Unit 21 : Rectangular Coordinates, Distance Formula ,Section Formula and Equation of Straight Line

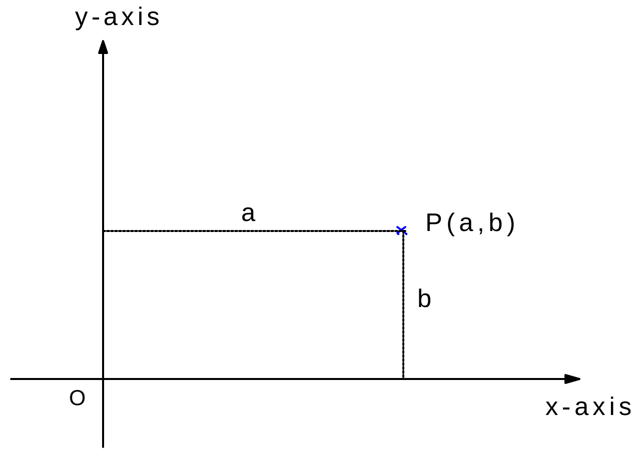
Learning Objectives

Students should be able to:

- Locate a point in a plane using rectangular coordinate system
- Calculate the distance between 2 points
- Determine the point of division of a line segment
- Determine the slope of a line
- State the condition for 2 lines to be parallel or perpendicular
- Understand that the equation of a straight line is of the first degree
- Determines the equation of a straight line
- Determines the slope and intercepts of a straight line
- Sketch a line when its equation is given

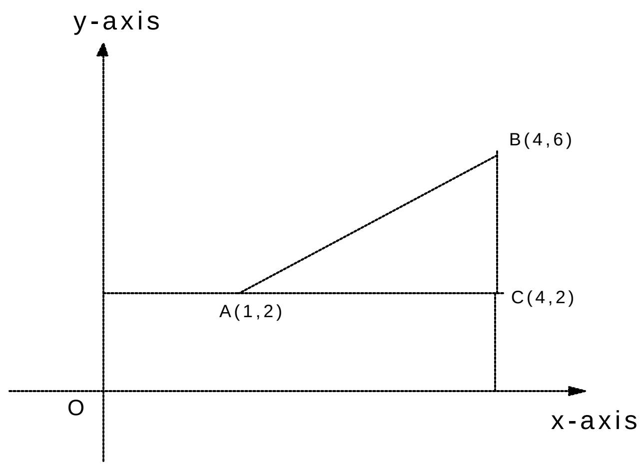
1. Review of coordinates

The coordinates of a point P referred to the perpendicular axes are the ordered pair (a, b) where
a (the x-coordinate) = the distance of P from the y-axis; and
b (the y-coordinate) = the distance of P from the x-axis.



Example 1: Plot the points A(1,2), B(4,6) and C(4,2); hence find the distances between:
a) A and C;
b) B and C;
c) A and B.

Soln:



From the figure,

a) AC = 4 – 1 = 3

b) BC = 6 – 2 = 4

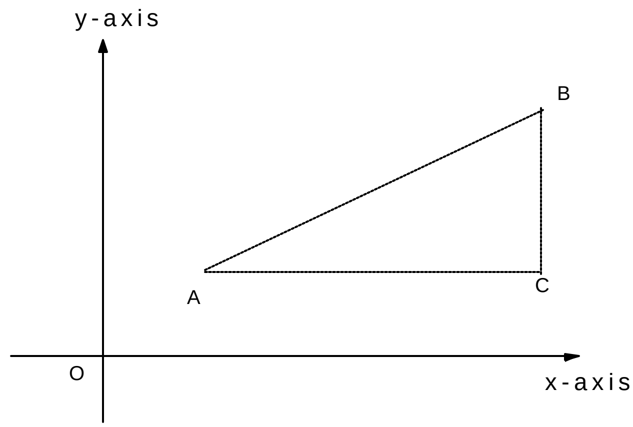
c) By Pythagoras theorem, AB = $\sqrt{AC^2 + BC^2}$

$= \sqrt{3^2 + 4^2}$

= 5 units

2. Distance formula

Now let us generalize Examples 1 and 2 by finding the distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$.



From the figure, $C = (x_2,y_1)$

$AC = x_2 - x_1$

$BC = y_2 - y_1$

By Pythagoras theorem, AB = $\sqrt{AC^2 + BC^2}$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 2: Find the distances between the pairs of points:

a) (3,4) and (15,9);

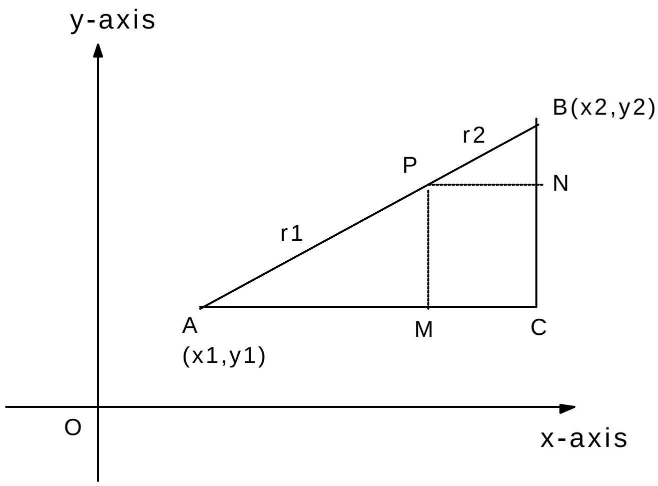
b) (5,1) and (-3,-4);

