Individual Events

I1	a	$\frac{2}{3}$	I2	a	12	I3	P	8	I4	n	9	I5	a	6
	b	0		b	36		Q	12		b	3		b	30
	c	3		c	12		R	4		c	8		c	4
	d	–6		d	2		S	70		d	62		d	4

Group Events

G1	a	180	G2	a	1	G3	m	–3	G4	a	99999919	G5	a	10	Spare (Group)	a	4
	b	7		b	2		b	1		b	1		b	9		k	2
	c	9		c	1		c	1.6		c	2		c	55		d	8.944
	d	4		d	120		d	2		d	1891		d	16		r	$\frac{25}{24}$

Individual Event 1

I1.1 Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in a and u. Solve for a.

3(1) + (2): $\frac{11}{a}=\frac{33}{2}$; $a=\frac{2}{3}$

I1.2 Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b.

$$\begin{cases} 3aq+b=1 \\ 9ap-q+2b=1 \\ 3aq=1 \end{cases}$$, sub. (3) into (1): $1 + b = 1 \Rightarrow b = 0$

I1.3 Find c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(-2, 2)$.

The 2 points are: (4, 5) and (–2, 2). The slope is $\frac{5-2}{4-(-2)}=\frac{1}{2}$.

The line $y=\frac{1}{2}x+c$ passes through (–2, 2): $2 = -1 + c \Rightarrow c = 3$

I1.4 The solution of the inequality $x^2 + 5x - 2c \le 0$ is $d \le x \le 1$. Find d.

$x^2 + 5x - 6 \le 0 \Rightarrow (x + 6)(x - 1) \le 0$

$-6 \le x \le 1$; $d = -6$

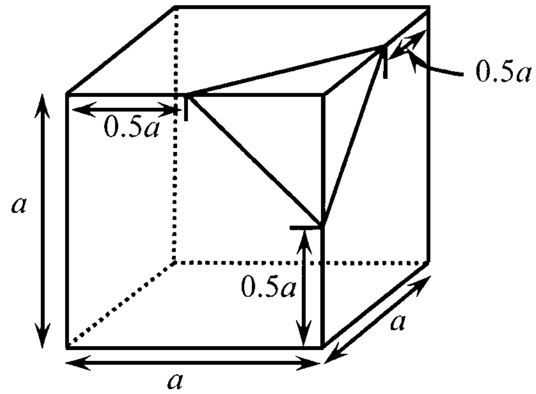
Individual Event 2

I2.1 By considering: $\frac{1^2}{1}=1$, $\frac{1^2+2^2}{1+2}=\frac{5}{3}$, $\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$, $\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$, find a such that $\frac{1^2+2^2+\cdots+a^2}{1+2+\cdots+a}=\frac{25}{3}$.

The given is equivalent to: $\frac{1^2}{1}=\frac{3}{3}$, $\frac{1^2+2^2}{1+2}=\frac{5}{3}$, $\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$, $\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=\frac{9}{3}$

and $2\times1+1=3$, $2\times2+1=5$, $2\times3+1=7$, $2\times4+1=9$; so $2a+1=25\Rightarrow a=12$

I2.2 A triangular pyramid is cut from a corner of a cube with side length a cm as the figure shown. If the volume of the pyramid is b cm^3, find b.



$b=\frac{1}{3}\text{base area}\times\text{height}=\frac{1}{3}\left(\frac{\frac{1}{2}a\times\frac{1}{2}a}{2}\right)\times\frac{1}{2}a$

$=\frac{1}{48}a^3=\frac{1}{48}\cdot12^3=36$

I2.3 If the value of x^2+cx+b is not less than 0 for all real number x, find the maximum value of c

$x^2+cx+36\ge0$

$\Delta=(4c)^2-4(36)\le0\Rightarrow c\le9$; The maximum value of $c=9$.

I2.4 If the unit digit of 19971997 is $c-d$, find d.

$9-d=7$; $d=2$

Individual Event 3

I3.1 The average of a, b c and d is 8. If the average of a, b, c, d and P is P, find P.

$\frac{a+b+c+d}{4}=8\ \Rightarrow a+b+c+d=32$

$\frac{a+b+c+d+P}{5}=P\ \Rightarrow 32+P=5P$; $P=8$

I3.2 If the lines $2x+3y+2=0$ and $Px+Qy+3=0$ are parallel, find Q.

