Unit 6: Simultaneous equations in two unknowns

Learning Objectives

The students should be able to:

- Solve simultaneous equations in two unknowns by elimination
- Solve simultaneous equations in two unknowns by substitution
- Solve simultaneous equations (one linear and one quadratic) by the method of substitution
- Solve simultaneous equations (one linear and one quadratic) by the graphical method
- Use intersecting graphs to solve quadratic equations

Simultaneous equations in two unknowns

1. What are simultaneous equations in two unknowns?

To solve simultaneous equations is to **find solutions** that **satisfy all** the given **equations**.

Consider the following equations:

$x - y = 0$ (1)

$x^2 - 3y + 2 = 0$ (2)

$x = 3$ and $y = 3$ is a solution of (1) but not of (2). $x = 5$ and $y = 9$ is a solution of (2) but not of (1). $x = 1$ and $y = 1$ is a solution for both of (1) and (2).

2. Solving simultaneous linear equations in two unknowns by elimination

How to solve $\begin{cases} ax + by = c & (1) \\ dx + ey = f & (2) \end{cases}$

Step 1 $d(1) - a(2)$

Step 2 solve the equation for y.

Step 3 Substitute the value of y into either of the equations.

Step 4 solve the equation for x.

(This method solves y first. How do we solve x first?)

E.g. 1	Solve $\begin{cases} 2x - y = 0 \\ x - 3y = -2 \end{cases}$	Explanation
Solution	$2x - y = 0$ (1) $x - 3y = -2$ (2) (1) – 2(2): $-y - 2(-3y) = 0 - 2(-2)$ $5y = 4$ $y = 0.8_{\#}$ subs. y into (2): $x - 3(0.8) = -2$ $x = -2 + 3(0.8)$ $= 0.4_{\#}$	$d = 1$ $a = 2$ Step 1 Step 2 Step 3 Step 4

E.g. 2	Solve $\begin{cases} 2x = 3y + 2 \\ 5x - 7y = -2 \end{cases}$	Explanation
Solution	$2x = 3y + 2$ $2x - 3y = 2 \quad (1)$ $5x - 7y = -2 \quad (2)$ $5(1) - 2(2)$: $5(\_\_\_\ y) - 2(-7y) = 5(2) - 2(\_\_\_)$ $-y = \_\_\_$ $y = \_\_\__{\#}$ subs. y into (1): $2x - 3(\_\_\_) = 2$ $2x = 2 - 3(\_\_\_)$ $x = \_\_\__{\#}$	Arrange the equations to the standard form $\begin{cases} ax + by = c & (1) \\ dx + ey = f & (2) \end{cases}$ $d =$ $a =$

3. Solving simultaneous linear equations in two unknowns by substitution

How to solve $\begin{cases} ax+by=c & (1) \\ dx+ey=f & (2) \end{cases}$

Step 1 use one of the equations to express x in terms of y.

Step 2 substitute the new equation into the other equation.

Step 3 solve the equation for y.

Step 4 substitute the value of y into either of the equations.

Step 5 solve the equation for x.

Note that the last three steps of both of the methods are the same.

(This method solves y first. How do we solve x first?)

E.g. 3	Solve $\begin{cases} 2x=3y+2 \\ 5x-7y=-2 \end{cases}$	Explanation
Solution	$2x=3y+2 \quad (1)$ $5x-7y=-2 \quad (2)$ by (1): $x=1.5y+1 \quad (3)$ subs. (3) into (2) $5(1.5y+1)-7y=-2$ $7.5y-7y=-2-5$ $y=-14_{\#}$ subs. y into (1): $2x=3(-14)+2$ $=-40$ $x=-20_{\#}$	Step 1: one of the equations is (1) Step 2: the other equation is (2)

Remark:

- In general, the method of elimination is more efficient in solving simultaneous linear equations.
- the method of substitution is usually used in solving one linear and one quadratic equations.

