Unit 5 : Linear and quadratic inequalities in one variable

Learning Objectives

Students should be able to :

State the basic properties of inequalities.

Solve a linear inequality in one variable.

Represent the solution graphically

Solve compound inequalities involving ‘and’ , ‘or’.

Solve a quadratic inequality in one variable.

Solve simple applied problems involving inequalities.

Activities

Teacher demonstration and students hand-on exercise.

Reference

Canotta 5A
Chapter 3

Linear and quadratic inequalities in one variable

1 Basic properties of inequalities

For any three real numbers a, b and c:

Property	Example
1. If $a>b$ and $b>c$, then $a>c$.	9>5 and 5>2, then 9>2
2. If $a>b$, then $a+c>b+c$.	3>1, then 3+5>1+5
3. If $a>b$, then $ac>bc$ when $c>0$; $ac<bc$ when $c<0$.	8>6, then 8(2)>6(2) for 2>0; 8(-2)<6(-2) for –2<0.
4. If $a>b$ (where $ab>0$), then $\frac{1}{a}<\frac{1}{b}$	4>2, then $\frac{1}{4}<\frac{1}{2}$
5. If $a \neq 0$, then $a^2>0$.	$-3 \neq 0$, then $(-3)^2>0$.

Class practice:

If $a < 0$, then a^2 ___ 0	If $a > 3$, then $a + 4 >$ ___
If $a < 3$, then $a - 1$ ___ 2	If $2a > 3$, then $a >$ ___
If $-3a < -6$, then a ___ 2	If $-\frac{1}{2}a < -6$, then a ___ 12
If $a > 3$, then $\frac{1}{a}$ ___ $\frac{1}{3}$	If $a < -4$, then $\frac{1}{a}$ ___ $-\frac{1}{4}$

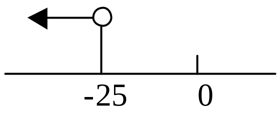
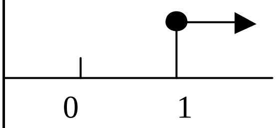
2 How to solve Linear inequality in one variable

1. Express the inequality as ax ($<, \leq, >, \geq$) b. (Use the technique of unit 1)
2. Use property 3 to solve the inequality

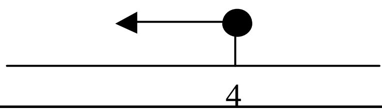
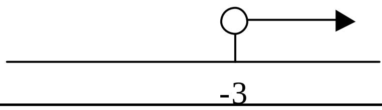
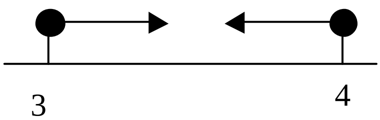
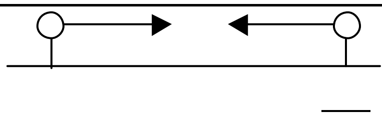
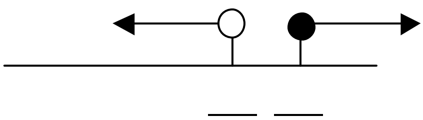
E.g. 1	Solve $2(x-5)<30$	Explanation
Solution	$2(x-5)<30$ $2x<30+2(5)$ <40 $x<20$	 Property 3

E.g. 2	Solve $2(x-5)<3(x+5)$	Explanation
Solution	$2(x-5)<3(x+5)$ $2x-\_\_\_x<3(5)+2(\_\_\_)$ $-x<\_\_\_$ $x>-\_\_\_$	 Property 3

3 Graphical Representations

Solution	Graph	Remark
$x < -25$		Put variable x on LHS. Put number on RHS. Arrow is in the same direction as the Inequality sign. ○ Means -25 is not included in the solution. ● Means 1 is included in the solution.
$x \geq 1$		

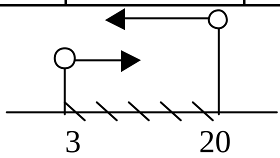
Class practice:

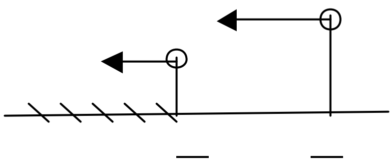
Inequality	Graphical Representation
$x \leq$ ___	
$x >$ ____	
___ $\leq x \leq$ ___	
$3 < x < 4$	
$x \geq 4$ or $x < -3$	

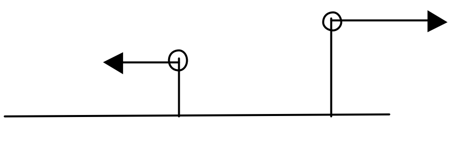
4 Compound inequalities involving 'and' , 'or'

Steps:
Solve each inequality separately.
Represent each solution graphically.
Shape the required region.:
  AND : The common region will give the solution of x.
  OR : Any region marked in step (2) will make up the solutions of x.
State the final answer.

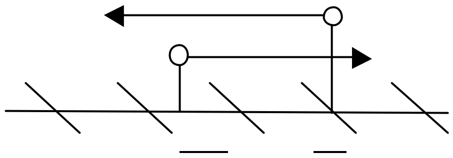
* usually, the answer can be simplified so that it does not contain "AND" or "OR".

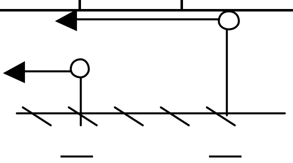
E.g. 3	Solve $x+1>4$ and $2(x-5)<30$			Explanation
Solution	$x+1>4$ $x>3$	and	$2(x-5)<30$ $2x<30+10$ <40 $x<20$	Solve each inequality separately
	 $3<x<20$			Represent the solution graphically and shading. State the final answer.

