Unit 8 : Angles Properties in Circles

Learning Objectives

The students should be able to:

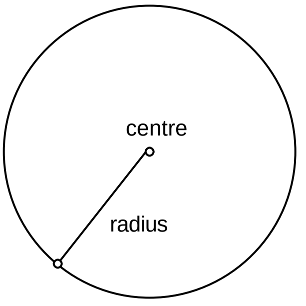
- recognize various parts of a circle.
- state the properties of chords of a circle.
- state and apply the property of angles at the centre.
- state and apply the property of angles in the same segment.
- recognize the property of angles in a semi-circle.
- explain the meaning of the concyclic points.
- state the properties of angles in a cyclic quadrilateral.
- state the definition of a tangent to a circle.
- recognize the properties of the tangents to a circle.
- state and apply the alternate segment theorem.

Circles

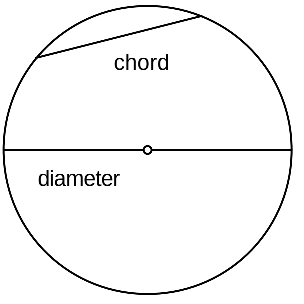
1. Parts of a circle

A circle is a closed curve in a plane such that all points on the curve are equidistant from a fixed point.

The given distance is called the radius of the circle.

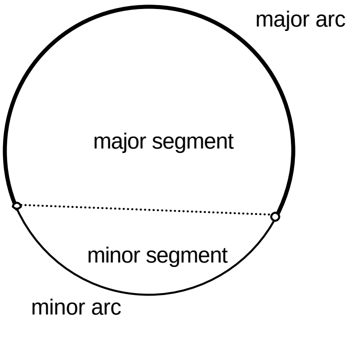


A chord is a line segment with its end points on the circle and a diameter is a chord passing through the centre.

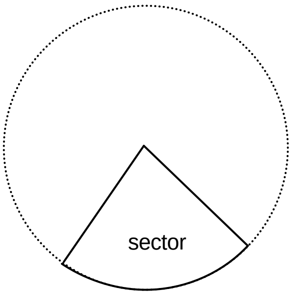


An arc is a part of the circle.

A segment is the region bounded by a chord and an arc of the circle.

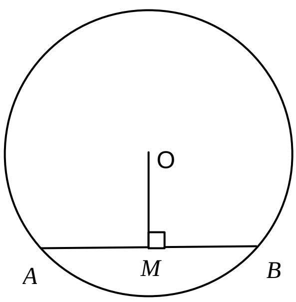
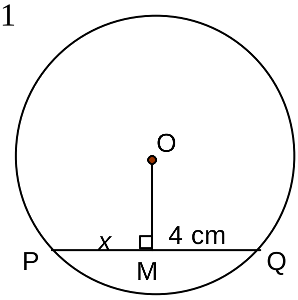
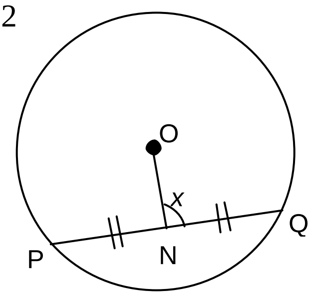
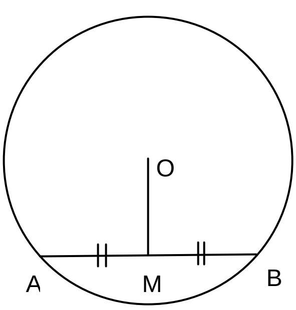
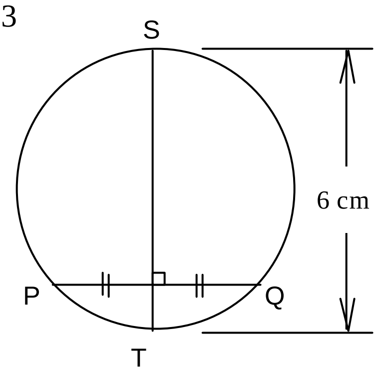
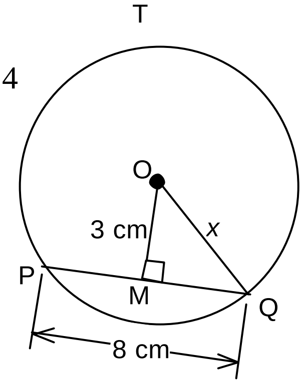


A sector is the region bounded by two radii and an arc.



