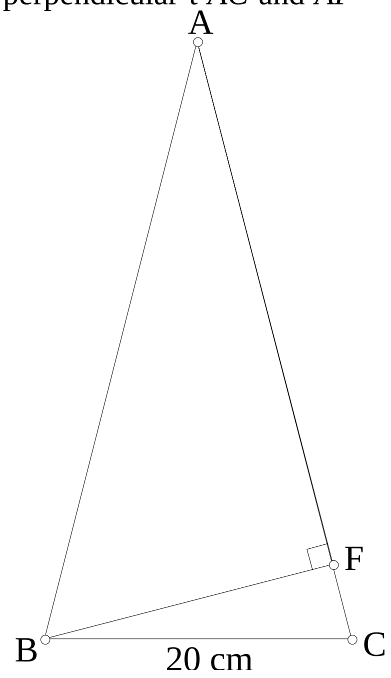
91-92 Individual	1	100	2	3	3	4	4	60	5	20
	6	C	7	±10	8	4	9	5	10	16
	11	12	12	35	13	1620	14	32	15	128
	16	3	17	10	18	$\frac{9}{10}$	19	8	20	$-\frac{4}{3}$

91-92 Group	1	102	2	–1	3	52	4	191	5	5
	6	10	7	42	8	3	9	1	10	7

Individual

12. In the figure, $AB = AC = 2BC$ and $BC = 20$ cm. If BF is perpendicular t AC and $AF = x$ cm, find x



Let $\angle ABC = \theta = \angle ACB$ (base $\angle$s isos. Δ)

$AB = AC = 40$

$$\cos\theta = \frac{\frac{1}{2}BC}{AC} = \frac{10}{40} = \frac{1}{4}$$

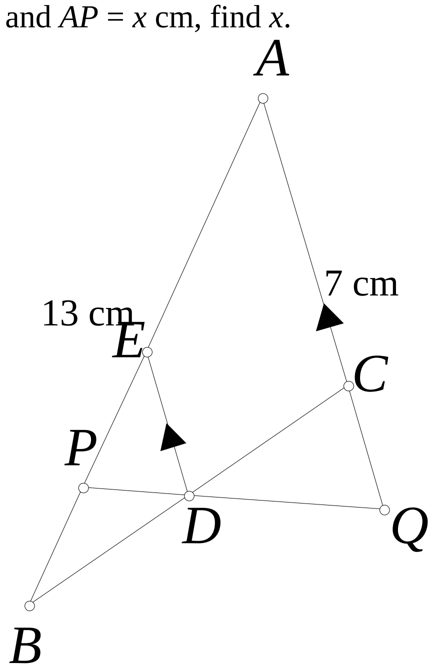
$$CF = BC\cos\theta = 20\times\frac{1}{4} = 5$$

$AF = AC - CF = 40 - 5 = 35$ cm

$x = 35$

Group

6. In the figure, $BD = DC$, $AP = AQ$. If $AB = 13$ cm, $AC = 7$ cm and $AP = x$ cm, find x.



From D, draw a parallel line $DE \parallel QA$

$\because D$ is the mid-point of BC.

$\therefore BE = EA$ (equal intercepts)

$= 13 \div 2 = 6.5$ cm

$DE = 7 \div 2 = 3.5$ cm (mid-point theorem on ΔABC)

$\angle APQ = \angle AQP$ (base $\angle$s. isos. Δ, $AP = AQ$)

$= \angle EDP$ (corr. $\angle$s, $AQ \parallel ED$)

$\therefore PE = DE$ (side opp. equal $\angle$s)

$= 3.5$ cm

$AP = AE + EP = 6.5 + 35 = 10$ cm

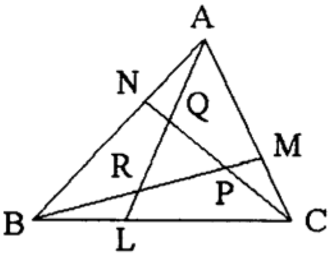
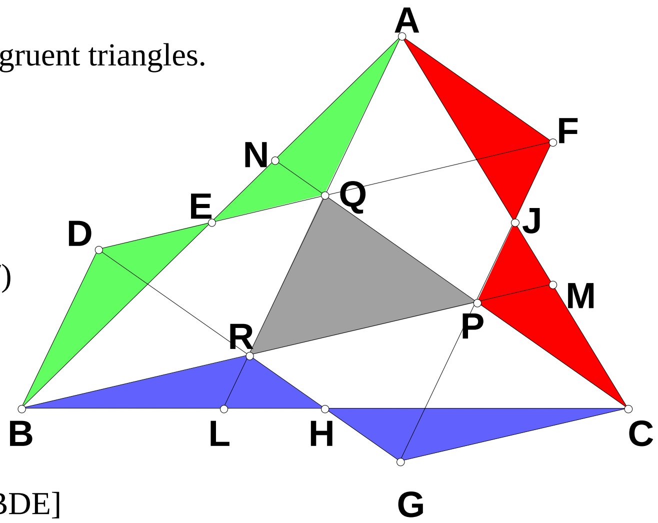


figure 2

7. In the figure, $BL = \frac{1}{3}BC$, $CM = \frac{1}{3}CA$ and $AN = \frac{1}{3}AB$. If the areas of ΔPQR and ΔABC are 6 cm^2 and x cm^2 respectively, find x.

Denote [ABC] = area of triangle ABC.
Draw BEQF // BM // GC, GHRD // CN // FA, FJPG // AL // DB as shown.
AC intersects GF at J, BC intersects DG at H, AB intersects DF at E.
Then AQPF, QRGP are congruent parallelograms.
BDQR, RQFP are congruent parallelograms. CGRP, PRDQ are congruent parallelograms.
AQF, PFQ, QRP, RQD, DBR, GPR, PGC are congruent triangles.



