Individual Events

SI	h	4	I1	a	5	I2	p	3	I3	a	1000	I4	a	5	I5	a	17
	k	32		b	4		q	36		b	8		b	12		b	5
	p	3		c	10		k	12		c	16		c	4		c	23
	q	16		d	34		m	150		d	1		d	12		d	9

Group Events

SG	a	2	G6	a	150	G7	C	47	G8	A	2	G9	S	1000	G10	A	1584
	b	–3		b	10		K	2		B	3		K	98		k	14
	p	60		k	37.5		A	1		C	7		t	20		x	160
	q	136		d	6		B	5		k	9		d	5		n	15

Sample Individual Event

SI.1 Given that $3x^2 - 4x + \frac{h}{3} = 0$ has equal roots, find h.

$\Delta = (-4)^2 - 4(3)\frac{h}{3} = 0$; $h = 4$

SI.2 If the height of a cylinder is doubled and the new radius is h times the original, then the new volume is k times the original. Find k.

Let the old height be x, old radius be r, then the old volume is $\pi r^2 x$.

The new height is $2x$, the new radius is $4r$, then the new volume is $\pi(4r)^2(2x) = 32\pi r^2 x$; $k = 32$

SI.3 If $\log_{10} 210 + \log_{10}k - \log_{10}56 + \log_{10}40 - \log_{10}120 + \log_{10}25 = p$, find p.

$p = \log_{10}\left(\frac{210\times 32\times 40\times 25}{56\times 120}\right) = \log_{10} 1000 = 3$

SI.4 If $\sin A = \frac{p}{5}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find q.

$\sin A = \frac{3}{5}$; $\frac{q}{15} = \frac{\cos A}{\tan A} = \frac{\cos^2 A}{\sin A} = \frac{1-\sin^2 A}{\sin A} = \frac{1-\left(\frac{3}{5}\right)^2}{\frac{3}{5}} = \frac{16}{15}$; $q = 16$

Individual Event 1

I1.1 Find a if $2t + 1$ is a factor of $4t^2 + 12t + a$.

Let $f(t) = 4t^2 + 12t + a$; $f\left(-\frac{1}{2}\right) = 0 \Rightarrow 4\left(-\frac{1}{2}\right)^2 + 12\left(-\frac{1}{2}\right) + a = 0$; $a = 5$

I1.2 $\sqrt{K}$ denotes the nonnegative square root of K, where $K \geq 0$. If b is the root of the equation $\sqrt{a-x} = x - 3$, find b.

$\left(\sqrt{5-x}\right)^2 = (x-3)^2 \Rightarrow 5 - x = x^2 - 6x + 9 \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow x = 1$ or 4

When $x = 1$, LHS $= 2 \neq -1 =$ RHS; when $x = 4$, LHS $= 1 =$ RHS. $\therefore x = b = 4$

I1.3 If c is the greatest value of $\frac{20}{b + 2\cos\theta}$, find c.

$\frac{20}{b+2\cos\theta} = \frac{20}{4+2\cos\theta} = \frac{10}{2+\cos\theta}$; $c =$ greatest value $= \frac{10}{2-1} = 10$

I1.4 A man drives a car at $3c$ km/h for 3 hours and then $4c$ km/h for 2 hours. If his average speed for the whole journey is d km/h, find d.

Total distance travelled = $(30\times 3 + 40\times 2)$ km = 170 km

$d = \frac{170}{3+2} = 34$

Individual Event 2

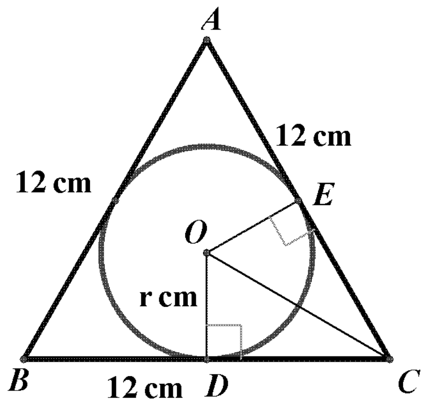
I2.1 If $0° \le \theta < 360°$, the equation in θ: $3\cos\theta + \frac{1}{\cos\theta} = 4$ has p roots. Find p.