4. Solving one linear equation and one quadratic equation by substitution

Step 1 use the linear equation to express x in terms of y.
Step 2 substitute the new equation into the quadratic equation.
Step 3 solve the quadratic equation for y.
Step 4 substitute the value of y into the new equation.
Step 5 solve the equation for x.

E.g. 4	Solve $\begin{cases} x - y = 0 \\ x^2 - 3y + 2 = 0 \end{cases}$	Explanation
Solution	$x - y = 0$ (1) $x^2 - 3y + 2 = 0$ (2) by (1): $x =$ ___ (3) subs. (3) into (2) $\text{\_\_\_}^2 - 3y + 2 = 0$ $(y - \text{\_\_\_})(y - \text{\_\_\_}) = 0$ $y =$ __, ___# subs. y into (3): when y = ___, x = 1 when y = 2, x = ___	 Step 1: use the linear equation Step 2. Step 4.

- *Summary:* the key idea is to create an equation with one unknown. Then we may solve the unknown (use the technique of the previous units) and then we may solve the other unknown.

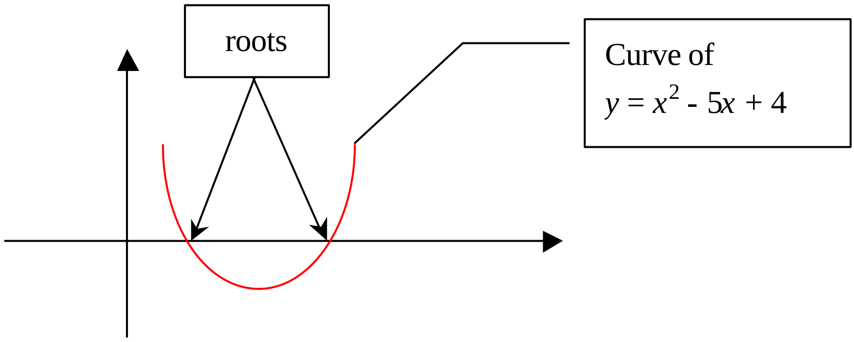
Question: $2x + 3y = 5$ and $4x^2 - 5x + 3xy = 0$. How to do the substitution?

5. Use intersecting graphs to solve quadratic equations

Basic theory:

How do we solve a quadratic equation by graphical method? For example, to solve $x^2 - 5x + 4 = 0$

We plot the graph of $y = x^2 - 5x + 4$, and the roots are x-intercepts.



On the other hand, we may say that the roots are the intersections of the curves of

$y = x^2 - 5x + 4$ (eq1)

and

$y = 0$ (the x-axis). (eq2)

How do the original equation and (eq1 and eq2) relate to each other? They are the LHS and RHS of the original equation. The important point is

Intersections of (y = LHS of the original equation) and (y = RHS of the original equation) are the roots of the original equation!

For example:

$x^2 - 5x + 4 = 0 \Leftrightarrow x^2 - 5x = -4$

The intersections of ($y = x^2 - 5x$) and ($y = -4$) gives the solutions of $x^2 - 5x + 4 = 0$ as well.

We may use a given quadratic graph to find the solutions of another quadratic equation.

E.g. 5	Given the curve of $y = x^2 - 5x + 4$, solve the following quadratic equations by drawing a suitable line a) $x^2 - 5x - 6 = 0$ b) $x^2 - x - 6 = 0$
Solution	a) $x^2 - 5x - 6 = 0$ $x^2 - 5x - 6 + 10 = 0 + 10$ $x^2 - 5x + 4 = 10$ Draw $y = 10$. The roots are – ___ and ___. a) $x^2 - x - 6 = 0$ $(x^2 - x - 6) -$ ___x + ___ = 0 - ___x + ___ $x^2 - 5x + 4 =$ - ___x + ___ Draw y = - ___x + ___. The roots are – ___, ___

y $y = x^2 - 5x + 4$