2. Chords of a circle

Following are properties on chords of a circle. All these facts can be proved by the properties of congruent triangles.

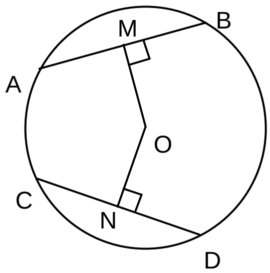
Theorem	Example
Theorem 1 The line joining the centre to the midpoint of a chord is perpendicular to the chord. i.e. If $OM \perp AB$ then $MA = MB$  Ref.: *line from centre ^ chord bisects chord*	O is the centre of the circle. Find the unknown in each of the following figures. 1.1  $x =$ <u>4 cm</u> 1.2  $\angle x =$ ____°
Theorem 2 The line joining the centre of a circle and the mid-point of a chord is perpendicular to the chord. i.e. If $MA = MB$ then $OM \perp AB$  *Ref.*: *line joining centre to mid-pt. of chord ^ chord*	1.3  $r =$ ___ cm 1.4  $x^2 = 3^2 +$ ___^{2} $x^2 =$ ___^{2} $x =$ ___ (cm)

Theorem	Example 2

Theorem 3

Equal chords are equidistant from the centre of a circle.

i.e. If $AB = CD$,
then $OM = ON$

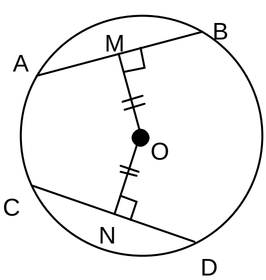


Ref.: *equal chords, equidistant from center*

Theorem 4

Chords which are equidistant from the centre of a circle are equal.

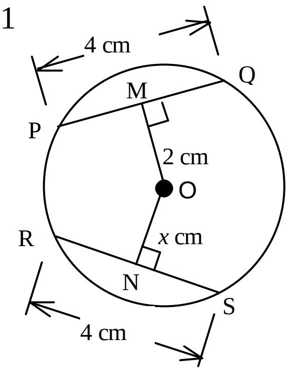
i.e. If $OM = ON$,
then AB = CD



Ref.: *chords equidistant from centre are eqaul*

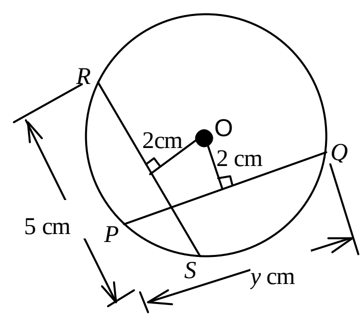
O is the centre of the circle. Find the unknown(s) in each of the following figures.