$3\cos^2\theta + 1 = 4\cos\theta \Rightarrow 3\cos^2\theta - 4\cos\theta + 1 = 0 \Rightarrow \cos\theta = \frac{1}{3}$ or 1; $p = 3$

I2.2 If $x - \frac{1}{x} = p$ and $x^3 - \frac{1}{x^3} = q$, find q.

$x - \frac{1}{x} = 3$; $\left(x - \frac{1}{x}\right)^2 = 9 \Rightarrow x^2 + \frac{1}{x^2} = 11$; $q = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = 3(11 + 1) = 36$

I2.3 A circle is inscribed in an equilateral triangle of perimeter q cm. If the area of the circle is $k\pi$ cm^2, find k.



Let the equilateral triangle be ABC, the centre of the inscribed circle is O, which touches the triangle at D and E, with radius r cm

Perimeter = 36 cm $\Rightarrow$ Each side = 12 cm

$\angle ACB = 60°$ ($\angle$s of an equilateral Δ)

$\angle ODC = 90°$ (tangent $\perp$ radius)

$\angle OCD = 30°$ (tangent from ext. pt.)

$CD = 6$ cm (tangent from ext. pt.)

$r = 6\tan 30° = 2\sqrt{3}$

Area of circle $= \pi\left(2\sqrt{3}\right)^2$ cm^2 = 12π cm^2; $k = 12$

I2.4 Each interior angle of a regular polygon of k sides is $m°$. Find m.

Angle sum of 12-sides polygon = $180°(12 - 2) = 1800°$

Each interior angle = $m° = 1800° \div 12 = 150°$; $m = 150$

Individual Event 3

I3.1 If $998a + 1 = 999^2$, find a.

$998a = 999^2 - 1 = (999 - 1)(999 + 1) = 998\times1000$; $a = 1000$

I3.2 If $\log_{10}a = \log_2 b$, find b.

$\log_{10} 1000 = \log_2 b$

$\log_2 b = 3 \Rightarrow b = 2^3 = 8$

I3.3 The area of the triangle formed by the x-axis, the y-axis and the line $2x + y = b$ is c sq. units. Find c.

$2x + y = 8$; x-intercept = 4, y-intercept = 8

$c = \text{area} = \frac{1}{2}\cdot 4\times 8 = 16$

I3.4 If $64t^2 + ct + d$ is a perfect square, find d.

$64t^2 + 16t + d$ has a double root

$\Delta = 16^2 - 4\times64d = 0$, $d = 1$

Individual Event 4

I4.1 Solve the equation $2^{a+1} + 2^a + 2^{a-1} = 112$ in a.

$112 = 64 + 32 + 16 = 2^6 + 2^5 + 2^4$; $a = 5$

I4.2 If a is one root of the equation $x^2 - bx + 35 = 0$, find b.

One root of $x^2 - bx + 35 = 0$ is $5 \Rightarrow 5^2 - 5b + 35 = 0 \Rightarrow b = 12$

I4.3 If $\sin\theta = \frac{-b}{15}$, where $180° < \theta < 270°$, and $\tan\theta = \frac{c}{3}$, find c.

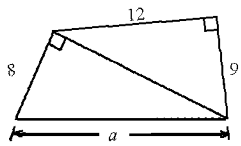
$\sin\theta = -\frac{12}{15} = -\frac{4}{5} \Rightarrow \tan\theta = \frac{4}{3} \Rightarrow c = 4$

I4.4 The probability of getting a sum of c in throwing two dice is $\frac{1}{d}$. Find d.

$P(\text{sum} = 4) = P((1,3), (2, 2), (3, 1)) = \frac{3}{36} = \frac{1}{12} = \frac{1}{d} \Rightarrow d = 12$

Individual Event 5

I5.1 In the figure, find a.



$a^2 = 12^2 + 9^2 + 8^2 = 289 = 17^2$ (Pythagoras' Theorem)

$a = 17$

I5.2 If the lines $ax + by = 1$ and $10x - 34y = 3$ are perpendicular to each other, find b.

$17x + by = 1$ is $\perp$ to $10x - 34y = 3 \Rightarrow$ product of slopes $= -1$

$-\frac{17}{b} \times \frac{10}{34} = -1 \Rightarrow b = 5$

I5.3 If the b^{th} day of May in a year is Friday and the c^{th} day of May in the same year is Tuesday, where $16 < c < 24$, find c.

