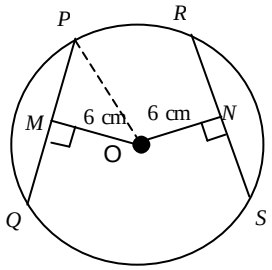
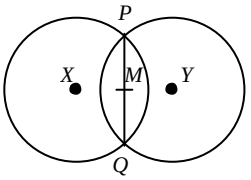
Tutorial 8 : Angles properties in cicles

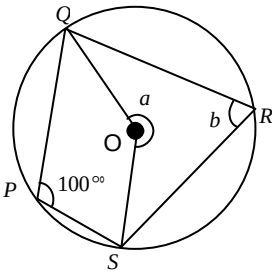
1. In the figure, $OM \perp PQ$, $ON \perp RS$, $PQ = (5x - 24)$ cm, $RS = (8 + x)$ cm and $OM = ON = 6$ cm. Find
   a) the value of x;
   b) the diameter of the circle.



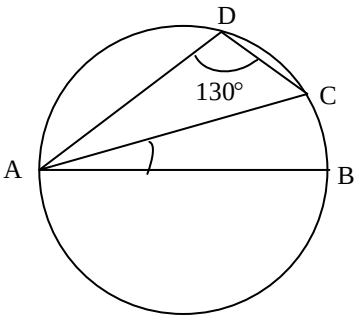
2. Two circles, having equal radii and with centres X and Y, intersect at P and Q. M is the mid-point of PQ.
   Find $\angle XMP$ and $\angle YMP$.
   If $XM = a^2$ cm and $YM = (a + 12)$ cm, find the value of a.



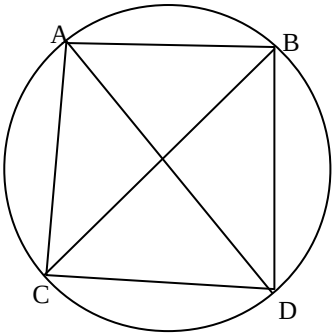
3. Find the unknowns of the following figure.



4. AB is a diameter of the circle shown below. CD are points on the circumference of the circle and $\angle ADC = 130°$.
   a) Join BD. Find $\angle BDC$.
   b) Find $\angle CAB$.



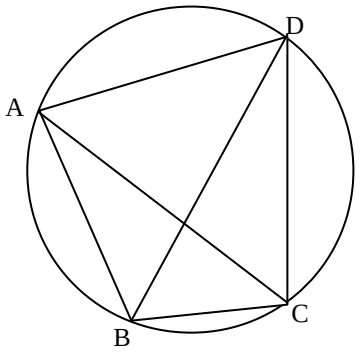
5. ABCD is a cyclic quadrilateral. $\angle BAC$ equals 101^o and $\angle ABC$ equals 51^o .
   Find $\angle ADB$.



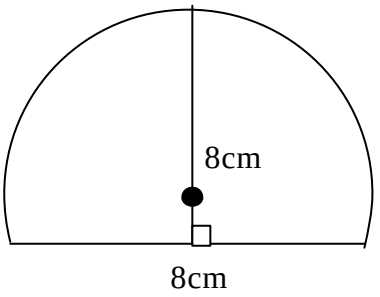
6. ABCD are points on the circumference of a circle. Given ∠BAC = 27° and ∠ADB = 46°.

   Find a) ∠BDC; and

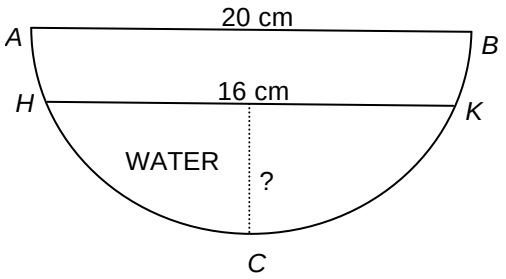
   b) ∠ABC.