2.1



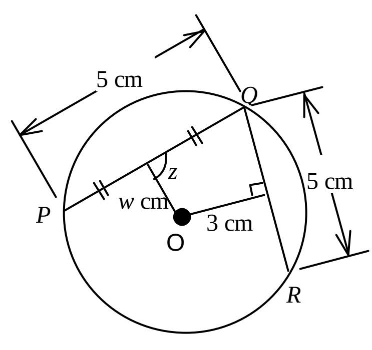
x = 2 cm

2.2



y = ____ cm

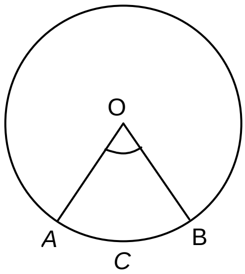
2.3

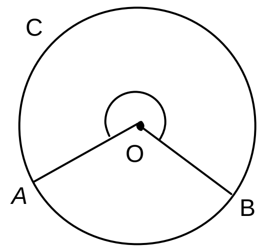


w = ____ cm, $\angle z$ = ____°

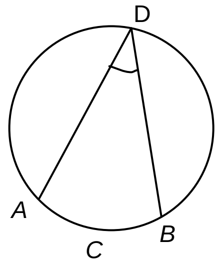
3. Angles in a circle

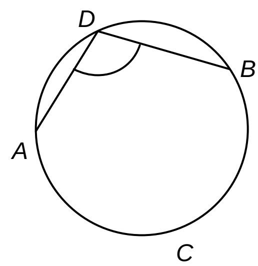
As shown in the figure, ∠*AOB* is the angle at the centre subtended by the arc *ACB*.



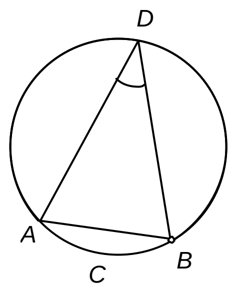


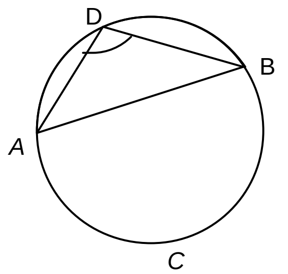
∠ADB is the angle at the circumference subtended by the arc ACB





∠ADB is also called the angle in the segment ADB.

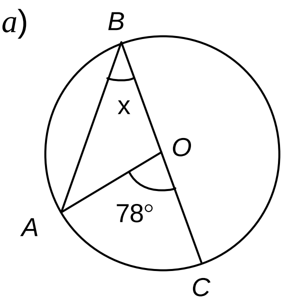




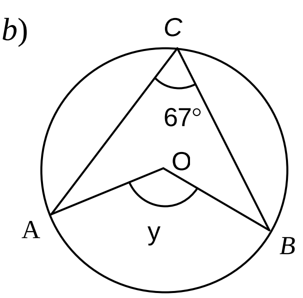
<u>Example 3.1</u>

In each of the following figures, find the angles marked:-

a)



b)



<u>Solution</u>

a) $OA = OB$

$\angle x = \angle ABO$

? $78° = 2\angle x$

or $\angle x = 39°$

b) Join *CO* and project to *D*

From *a*), $\angle y = 2 \times$ ___°

$=$ ___°

Theorem	Example 4

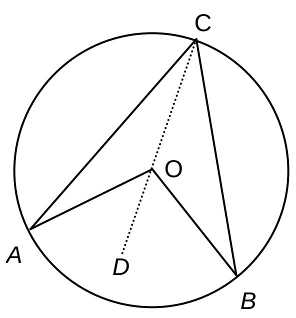
Theorem 5 (Angle at the centre theorem)

The angle that an arc of a circle subtends at the centre is twice the angle that it subtends at any point on the remaining part of the circumference.

i.e.