5^{th} May is a Friday$\Rightarrow$ 9^{th} May is Tuesday$\Rightarrow$ 16^{th} May is Tuesday$\Rightarrow$ 23^{th} May is Tuesday; c=23

I5.4 c is the d^{th} prime number. Find d.

The first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23; 23 is the 9^{th} prime number; d=9

Sample Group Event

SG.1 The sum of two numbers is 50, and their product is 25. If the sum of their reciprocals is a, find a.

Let the 2 numbers be x, y. $x + y = 50$, $xy = 25 \Rightarrow a = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{50}{25} = 2$

SG.2 If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular, find b.

$2x + 2y + 1 = 0$ is $\perp$ to $3x + by + 5 = 0 \Rightarrow$ product of slopes $= -1$

$-\frac{2}{2} \times \frac{-3}{b} = -1 \Rightarrow b = -3$

SG.3 The area of an equilateral triangle is $100\sqrt{3}$ cm^2. If its perimeter is p cm, find p.

Let the length of one side be x cm.

$\frac{1}{2}x^2 \sin 60° = 100\sqrt{3} \Rightarrow x = 20 \Rightarrow p = 60$

SG.4 If $x^3 - 2x^2 + px + q$ is divisible by $x + 2$, find q.

Let $f(x) = x^3 - 2x^2 + 60x + q$; $f(-2) = -8 - 8 - 120 + q = 0$

$q = 136$

Group Event 6

G6.1 If $a = \dfrac{(68^3 - 65^3)\cdot(32^3 + 18^3)}{(32^2 - 32\times 18 + 18^2)\cdot(68^2 + 68\times 65 + 65^2)}$, find a.

$$a = \frac{(32+18)(32^2 - 32\times 18 + 18^2)\cdot(68-65)(68^2 + 68\times 65 + 65^2)}{(32^2 - 32\times 18 + 18^2)\cdot(68^2 + 68\times 65 + 65^2)} = 50\cdot 3 = 150$$

G6.2 If the 3 points (a, b), $(10, -4)$ and $(20, -3)$ are collinear, find b.

The slopes are equal: $\dfrac{b+4}{150-10} = \dfrac{-3+4}{20-10} \Rightarrow b = 10$

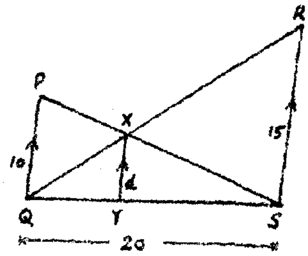
G6.3 If the acute angle formed by the hands of a clock at 4:15 is $k°$, find k.

$k = 30 + 30\times\dfrac{1}{4} = 37.5$

G6.4 In the figure, $PQ = 10$, $RS = 15$, $QS = 20$. If $XY = d$, find d.

$\dfrac{1}{d} = \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{25}{150} = \dfrac{1}{6}$

$d = 6$



Group Event 7

G7.1 2 apples and 3 oranges cost 6 dollars.

4 apples and 7 oranges cost 13 dollars.

16 apples and 23 oranges cost C dollars. Find C.

Let the cost of one apple be \$$x$ and one orange be \$$y$.

$2x + 3y = 6$ …… (1)

$4x + 7y = 13$…… (2)

(2) – 2(1): $y = 1$, $x = 1.5$

$C = 16x + 23y = 24 + 23 = 47$

G7.2 If $K = \dfrac{6\cos\theta + 5\sin\theta}{2\cos\theta + 3\sin\theta}$ and $\tan\theta = 2$, find K.

$$K = \frac{6\frac{\cos\theta}{\cos\theta} + 5\frac{\sin\theta}{\cos\theta}}{2\frac{\cos\theta}{\cos\theta} + 3\frac{\sin\theta}{\cos\theta}} = \frac{6+5\tan\theta}{2+3\tan\theta} = \frac{6+5\times 2}{2+3\times 2} = 2$$

G7.3 and G7.4 A, B are positive integers less than 10 such that $21A104 \times 11 = 2B8016 \times 9$.

G7.3 Find A.

11 and 9 are relatively prime, $21A104$ is divisible by 9.

$2 + 1 + A + 1 + 0 + 4 = 9m \Rightarrow 8 + A = 9m \Rightarrow A = 1$

G7.4 Find B.