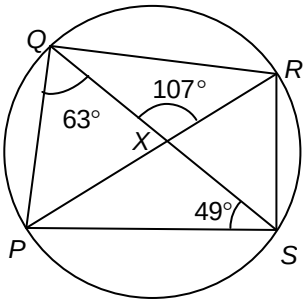
7. The cross section of a tunnel is in the shape of a segment of a circle. The road surface measures 8 m wide and the highest point of the tunnel is 8 m from the road surface. Calculate the radius of the segment.



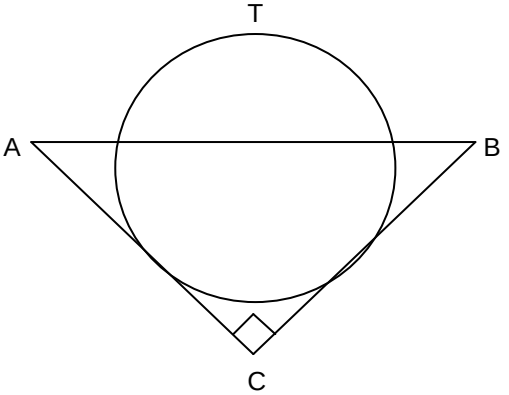
8. As shown, *ABC* is the cross section of a hemispherical bowl of diameter 20 cm. *HK*, the width of the water surface, is 16 cm. What is the greatest depth of water in the bowl?



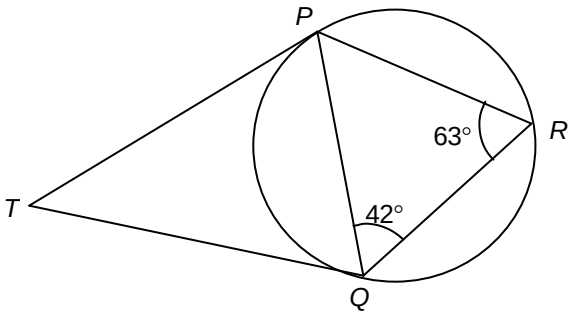
9 In the figure, *PQRS* is a cyclic quadrilateral whose diagonals *PR, QS* intersect at *X*. ∠*PSQ*=49º, ∠*PQS*=63º and ∠*QXR*=107º. Find the angles of the quadrilateral.



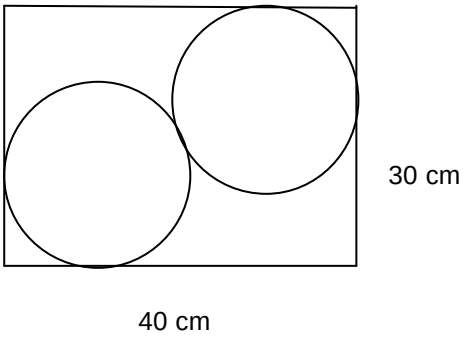
10. As shown in the diagram, a sphere of radius 5 cm rests on an upright conical funnel ACB. If the top of the sphere T is 3 cm above AB and ∠ACB=90º, calculate AB, the diameter of the funnel.



11. In the figure, TP and TQ are tangents to the circle. $\angle PQR = 42^o$ and $\angle PRQ = 63^o$, find $\angle QTP$.

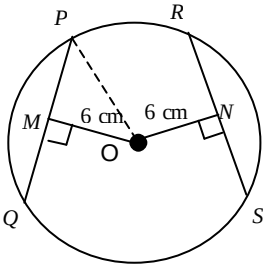


12. In the figure, two identical circles are drawn touching each other while each touches the adjacent sides of a rectangle of 40 cm by 30 cm. Find the radius of each circle.



Solution to Tutorial 8

1. In the figure, $OM \perp PQ$, $ON \perp RS$, $PQ = (5x - 24)$ cm, $RS = (8 + x)$ cm and $OM = ON = 6$ cm. Find
   a) the value of x;
   b) the diameter of the circle.



$OM = ON = 6$ cm

$PQ = RS$; *chord equidistant from centre are equal*

$5x - 24 = 8 + x$

$4x = 32$

$x = 8$

$PQ = 5x - 24$

$= 5(8) - 24$

$= 16$

$OM \perp PQ$

$PM = \frac{1}{2} PQ$; *line from centre ^ to chord bisects chord*

$= \frac{1}{2} \times 16$

$= 8$

$OP^2 = OM^2 + PM^2$; pyth. th.