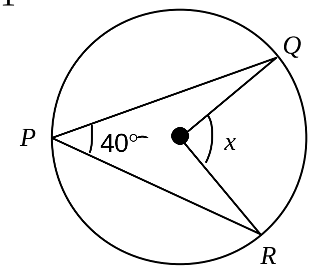
If O is the centre of the circle,

then $\angle AOB = 2\angle ACB$



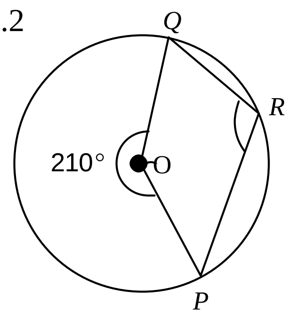
Ref.: *Ð at centre twice Ð at ?* ce

4.1



$x = 80°$

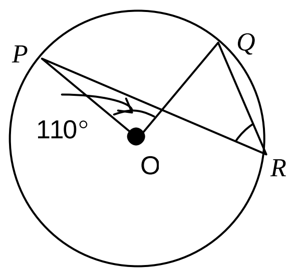
4.2



___° = $2\times\angle Q$

$\angle Q$ = ___°

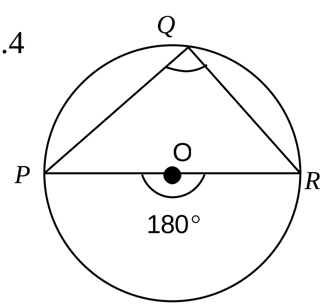
4.3



___° = $2y$

$\angle R$ = ___°

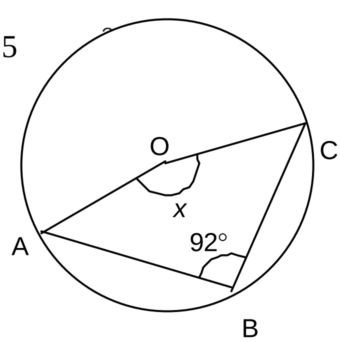
4.4



___° = $2\times\angle Q$

$\angle Q$ = ___°

4.5



____° − x

= 2 × ___°

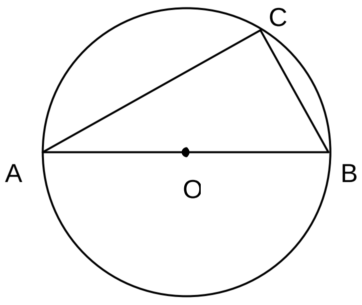
x = ____°

Theorem	Example 5

Theorem 6 (Angles in a semi-circle theorem)

The angle in a semi-circle is a right angle.

i.e. If AB is a diameter,
then $\angle ACB = 90°$.



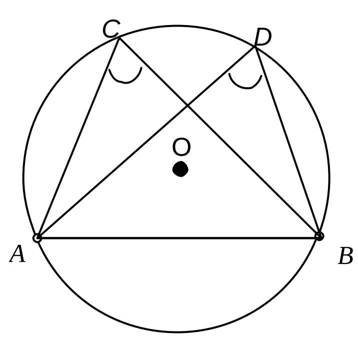
Ref.: *Ð in semi-circle*

Theorem 7 (Angle in the same segment theorem)

Angles in the same segment of a circle are equal.

i.e. If $\angle ADB$ and $\angle ACB$ are
in the same segment $ABDC$,

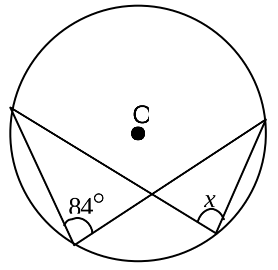
then $\angle ADB = \angle ACB$



Ref.: *Ðs in the same segment*

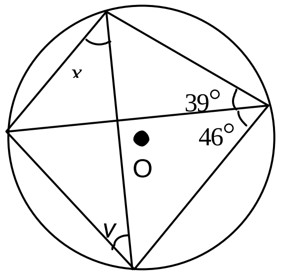
O is the centre of the circle. Find the unknown(s) in each of the following figures.