Soln: a) $\sqrt{12^2 + 5^2} = 13$ units

b) $\sqrt{(-8)^2 + (-5)^2} = \sqrt{89}$ units

3. Section formula

A point P on the joining of AB is said to divide AB in the ratio $r_1 : r_2$ if $AP : PB = r_1 : r_2$. In the figure below, if P divides AB in the ratio $r_1 : r_2$



then $\frac{AP}{PB} = \frac{AM}{PN} = \frac{MP}{NB} = \frac{r_1}{r_2}$

i.e. $\frac{x - x_1}{x_2 - x_1} = \frac{r_1}{r_2}$

hence $x = \frac{r_1 x_2 + r_2 x_1}{r_1 + r_2}$

Similarly, $y = \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}$

Hence, the <u>section formula</u> concludes that:

The coordinates of the point P which divides the joining of $A(x_1,y_1)$ and $B(x_2,y_2)$ is

$$x = \frac{r_1 x_2 + r_2 x_1}{r_1 + r_2} \quad \text{and} \quad y = \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}$$

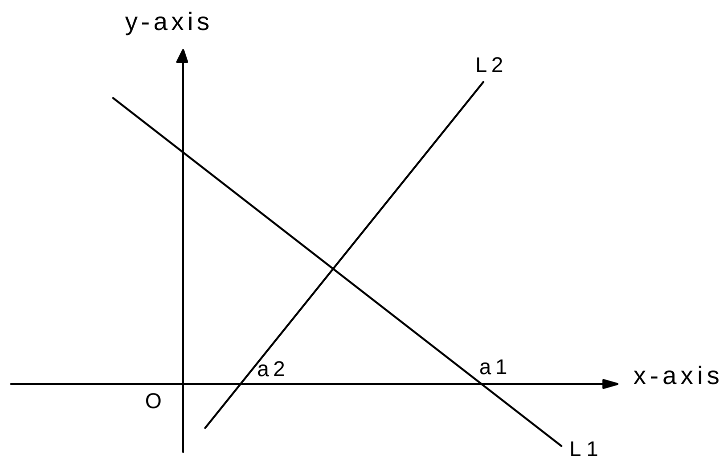
<u>Example 3</u>: Write down the mid-points of the lines joining:
a) (1,1) and (5,4);
b) (6,1) and (-8,3);

<u>Soln</u>: a) (3, 5/2)
b) (-1, 2)

Example 4: Given A = (4,16) and B = (11,-5), find H if H divides AB internally in the ratio 1:6

Soln: (5, 13)

4. <u>Inclination and slope of a line</u>



The inclination of a line is the angle made between the line and the positive x-axis.

The slope of a line is defined as the tangent of the inclination of the line.
i.e. $m = \tan a$

<u>Example 5</u>: Find the slopes of the lines with inclinations:
a) $a = 30^o$;
b) $a = 45^o$;
c) $a = 135^o$.
What conclusion can you make if the two lines are perpendicular?

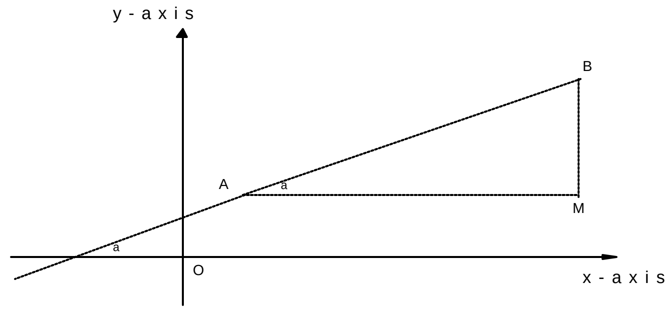
Soln: (a) $m = \tan 30^o = 0.5771$
(b) $m = \tan 45^o = 1$
(c) $m = \tan 135^o = -1$
from (b) and (c), $\mathbf{m_1 m_2 = -1}$ **and these two lines are perpendicular.**

In general,

(i) **if** $L_1 // L_2$ **then** $\mathbf{m_1 = m_2}$ **;**

(ii) **if** $L_1 \perp L_2$ **then** $\mathbf{m_1 m_2 = -1}$ **.**

5. Slope of the line passing through two points



From the above figure, slope of the line passing through the points A and B is

$$m = \tan a$$
$$= \frac{BM}{AM}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 6: Find the slopes of the lines passing through:

a) (5,2) and (-2,2);
b) (5,1) and (-3,-4);
c) (1,2) and (-1,4);
d) (4,7) and (4,1).