$= 6^2 + 8^2$

$OP = 10$

The diameter of the circle $= 2 \times 10$

$= 20$ cm

2. Two circles, having equal radii and with centres X and Y, intersect at P and Q. M is the mid-point of PQ.
   a) Find $\angle XMP$ and $\angle YMP$
   b) If $XM = a^2$ cm and $YM = (a + 12)$ cm, find the value of a.

Join XM and YM

a) $\because$ M is the mid point of AB

$\therefore \angle XMP = \angle YMP = 90°°$; line joining centre to mid-pt.of chord $\perp$ to chord

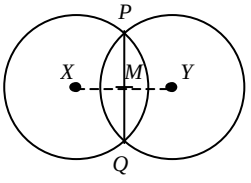
b) $\because$ $\angle XMP = \angle YMP$

$XP = YP$ *; radii*

$PM = PM$ *; common side*

$\therefore \Delta XPM \cong \Delta YPM$ *; RHS*

$XM = YM$ *; corr. sides,* $\cong \Delta$s



$a^2 = a + 12$

$a^2 - a - 12 = 0$

$(a - 4)(a + 3) = 0$

$a = 4$ or $a = -3$

3 Find the unknowns of the following figures

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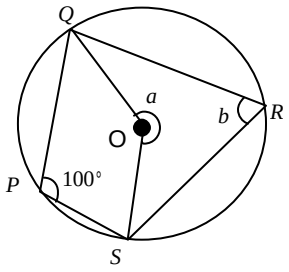


$\angle a = 2\angle QPS$; Ð at the centre twice Ð at ? ce

$= 2 \times 100$

$= 200°$

$\angle QOS = 360° - 200°$; Ðs at a pt.

$= 160°$

$\angle QOS = 2b$; Ð at the centre twice Ð at ? ce

$b = ½ \times 160°$

$= 80°$

4. a) $\angle BDC = \angle BAC = 27°$

b) $\angle ABC = 180° - (46° + 27°) = 107°$

5. $\angle BOC = 2\angle BAC$

$= 2(44°)$

$= 88°$

$\angle BDC + \angle BAC = 180°$

$\angle BDC = 180° - 44°$

$= 136°$

6. a) $\angle BDC = 130° - 90° = 40°$

b) $\angle CAB = \angle BDC = 40°$

7. Let the radius of the segment be r cm, then

$r^2 = 4^2 + (8 - r)^2$

$16r = 80$

$r = 5(cm)$

8. Let the greatest depth be x cm, then

$10^2 = 8^2 + (10 - x)^2$

$64 - 20x + x^2 = 0$

$(x - 4)(x - 16) = 10$

$x = 4$ or $x = 16(rejected)$

9. $\angle PRQ = 49°$

$\angle PRS = 63°$

$\angle R = 49° + 63° = 112°$

$\angle P = 180° - 112° = 68°$

$\angle SQR = 107° - 49° = 58°$

$\angle Q = 63° + 58° = 121°$

$\angle S = 59°$

10 $OC = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$AD = CD = 2 + \sqrt{5}$

$AB = 2(2 + \sqrt{5}) = 4 + 10\sqrt{2}$

11 $\angle TPQ = \angle TQP = 63°$

$\angle QTP = 180° - 2 \times 63° = 54°$

12. Let the radius be r, then

$(2r)^2 = (40 - 2r)^2 + (30 - 2r)^2$

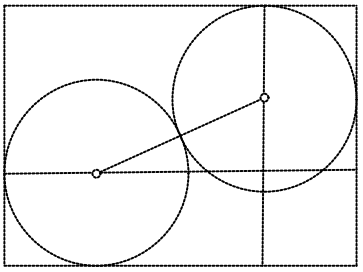
$4r^2 - 280r + 2500 = 0$

$r^2 - 70r + 625 = 0$

$\Delta = (-70)^2 - 4 \times 1 \times 625 = 2400$

$r = \dfrac{70 + \sqrt{2400}}{2} (rejected)$

$r = \dfrac{70 - \sqrt{2400}}{2}$



30 cm

40 cm