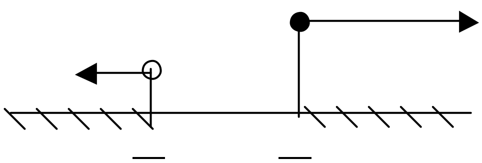
E.g. 4	Solve $2(x-5)<30$ and $3(x-5)<15$			Explanation
Solution	$2(x-5)<30$ $2x<30+2(\_\_\_)$ $<\_\_\_$ $x<\_\_\_$	and	$3(x-5)<15$ $3x<15+3(\_\_\_)$ $<\_\_\_$ $x<\_\_\_$	Solve each inequality separately
	 $x<\_\_$			represent the solution graphically and shading. State the final answer.

E.g. 5	Solve $x+1<2(5-x)<-10$			Explanation
Solution	$x+1<2(5-x)$ $x+\_\_x<2(5)-\_$ $\_\_x<\_\_$ $x<\_\_$	and	$2(5-x)<-10$ $-\_\_x<-10-2(5)$ $<-\_\_$ $x>\_\_$	The question is another style of writing an "AND" inequality.
	 ___ solution.			There is no common region. State the final answer.

E.g. 6	Solve $2(-x-5)<10$ or $3-3(x-6)>9$			Explanation
Solution	$2(-x-5)<10$ $-2x<10+2(\_\_)$ $<\_\_\_$ $x>-\_\_\_$	or	$3-3(x-6)>9$ $-3x>9-3-3(\_\_)$ $-3x>-\_\_\_$ $x<\_\_\_$	

	 x can be any ___ number.	State the final answer.

E.g. 7	Solve $2(x-5)<30$ or $3-(x-6)>11$			Explanation
Solution	$2(x-5)<30$ $2x<30+2(\_\_\_)$ $<\_\_\_$ $x<\_\_\_$	or	$3-(x-6)>11$ $-x>11-3-\_\_\_$ $x<\_\_\_$	
	 $x<\_\_\_$			State the final answer.

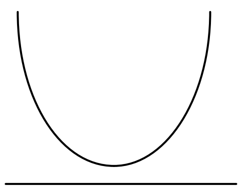
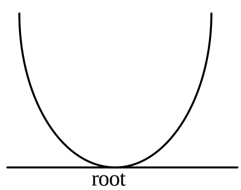
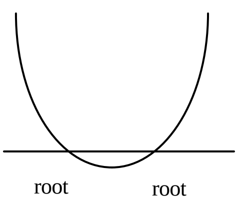
E.g. 8	Solve $2(-x-5)\le -16$ or $3-(x-6)>11$			Explanation
Solution	$2(-x-5)\le -16$ $-2x\le -16+2(\_\_\_)$ $\le \_\_\_$ $x\ge \_\_\_$	or	$3-(x-6)>11$ $-x>11-\_\_\_-6$ $x<\_\_\_$	
	 $x<\_\_\_$ or $x\ge \_\_\_$			Further simplification not possible.

5 Quadratic inequality in one variable.

Steps:

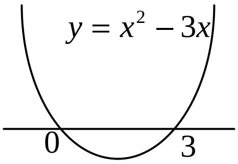
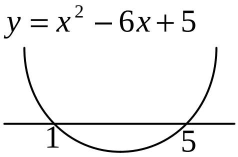
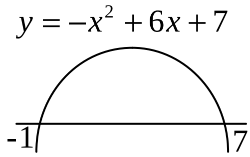
(1) Express the inequality as "ax^2+bx+c ($<,\le,>,\ge$) 0" with $a>0$.

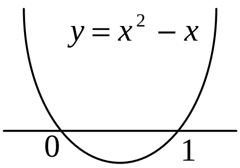
(2) Find the roots of $ax^2+bx+c=0$ and sketch the graph of $y=ax^2+bx+c$.

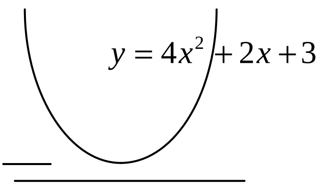
No real roots	Double root	Two distinct roots
		

(3) Read the answer from the graph.

Class practice: use step 3 to do the following.

$x^2 - 3x > 0$  Ans: $x <$ ___ or $x >$ ___	$x^2 - 6x + 5 < 0$  Ans: ___ $< x <$ ___	$-x^2 + 6x + 7 > 0$  Ans: ___ $< x <$ ___

E.g. 9	Solve $x - x^2 > 0$.	Explanation
Solution	$x - x^2 > 0$ $x^2 - x < 0$ ___$(x - 1) < 0$ ___ $< x <$ ___. 	Step 1 Step 2: roots found by factorization. Step 3: the region of x such that $y < 0$

E.g. 10	Solve $(x+1)(2x-3) \geq (2x+3)(3x-4) + 12$.	Explanation
Solution	$(x+1)(2x-3) \geq (2x+3)(3x-4) + 12$ $2x^2 - x -$ ___ $\geq 6x^2 + x -$ ___ $+ 12$ $4x^2 +$ ___ $x + 3 \leq 0$  $\Delta = 2^2 - 4($___$)($___$) =$ ___ $<$ ___ no real solution for x.	Note that there is no real root as the discriminant is less than 0. None of the curve is below 0.