5.1



$x = 84°$

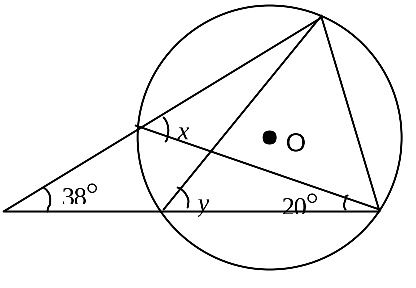
5.2



x = ____°

y = ____°

5.3



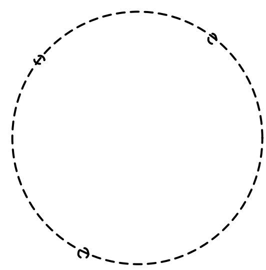
x = 20° + ____°
= ____°

y = ____
= ____°

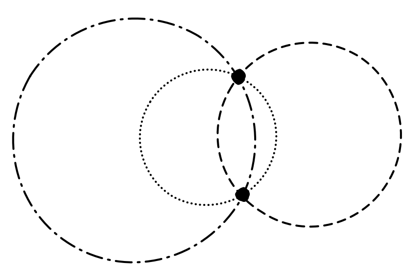
4. Cyclic quadrilaterals

4.1 Concyclic points

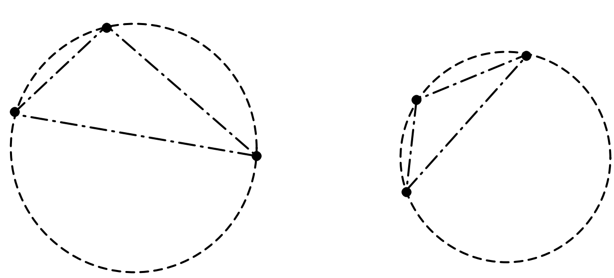
Points are concyclic if they all lie on a circle, i.e. a circle can be drawn to pass through all of them.



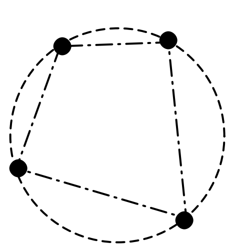
An infinite number of circles can be drawn to pass through any two points.



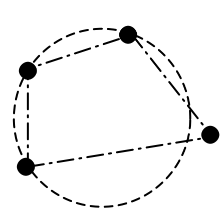
If three points are not collinear, then one and only one circle can be drawn to pass through them.



If four points are concyclic, a circle can be drawn, but if they are not concyclic, no circle can be drawn to pass through all of them.



concyclic points



non-concyclic points

4.2 Cyclic quadrilateral

There are two important facts about a cyclic quadrilateral:

i) A quadrilateral is called cyclic if a circle can be drawn to pass through all the four vertices.

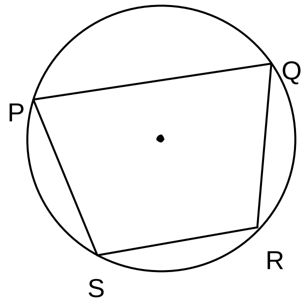
ii) All triangles are cyclic, but it is not true for quadrilateral..

Theorem	Example 6

Theorem 8

The opposite angles of a cyclic quadrilateral are supplementary.

i.e. If *P*, *Q*, *R*, *S* are concyclic,
then $\angle P + \angle R = 180°$,
and $\angle S + \angle Q = 180°$

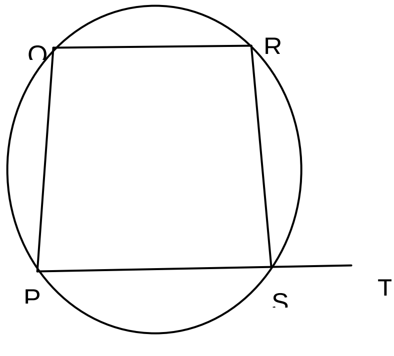


Ref.: *opp. Ðs , cyclic quad.*

Theorem 9

If one side of a cyclic quadrilateral is extended, the exterior angle equals the interior opposite angle.

i.e. If *PQRS* is a cyclic quadrilateral
and *PS* is extended to *T*,
then $\angle RST = \angle PQR$.

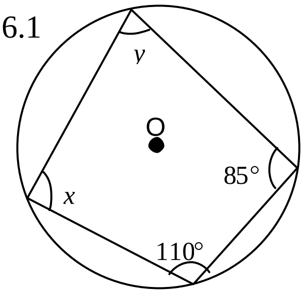


Ref.: *ext. Ð , cyclic quad.*

Example 6

O is the centre of the circle. Find the unknown(s) in each of the following figures