Consider triangles AFJ and CPJ:
AF = QP opp. sides of //-gram
= RG opp. sides of //-gram
= PC opp. sides of //-gram
AF // PC by construction
AFCP is a parallelogram (Two sides are eq. and //)
AJ = JC diagonal of a //-gram
$\angle$AJF = $\angle$CJP vert. opp. $\angle$s
$\angle$AFJ = $\angle$CPJ alt. $\angle$s AF // PC
$\therefore \Delta$AFJ $\cong$ ΔCPJ (AAS)
Areas [CPJ] = [AFJ]
In a similar manner, [BRH] = [CGH], [AQE] = [BDE]
[ABC] = [PQR] + [AQC] + [CPQ] + [BRA]
= [PQR] + [AQPF] + [CPRG] + [BRQD]
= 7 [PQR] ($\because$ they are congruent triangles, so areas equal)
= $7 \times 6 = 42$

method 2

By considering the areas of ΔACL and ΔABL

$$\frac{\frac{1}{2}AC \cdot AL \sin\angle CAL}{\frac{1}{2}AB \cdot AL \sin\angle BAL} = \frac{2}{1} \Rightarrow \frac{AC\sin\angle CAL}{AB\sin\angle BAL} = 2 \text{ ...... (1)}$$

By considering the areas of ΔAMR and ΔABR

$$\frac{\frac{1}{2}AM \cdot AR \sin\angle CAL}{\frac{1}{2}AB \cdot AR \sin\angle BAL} = \frac{MR}{BR}$$

$$\frac{AM\sin\angle CAL}{AB\sin\angle BAL} = \frac{MR}{BR}$$

$$\frac{\frac{2}{3}AC\sin\angle CAL}{AB\sin\angle BAL} = \frac{MR}{BR}$$

By (1), $\frac{2}{3}\times 2 = \frac{MR}{BR} \Rightarrow \frac{MR}{BR} = \frac{4}{3}$ .... (2)

By considering the areas of ΔACN and ΔBCN

$$\frac{\frac{1}{2}AC \cdot CN \sin\angle ACN}{\frac{1}{2}BC \cdot CN \sin\angle BCN} = \frac{1}{2} \Rightarrow \frac{AC\sin\angle ACN}{BC\sin\angle BCN} = \frac{1}{2} \text{ ...... (3)}$$

By considering the areas of ΔMCP and ΔBCP

$$\frac{\frac{1}{2}CM \cdot CP \sin\angle ACN}{\frac{1}{2}BC \cdot CP \sin\angle BCN} = \frac{MP}{BP}$$

$$\frac{CM\sin\angle ACN}{BC\sin\angle BCN} = \frac{MP}{BP}$$

$$\frac{\frac{1}{3}AC\sin\angle ACN}{BC\sin\angle BCN}=\frac{MP}{BP}$$

By (3), $\frac{1}{3}\times\frac{1}{2}=\frac{MP}{BP}\Rightarrow\frac{MP}{BP}=\frac{1}{6}$ .... (4)

By (2) and (4), $MP : PR : RB = 1 : 3 : 3$

By symmetry $NQ : QP : PC = 1 : 3 : 3$ and $NR : RQ : QA = 1 : 3 : 3$

Let s stands for the area, x = area of ΔABC.

$s_{\Delta ABL} = s_{\Delta BCM} = s_{\Delta ACN} = \frac{x}{3}$

and $s_{\Delta ANQ} = s_{\Delta BLR} = s_{\Delta CMP} = \frac{1}{7}\times\frac{x}{3}=\frac{x}{21}$ ($\because NQ = QC = 1 : 6 \Rightarrow NQ = \frac{1}{7}CN$)

The total area of ΔABC: $x = s_{\Delta ABL} + s_{\Delta BCM} + s_{\Delta ACN} + s_{\Delta PQR} - 3\, s_{\Delta ANQ}$

$x = \frac{x}{3} + \frac{x}{3} + \frac{x}{3} + 6 - 3\times\frac{x}{21}$

$0 = 6 - \frac{1}{7}x$

$x = 42$

method 3 (Vector method)

Let $\overrightarrow{AC}=\vec{c}$, $\overrightarrow{AB}=\vec{b}$

Suppose $BR : RM = r : s$

By ratio formula, $\overrightarrow{AR}=\frac{r\left(\frac{2}{3}\vec{c}\right)+s\vec{b}}{r+s}$; $\overrightarrow{AL}=\frac{\vec{c}+2\vec{b}}{3}$

$\because AR \,//\, AL \therefore \frac{\frac{s}{r+s}}{\frac{2}{3}}=\frac{\frac{2r}{3(r+s)}}{\frac{1}{3}}$ (their coefficients are in proportional)

$3s = 4r$

$r : s = 3 : 4$

Suppose $BP : PM = m : n$, let $\overrightarrow{CB}=\vec{a}$

By ratio formula, $\overrightarrow{CP}=\frac{n\vec{a}+m\left(-\frac{1}{3}\vec{c}\right)}{m+n}$; $\overrightarrow{CN}=\frac{\vec{a}+2\left(-\vec{c}\right)}{3}$

$\because CP \,//\, CN \therefore \frac{\frac{n}{m+n}}{\frac{1}{3}}=\frac{-\frac{m}{3(m+n)}}{-\frac{2}{3}}$ (their coefficients are in proportional)

$6n = m$

$m : n = 6 : 1$

$\because r : s = 3 : 4$ and $m : n = 6 : 1$

$\therefore MP : PR : RB = 1 : 3 : 3$

By symmetry $NQ : QP : PC = 1 : 3 : 3$ and $NR : RQ : QA = 1 : 3 : 3$

The remaining steps are similar, so is omitted.