$2B8016$ is divisible by 11.

$2 + 8 + 1 - (B + 0 + 6) = 11n \Rightarrow 11 - (B + 6) = 11n \Rightarrow B = 5$

Group Event 8

In the multiplication shown, the letters A, B, C and K ($A < B$) represent different integers from 1 to 9.

$$\begin{array}{r} A\ \ C \\ \times)\ \ B\ \ C \\ \hline K\ \ K\ \ K \end{array}$$

G8.1 Find A.

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$

Possible $K = 1, 4, 5, 6, 9$

$100K + 10K + K = 111K = 3\times37K$, 37 is a prime number

Either $10A + C$ or $10B + C$ is divisible by 37

$10B + C = 37$ or 74

When $B = 3$, $C = 7$, $K = 9$

$999 \div 37 = 27$

$\therefore A = 2$

G8.2 Find B.

$B = 3$

G8.2 Find C.

$C = 7$

G8.4 Find K.

$K = 9$

Group Event 9

G9.1 If $S = ab - 1 + a - b$ and $a = 101$, $b = 9$, find S.

$S = (a - 1)(b + 1) = 100\times10 = 1000$

G9.2 If $x = 1.9\dot{8}\dot{9}$ and $x - 1 = \frac{K}{99}$, find K.

$x = 1.9 + \frac{89}{990}$

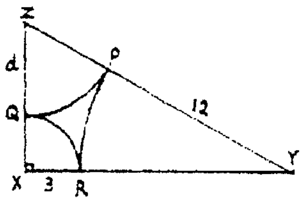
$x - 1 = \frac{K}{99} = \frac{9}{10} + \frac{89}{990} = \frac{9\times99+89}{990} = \frac{980}{990} = \frac{98}{99}$; $K = 98$

G9.3 The average of p, q and r is 18. The average of $p + 1$, $q - 2$, $r + 3$ and t is 19. Find t.

$\frac{p+q+r}{3} = 18 \Rightarrow p + q + r = 54$

$\frac{p+1+q-2+r+3+t}{4} = 19 \Rightarrow p + q + r + 2 + t = 76 \Rightarrow 54 + 2 + t = 76$; $t = 20$

G9.4 In the figure, $\overset{\frown}{QR}$, $\overset{\frown}{RP}$, $\overset{\frown}{PQ}$ are 3 arcs, centres at X, Y and Z respectively, touching one another at P, Q and R. If $ZQ = d$, $XR = 3$, $YP = 12$, $\angle X = 90°$, find d.



$XZ = 3 + d$, $XY = 3 + 12 = 15$, $YZ = 12 + d$

$XZ^2 + XY^2 = YZ^2$ (Pythagoras' Theorem)

$(3 + d)^2 + 15^2 = (12 + d)^2$

$9 + 6d + d^2 + 225 = 144 + 24d + d^2$

$18d = 90 \Rightarrow d = 5$

Group Event 10

G10.1 If $A = 1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \dots + 97 + 98 - 99$, find A.

$A = (1 + 2 - 3) + (4 + 5 - 6) + (7 + 8 - 9) + \dots + (97 + 98 - 99)$

$A = 0 + 3 + 6 + \dots + 96 = \dfrac{3+96}{2} \times 32 = 99\times 16 = 1584$

G10.2 If $\log_{10}(k - 1) - \log_{10}(k^2 - 5k + 4) + 1 = 0$, find k.

$10(k - 1) = k^2 - 5k + 4$

$k^2 - 15k + 14 = 0$

$k = 1$ or 14

When $k = 1$, LHS is undefined, $\therefore$ rejected

When $k = 14$, LHS $= \log_{10} 13 - \log_{10}(14 - 1)(14 - 4) + 1 =$ RHS

$\therefore k = 14$

G10.3 and G10.4 One interior angle of a convex n-sided polygon is $x°$. The sum of the remaining interior angles is $2180°$.

G10.3 Find x.

$2180 + x = 180(n - 2)$ ($\angle$s sum of polygon)

$2160 + 20 + x = 180\times 12 + 20 + x = 180(n - 2)$

$\because x < 180$

$\therefore 20 + x = 180; x = 160$

G10.4 Find n.

$n - 2 = 12 + 1$

$n = 15$