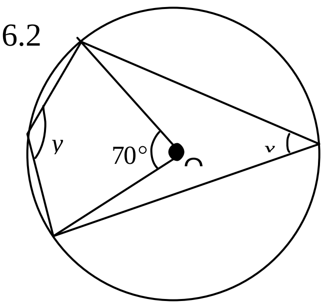
6.1



$\angle x = 95°$

$\angle y = 70°$

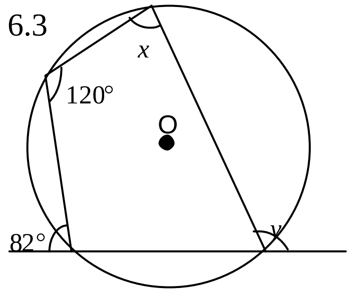
6.2



$\angle x =$ ____°

$\angle y =$ ____°

6.3



$\angle x =$ ____°

$\angle y$= ____°

5. Tangents to a circle

5.1. Definition of a tangent to a circle

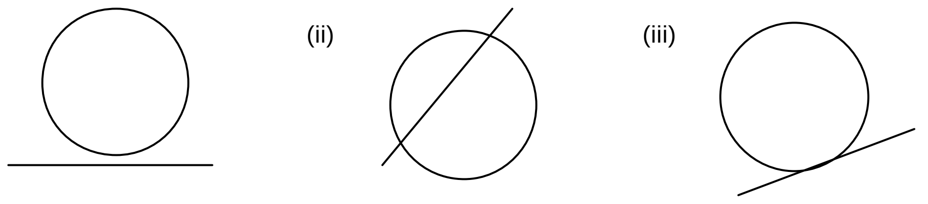
Figure 5.1 shows the three possibilities that a straight line

(i) does not intersect a circle;

(ii) intersects a circle at two points;

(iii) touches a circle (i.e. intersects at one and only point).

Fig. 5.1

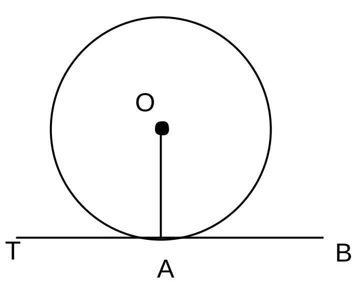


When a straight line touches a circle, it is called a tangent to the circle at that point. The following theorem states a basic property of a tangent to a circle.

Theorem 10

The tangent to a circle at a point is perpendicular to the radius at that point.

i.e. If *TAB* is a tangent at A,
then $OA \perp TA$

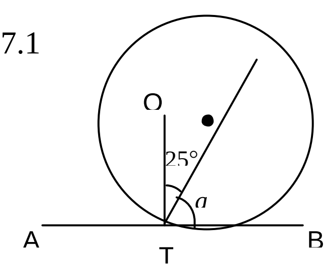


Ref.: *tangent ^ radius*

Example 7

AB is the tangent to the circle at T. Find the unknown

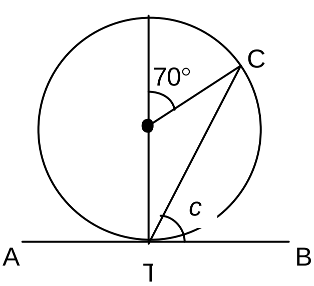
7.1



$25° + a = 90°$

$a = 55°$

7.2



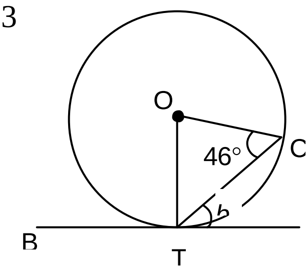
$\angle OTC = ½ \times$ ___°

= ___°

$y = 90° -$ ___°

= ___°

7.3



$OC = OT$

$\angle OTC = \angle$____

= ___°

$b =$ ___° $- \angle OTC$

= ____°

5.2. Tangents from an external point to a circle

Theorem 11

If two tangents are drawn to a circle from an external point,

a) the tangents are equal;

b) the tangents subtend equal angles at the centre;

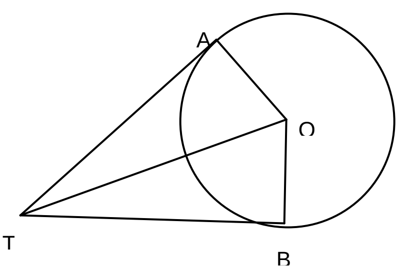
c) the line joining the external point to the centre bisects the angle between the tangents.

