Tutorial 22 : Equation of a circle & its intersection with a straight line

1. Write down the equations of the following circles with the given centres and radii
   (a) Centre (2, 3) , radius = 7
   (b) Centre (0, -6) , radius = $\frac{3\sqrt{2}}{2}$
   (c) Centre $(-\frac{1}{2}, -\frac{1}{3})$, radius = 1

2. Find the coordinates of the centre and the radius of each of the following circles.

   (a) $x^2 + y^2 - 16 = 0$
   (b) $x^2 + y^2 - 12x + 18y + 17 = 0$
   (c) $9x^2 + 9y^2 - 6x - 72y + 28 = 0$

3. Find the intersection point(s) between the circle, C, and the line, L.

   Given $\begin{cases} C: & x^2 + y^2 - 5x + 6y - 85 = 0 \\ L: & 2x - 3y + 15 = 0 \end{cases}$

4. The line L: $y = 3x + (k + 1)$ (where k is a constant) is tangent to the circle C: $(x-3)^2 + (y-1)^2 = 9$. Find the values of k.

   (Note that a circle should have 2 different tangents of same slope, therefore, 2 different values of k can be found)

5. Given a circle C: $(x - 7)^2 + (y + 6)^2 = k$ (where k is a constant)

   Find the range of values of k such that:

   a) x-axis is a tangent of C
   b) x-axis intersect at 2 distinct points with C
   c) x-axis does not intersect at any points with C

   (Think about this problem graphically, you may solve it quickly without tedious calculation)

Solution to Tutorial 25

1. (a) $(x - 2)^2 + (y - 3)^2 = 7^2$

$x^2 + y^2 - 4x - 6y - 36 = 0$

(b) $(x - 0)^2 + (y + 6)^2 = (\frac{3\sqrt{2}}{2})^2$

$x^2 + y^2 + 12y + 36 - \frac{9}{2} = 0$

$2x^2 + 2y^2 + 24y + 63 = 0$

(c) $(x + \frac{1}{2})^2 + (y + \frac{1}{3})^2 = 1$

$x^2 + y^2 + x + \frac{2}{3}y - \frac{23}{36} = 0$

$36x^2 + 36y^2 + 36x + 24y - 23 = 0$

2. (a) Centre is (0, 0) , $r = 4$

(b) Centre is (6, -9) , $r = \sqrt{(-6)^2 + 9^2 - 17} = 10$

(c) $9x^2 + 9y^2 - 6x - 72y + 28 = 0$

$x^2 + y^2 - \frac{2}{3}x - 8y + \frac{28}{9} = 0$

Centre is $(\frac{1}{3}, 4)$, $r = \sqrt{(-\frac{1}{3})^2 + (-4)^2 - \frac{28}{9}} = \sqrt{13}$

3. Solve the simultaneous equation system

$$\begin{cases} C: \ x^2 + y^2 - 5x + 6y - 85 = 0 & \cdots\cdots(1) \\ L: \ 2x - 3y + 15 = 0 & \cdots\cdots(2) \end{cases}$$

from (2), $x = (3/2)y - (15/2)$ ....(3)

Subs (3) into (1),

$((3/2)y - (15/2))^2 + y^2 - 5((3/2)y - (15/2)) + 6y - 85 = 0$

$13y^2 - 96y + 35 = 0$

$y = 7$ or $5/13$

Subs this results into (3),

When $y = 7$, $x = 3$

When $y = 5/13$, $x = -90/13$

L cuts C at 2 distinct points:(7, 3) & (5/13, -90/13)

4. Substitute L into C:

$(x - 3)^2 + (3x + k)^2 = 9$

$x^2 - 6x + 9 + 9x^2 + 6kx + k^2 - 9 = 0$

$10x^2 + (6k - 6)x + k^2 = 0$

For L is a tangent, the above quadratic equation should has only 1 root

Discriminant = 0

$(6k - 6)^2 - 4(10)(k^2) = 0$

$36k^2 - 72k + 36 - 40k^2 = 0$

$-4k^2 - 72k + 36 = 0$

$k^2 + 18k - 9 = 0$

k = 0.4868 or –18.49 (4 sign. fig.)

5. Note that C is circle centered at (7, -6) with radius $r = \sqrt{k}$ (See Figure 1 below)

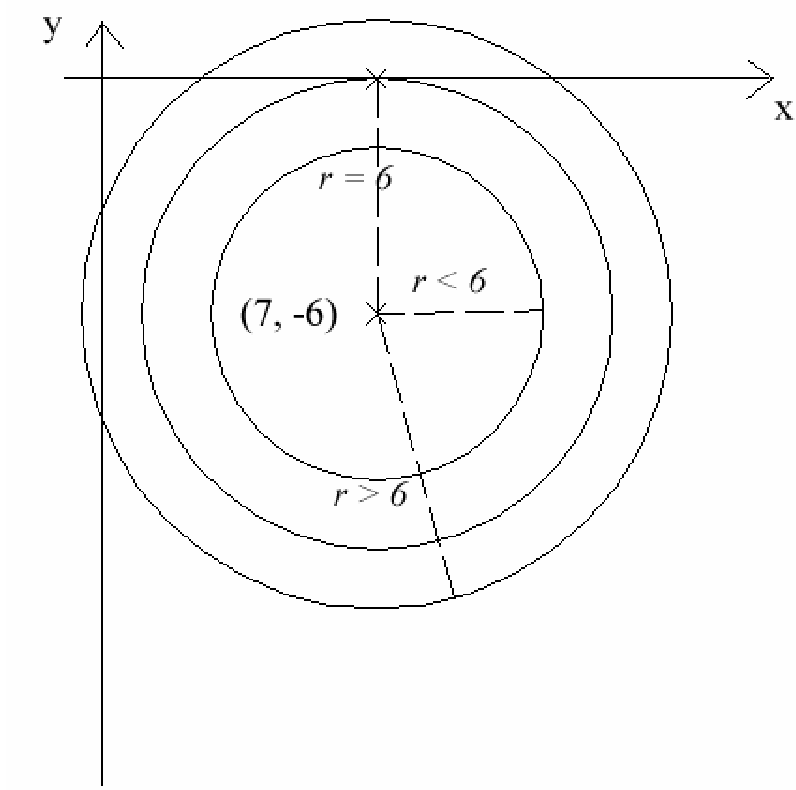

Figure 1

Obviously, when $r = 6$, x-axis is a tangent to C

when $r > 6$, x-axis intersect C at 2 distinct points

when $r < 6$, x-axis doesn't intersect with C

Therefore, we conclude, (since $r = \sqrt{k}$)

when $k = 36$, x-axis is a tangent to C

when $k > 36$, x-axis intersect C at 2 distinct points

when $k < 36$, x-axis doesn't intersect with C