Soln: a) 0
b) 5/8
c) –1
d) undefined.

Example 7 A and B are the points (-4, 1) and (8, 6) respectively.

(a) Find the slope of AB.

(b) Find the length of AB.

(c) Write down the coordinates of the mid-point of AB.

Soln: (a) slope m = $\frac{6-1}{8-(-4)} = \frac{5}{12}$

(b) length of AB = $\sqrt{(8-(-4))^2 + (6-1)^2}$

$= \sqrt{12^2 + 5^2} = 13$ units

(c) coordinates of mid-point $= (\frac{-4+8}{2}, \frac{1+6}{2}) = (2, \frac{7}{2})$

6. <u>Equation of the straight line</u>

For every point P(x,y) to lie on a straight line, its coordinates x and y must satisfy a certain relation. This relation is called the equation of the straight line.

For example, to find the equation of the line of slope 2 and through the point A(2,1) is to find the connection such that every point P(x,y) on the line should satisfy, i.e. slope of PA = given slope 2

$$\frac{y-1}{x-2} = 2$$

or 2x – y = 3 …………..(*)

Hence, in order that P(x,y) be a point on the line, its coordinates x and y must satisfy (*), and (*) is called the equation of the line.

In fact, the equations of straight lines are of 1st degree, and equations in x, y of 1st degree are called linear equations.

7. <u>Point-slope form</u>

We have already found in the last section that the line through the **point (x_1,y_1)** of **slope m** is:

$$\frac{y-y_1}{x-x_1} = m$$

<u>Example 8</u>: Find the equations of lines through the following given point(1,-2) and with slopes –1.

Solution: The equation of the line through (1,-2) with slope = -1 is :

$$\frac{y+2}{x-1} = -1$$

x + y + 1 = 0.

8. <u>Two points form</u>

From our last section, the line through (x_1,y_1) and of slope m is

$$\frac{y-y_1}{x-x_1} = m \ .$$

If the line also passes through (x_2,y_2), its slope is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

Therefore, **the equation of the line joining (x_1,y_1) and (x_2,y_2)** is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 9: Find the equation of the line joining (4,1) to (5,-2).

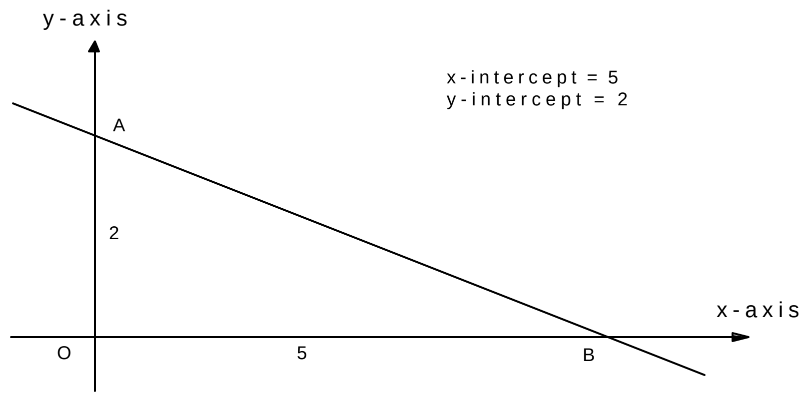
Soln: The equation is:

$$\frac{y-1}{x-4} = \frac{-2-1}{5-4}$$

or 3x + y = 13.

Example 10: Find the equation of the line which makes x-intercept 5 and y-intercept 2 on the axes.

Solution



As the line makes y-intercept 2 and x-intercept 5

From the figure, A = (0,2) and B = (5,0)

The equation of the line is $\dfrac{y-2}{x-0} = \dfrac{0-2}{5-0}$ or 2x + 5y = 10

Example 11: Find the equation of the line passing through (2,3) and is parallel to the line 3x + 4y =12.

Soln: When x=0, y=3

When y=0, x=4

The line 3x + 4y =12 passes through (0,3) and (4,0)

and its slope is $m=\frac{0-3}{4-0}=\frac{-3}{4}$

Equation of the line is: $\frac{y-3}{x-2}=\frac{-3}{4}$

Or 3x + 4y = 18.

9. Slope -intercept form

The equation of the line with **slope m** and **y-intercept b** is the line through (0,b), so its equation is:

$\frac{y-b}{x-0}=m$ or

y=mx+b.

By writing the equation of a line into this form, we can read the slope m of the line easily.

Example 12: Find the slope of each of the line 3x + 4y = 10.

Soln: 4y = -3x + 10

$y = \frac{-3}{4}x+10$ gives slope $m = \frac{-3}{4}$

Example 13: Find the equation of the line through (5,7) and is perpendicular to another line 3x + 2y – 7 = 0.

Soln: As 3x + 2y – 7 = 0

$y = \frac{-3}{2}x +7$

It's slope $m = \frac{-3}{2}$

Hence equation of the line is $\frac{y-7}{x-5}=\frac{2}{3}$

or 2x – 3y + 11 = 0