i.e. If TA, TB are tangents from T,

then $TA = TB$; and

$\angle TOA = \angle TOB$; and

$\angle ATO = \angle BTO$

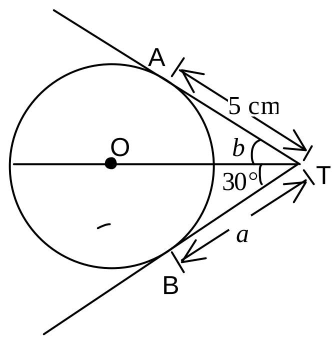


Ref.: *tangent properties*

Example 8

TA and TB are tangents to the circle at points A and B respectively. Find the unknowns.

8.1

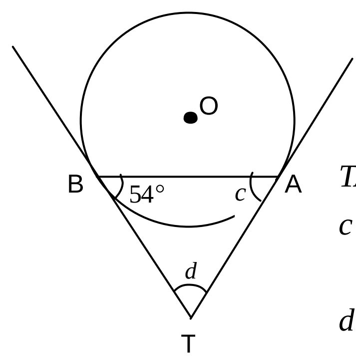


$TA = a$

$= 5$ cm

$b = \angle OTA$

8.2



$TA = TB$

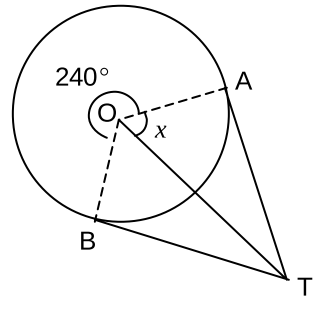
$c = \angle ABT$

$= \_\_\_\_°$

$d = 180° - 2 \times \_\_\_\_°$

$= \quad °$

8.3



$\angle TOB = e$

$2x = 360° - \_\_\_°$

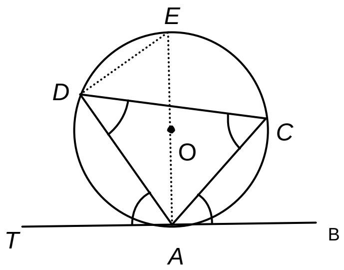
$x = \_\_\_\_°$

5.3. Alternate Segment Theorem

Theorem 12 (Alternate segment theorem)

The angles between a tangent and a chord through the point of contact are equal respectively to the angles in the alternate segment.

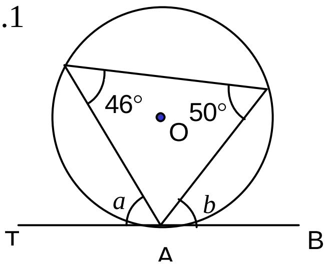
i.e. If *TAB* is a tangent at *A*,
then $\angle TAD = \angle ACD$; and
$\angle BAC = \angle ADC$



Ref.: *Ð in alt. Segment*

Example 9

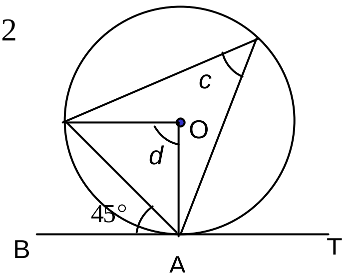
9.1



a = 50°

b = 46°

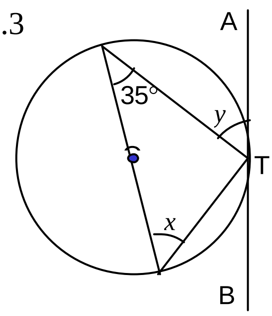
9.2



c = _____°

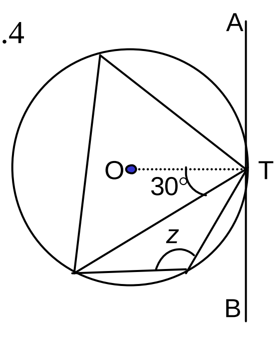
d = _____°

9.3



y = *x*
= 180° - ____° - ____°
= _____°

9.4



Z = _____° + ____°
= _____°