Unit 7 : Graphical Method and Bisection Method

Learning Objectives

Students should be able to

- Use graphical method to solve an equation to desired accuracy
- Use bisection method to solve an equation to desired accuracy

1. Introduction

There are equations that can be solved exactly. For example, $ax^2 + bx + c = 0$ can be solved for any values of a, b and c. On the other hand, there are lots of equations that cannot be solved by algebraic methods. For example, $x^5 - 2x^4 + 3x^3 - 4x^2 + 5x - 6 = 0$ cannot be solved exactly. The equation $2x + 1 = 3\tan x$ is another example.

In here, we introduce 2 methods to solve equations. Then we shall extend the method to solve simultaneous equations.

2. Graphical Method

Aim: To solve $f(x) = 0$ in a given interval of $a < x < b$

How:

- Define y = $f(x)$ and tabulate a table of (x, y). We need about 10 points.
- plot the points and join them *smoothly*
- find the points that $y = 0$
- the corresponding values of x are the roots of $f(x) = 0$

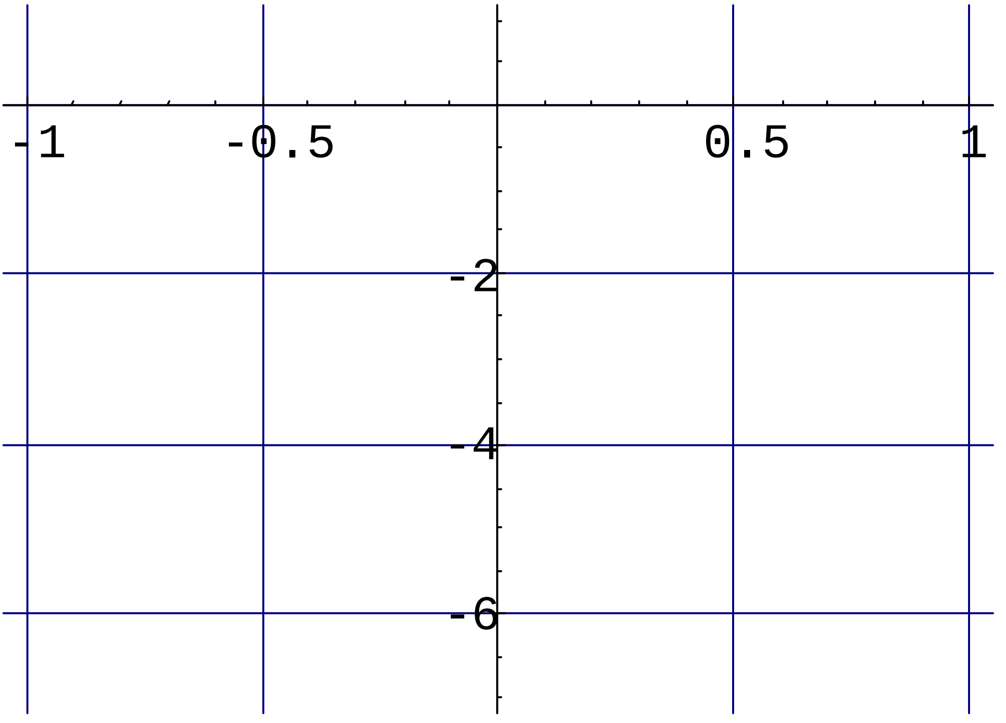
E. g. 1	Solve $x^3 - 2x^2 + 3x - 1 = 0$ by graphical method in the interval -1 to 1 (correct to 1dp).

Solution:

Let $y = x^3 - 2x^2 + 3x - 1$

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y											

[Plot your graph on graph paper and paste it on this page]



The root is at x =

Magnification:

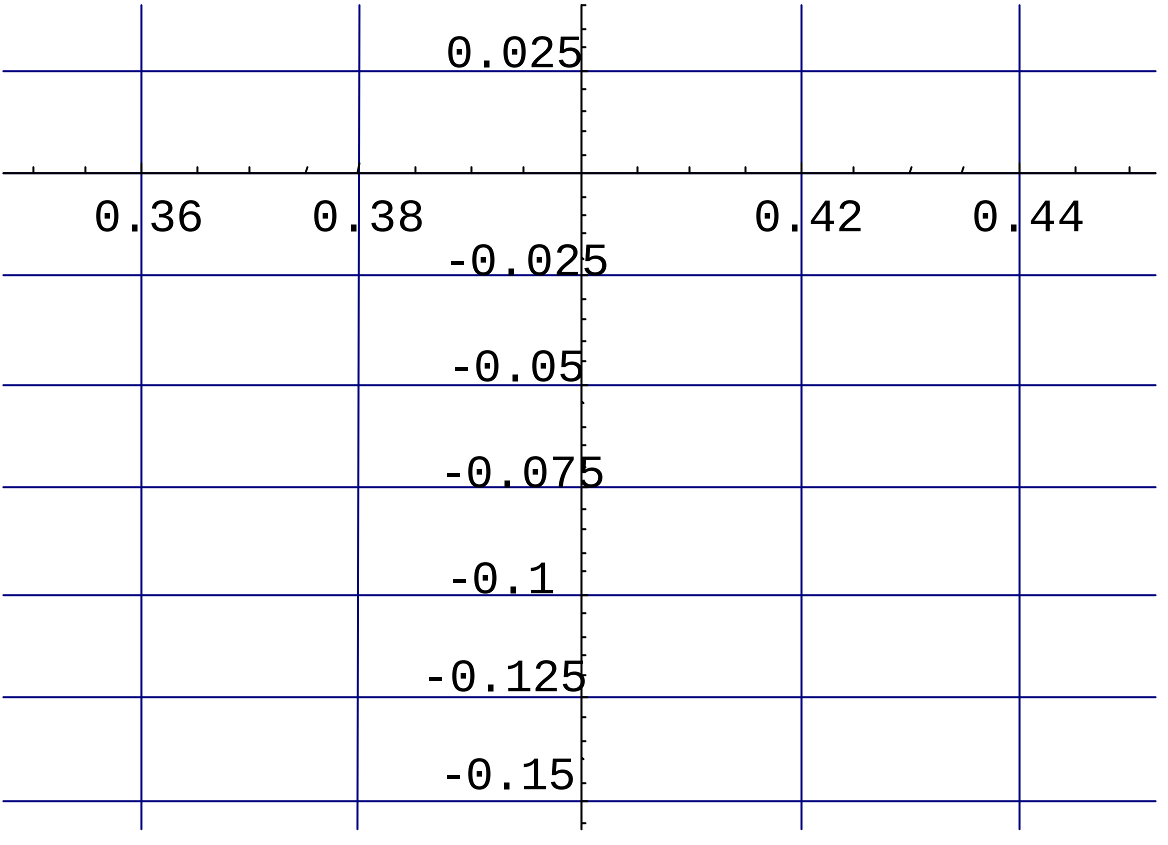
We may increase the accuracy of the roots.

E. g. 2	Use the results of the last example, find the root with 2dp accuracy.

Solution:
The root is in between 0.35 to 0.45

x	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44	0.45
y											

[Plot your graph on graph paper and paste it on this page]



The root is at x = ______ (2 dp)

- By applying magnification, we may find the location of the root correct to 3dp, 4dp… etc.

This method can be used to find multiple roots.

