial 21 : straight lines

1. Prove that the 3 points A(3,5), B(7,13), C(-4,-9) are collinear.

2. Given: A(2,-3), B(8,11). Find the coordinates of P, such that P divide the line segment $\overline{AB}$ internally in the ratio $\overline{AP} : \overline{PB} = 7 : 5$.

3. Given the straight line L: $3x - 4y + 24 = 0$
   (a) Find the slope of L
   (b) Find the x and y intercepts of L.

4. Find the equation of the straight line
   (a) which passes through the points (-2, 4) and (3, -1),
   (b) which passes through (-5, 6) and is perpendicular to the line $2x - 3y + 4 = 0$

5. Find the equation of the straight line that passes through the mid-point of the line segment joining A(2, 9) and B(6, -3), and is parallel to the line $2x - 6y + 13 = 0$

6. Find the equation of the perpendicular bisector of the line segment joining P(-7, 10) and Q(1, 6).

7. A straight line L makes an angle of 120^o with the positive x-axis.
   The x-intercept of L is 5. Find the equation of L.

8. When you watch TV, in the weather report, you will find that, in China/Hong Kong, we are using Celsius scale in measuring temperature. However, another scale (Fahrenheit scale) is being used in USA.
   Actually, these 2 scales have a linear relationship. Given:
   boiling point of water: 100 ^{O}C or 212 ^{O}F
   melting point of ice: 0 ^{O}C or 32 ^{O}F
   Using X for readings in ^{O}C and Y for readings in ^{O}F, write out the equation describing the relationship between X, Y.
   One day, a relative in NY told me, over the phone, that it was very hot there and it measured 97 ^{O}F. Was his saying justifiable?
   (Usually we feel very hot when the temperature reaches 33 ^{O}C in HK)

Solution :

1. Slope of AB = (13-5)/(7-3) = 2

   Slope of AC = (-9 – 5)/(-4 – 3) = 2

   Since slope of AB = Slope of AC, A, B, C are collinear

2. Let P:(x, y), using formulae

   x = (5x2 + 7x8)/(7 + 5) = 11/2

   y = (5x(-3) + 7x11)/(7+5) = 31/6

   P= (11/2, 31/6)

3. (a) $3x - 4y + 24 = 0$

   $y = -\frac{3}{4}x + 6$

   $\therefore \quad m = -\frac{3}{4}$

   (b) from the equation, y-intercept = 6

   when y = 0, $3x - 4(0) + 24 = 0$

   $x = -8$

   $\therefore$ x-intercept = -8

4. (a) $\frac{y-4}{x-(-2)} = \frac{4-(-1)}{-2-3}$

   $\frac{y-4}{x+2} = -1$

   $x + y - 2 = 0$

   (b) $2x - 3y + 4 = 0$

   $y = \frac{2}{3}x + \frac{4}{3}$

   $m = \frac{2}{3}$

   The slope of the required line is $\frac{-1}{\frac{2}{3}} = -\frac{3}{2}$

   $\therefore$ Equation of the required line is $\frac{y-6}{x-(-5)} = \frac{-3}{2}$

$2y - 12 = -3x - 15$

$3x + 2y + 3 = 0$

5. Coordinates of mid-point is $(\frac{2+6}{2}, \frac{9-3}{2}) = (4,3)$

slope of $2x - 6y + 13 = 0$ is $\frac{-2}{-6} = \frac{1}{3}$

Equation of the required line is

$\frac{y-3}{x-4} = \frac{1}{3}$

$3y - 9 = x - 4$

$x - 3y + 5 = 0$

6. Mid-point of PQ is $(\frac{-7+1}{2}, \frac{10+6}{2}) = (-3,8)$

Slope of PQ = $\frac{10-6}{-7-1} = -\frac{1}{2}$

Slope of the perpendicular bisector = 2

Equation of the perpendicular bisector is

$\frac{y-8}{x-(-3)} = 2$

$y - 8 = 2x + 6$

$2x - y + 14 = 0$

7. $m = \tan 120^\circ = -\sqrt{3}$

Equation of L is $y = m x + c$

$y = -\sqrt{3}\, x + c$

Since (5, 0) lies on L, $\therefore \quad 0 = -\sqrt{3}\,(5) + c$

$c = 5\sqrt{3}$

Equation of L is $y = -\sqrt{3}\, x + 5\sqrt{3}$

8. Using slope-intercept form of straight line: $y = mx + c$

since when $x = 100$, $y = 212$

$\therefore \quad 212 = m(100) + c$ ……(1)

since when $x = 0$, $y = 32$

$\therefore \quad 32 = m(0) + c$ ……(2)

Solving (1) & (2),we have

$c = 32$, $m = 9/5$

$\therefore$ Equation relating x, y is: $y = (9/5)x + 32$

Moreover, when $y = 97$,

$x = (97 - 32)/(9/5) = 36.11\ ^{O}C$

It's certainly a hot day.