Their slopes are equal: $-\frac{2}{3}=-\frac{8}{Q}$; $Q=12$.

I3.3 The perimeter and the area of an equilateral triangle are Q cm and $\sqrt{3}R$ cm^2 respectively. Find R.

Perimeter = 12 cm, side = 4 cm

Area $=\frac{1}{2}\cdot4^2\sin60^\circ=4\sqrt{3}$; $R=4$

I3.4 If $(1+2+\ldots+R)^2=1^2+2^2+\ldots+R^2+S$, find S.

$(1+2+3+4)^2=1^2+2^2+3^2+4^2+S$

$100=30+S$

$S=70$

Individual Event 4

I4.1 If each interior angle of a n-sided regular polygon is 140°, find n.

Each exterior angle is 40° (adj. ∠s on st. line)

$\frac{360^\circ}{n} = 40^\circ$; $n = 9$

I4.2 If the solution of the inequality $2x^2 - nx + 9 < 0$ is $k < x < b$, find b.

$2x^2 - 9x + 9 < 0$

$(2x - 3)(x - 3) < 0$

$\frac{3}{2} < x < 3 \Rightarrow b = 3$

I4.3 If $cx^3 - bx + x - 1$ is divided by $x + 1$, the remainder is -7, find c.

$f(x) = cx^3 - 3x + x - 1$

$f(-1) = -c + 3 - 1 - 1 = -7$

$c = 8$

I4.4 If $x + \frac{1}{x} = c$ and $x^2 + \frac{1}{x^2} = d$, find d.

$x + \frac{1}{x} = 8 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 64$

$x^2 + \frac{1}{x^2} + 2 = 64$; $d = 62$

Individual Event 5

I5.1 The volume of a hemisphere with diameter a cm is 18π cm^3, find a.

$\frac{1}{2} \cdot 4\pi\left(\frac{a}{2}\right)^2 = 18\pi$; $a = 6$

I5.2 If $\sin 10a^\circ = \cos(360^\circ - b^\circ)$ and $0 < b < 90$, find b.

$\sin 60^\circ = \cos(360^\circ - b^\circ)$

$360^\circ - b^\circ = 330^\circ$

$b = 30$

I5.3 The triangle is formed by the x-axis and y-axis and the line $bx + 2by = 120$. If the bounded area of the triangle is c, find c.

$30x + 60y = 120 \Rightarrow x + 2y = 4$

x-intercept = 4, y-intercept = 2

$c = \frac{1}{2} \cdot 4 \cdot 2 = 4$

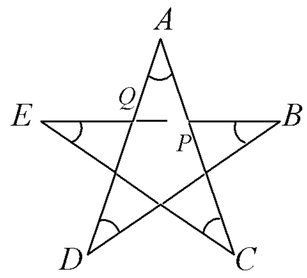
I5.4 If the difference of the two roots of the equation $x^2 - (c + 2)x + (c + 1) = 0$ is d, find d.

$x^2 - 6x + 5 = 0 \Rightarrow (x - 1)(x - 5) = 0$

$x = 1$ or $5 \Rightarrow d = 5 - 1 = 4$

Group Event 1

G1.1 In the given diagram, $\angle A + \angle B + \angle C + \angle D + \angle E = a°$, find a.



In ΔAPQ, $\angle B + \angle D = \angle AQP$ ……(1) (ext. $\angle$ of Δ)

$\angle C + \angle E = \angle APQ$ ……(2) (ext. $\angle$ of Δ)

$\angle A+\angle B + \angle C + \angle D +\angle E = \angle A + \angle AQP +\angle APQ$ (by (1) and (2))

$= 180°$ ($\angle$s sum of Δ)

$\therefore a = 180$

G1.2 There are x terms in the algebraic expression $x^6 + x^6 + x^6 + \ldots + x^6$ and its sum is x^b. Find b.

$x \cdot x^6 = x^b$

$x^7 = x^b$; $b = 7$

G1.3 If $1 + 3 + 3^2 + 3^3 + \ldots +3^8 = \frac{3^c - 1}{2}$, find c.

$\frac{3^9 - 1}{2} = \frac{3^c - 1}{2}$; $c = 9$

G1.4 16 cards are marked from 1 to 16 and one is drawn at random. If the chance of it being a perfect square number is $\frac{1}{d}$, find d.