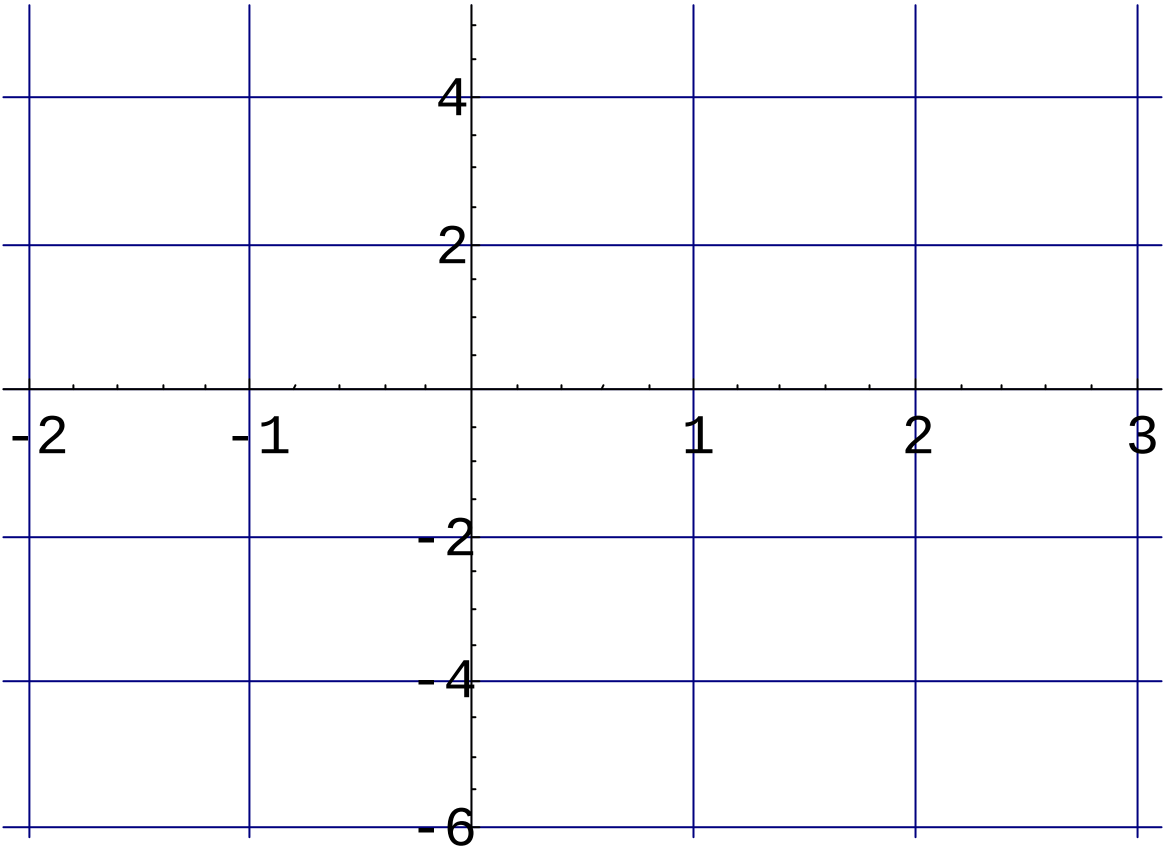
E. g. 3	Solve $x^3 - x^2 - 4x - 1 = 0$ by graphical method in the interval -2 to 3 (correct to 1dp).

Solution:

Let $y = x^3 - x^2 - 4x - 1$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y											

[Plot your graph on graph paper and paste it on this page]



The roots are –___, -___ and ___

Bisection Method

The basic idea:
If $f(a) > 0$ and $f(b) < 0$ (or vice versa), there is a root between a and b. If we approximate the root by the value of $\frac{a+b}{2}$, the precision of the root will be $\frac{b-a}{2}$. By making a and b closer and closer, we have a better and better approximation of the true root.

How:

1. Calculate $c = \frac{a+b}{2}$ and $f(c)$
2. If the sign of $f(a)$ = the sign of $f(c)$, replace a by c; If the sign of $f(b)$ = the sign of $f(c)$, replace b by c.
3. precision $= \frac{b-a}{2}$. If the current precision is not good enough, go to step 1
4. Use the values of $f(a)$ and $f(b)$ to choose the root.
   Choose the one that is closer to zero.

E. g. 4 Solve $x^3 - 2x^2 + 3x - 1 = 0$ by bisection method in the interval –1 to 1 (correct to 2dp).

Solution:

Iteration	a	$f(a)$	b	$f(b)$	precision
	-1	___	1	___	
1					1
2					0.5
3					______
4					0.125
5					0.0625
6					______
7					0.015625
8					______

$x =$ _____

- There may be many roots between a and b, bisection method can only find one of them

E. g. 5 Solve $x^3 - x^2 - 4x - 1 = 0$ by bisection method in the interval –2 to 3 (correct to 2dp).

Iteration	a	$f(a)$	b	$f(b)$ precision
	-2	_____	3	_____
1				_____
2				____
3				0.625
4				0.3125
5				____
6				0.078125
7				0.039063
8				____
				0.009766

$x =$ _____