Perfect square numbers are 1, 4, 9, 16.

Probability $= \frac{4}{16} = \frac{1}{d}$; $d = 4$.

Group Event 2

G2.1 If the sequence 1, $6 + 2a$, $10 + 5a$, … forms an A.P., find a.

$6 + 2a = \frac{1 + 10 + 5a}{2}$

$12 + 4a = 11 + 5a \Rightarrow a = 1$

G2.2 If $(0.0025\times40)^b = \frac{1}{100}$, find b.

$\left(\frac{1}{400}\times 40\right)^b = \frac{1}{100} \Rightarrow \frac{1}{10^b} = \frac{1}{10^2}$; $b = 2$

G2.3 If c is an integer and $c^3 + 3c + \frac{3}{c} + \frac{1}{c^3} = 8$, find c.

$\left(c + \frac{1}{c}\right)^3 = 8 \Rightarrow c + \frac{1}{c} = 2$

$c^2 - 2c + 1 = 0 \Rightarrow c = 1$

G2.4 There are d different ways for arranging 5 girls in a row. Find d.

First position has 5 choices; 2nd position has 4 choices, …, the last position has 1 choice.

$d = 5\times4\times3\times2\times1 = 120$

Group Event 3

G3.1 Let m be an integer satisfying the inequality $14x - 7(3x - 8) < 4(25 + x)$.
Find the least value of m.
$14x - 21x + 56 < 100 + 4x$
$-44 < 11x \Rightarrow -4 < x$; $m = -3$

G3.2 It is given that $f(x)=\frac{1}{3}x^3-2x^2+\frac{2}{3}x^3+3x^2+5x+7-4x$. If $f(-2) = b$, find b.
$f(x) = x^3 + x^2 + x + 7$
$b = f(-2) = -8 + 4 - 2 + 7 = 1$

G3.3 It is given that $\log\frac{x}{2}=0.5$ and $\log\frac{y}{5}=0.1$. If $\log xy = c$, find c.
$\log\frac{x}{2}+\log\frac{y}{5}=0.5+0.1$
$\log xy - 1 = 0.6 \Rightarrow c = \log xy = 1.6$

G3.4 Three prime numbers d, e and f which are all less than 10, satisfy the two conditions $d + e = f$ and $d < e$. Find d.
Possible prime numbers are 2, 3, 5, 7.
$\therefore d = 2, e = 3, f = 5$

Group Event 4

G4.1 It is given that $a = 103\times97\times10009$, find a.
$a = (100 + 3)(100 - 3)\times10009$
$=(10000 - 9)\times(10000 + 9) = 100000000 - 81$
$a = 99999919$

G4.2 It is given that $1 + x + x^2 + x^3 + x^4 = 0$. If $b = 2 + x + x^2 + x^3 + x^4 + \dots + x^{1989}$, find b.
$b = 1 + (1 + x + x^2 + x^3 + x^4) + x^5(1 + x + x^2 + x^3 + x^4) + \dots + x^{1985}(1 + x + x^2 + x^3 + x^4) = 1$

G4.3 It is given that m and n are two national numbers and both are not greater that 10. If c is the number of pairs of m and n satisfying the equation $mx = n$, where $\frac{1}{4}<x<\frac{1}{3}$, find c.

$\frac{1}{4}<\frac{m}{n}<\frac{1}{3} \Rightarrow \frac{n}{4}<m<\frac{n}{3}$

$\begin{cases}4m-n>0\\3m-n<0\end{cases}$

$3m < n < 4m$

$1 \le m \Rightarrow 3 \le 3m < n < 4m \le 4\times10 = 40$

Possible $n = 4, 5, 6, \dots, 10$

when $n = 4$, $\frac{4}{4}<m<\frac{4}{3}$ no solution

when $n = 5$, $\frac{5}{4}<m<\frac{5}{3}$ no solution

when $n = 6$, $\frac{6}{4}<m<\frac{6}{3}$ no solution

when $n = 7$, $\frac{7}{4}<m<\frac{7}{3} \Rightarrow m = 2, x = \frac{2}{3}$

when $n = 8$, $\frac{8}{4}<m<\frac{8}{3}$ no solution

when $n = 9$, $\frac{9}{4}<m<\frac{9}{3}$ no solution

when $n = 10$, $\frac{10}{4}<m<\frac{10}{3} \Rightarrow m = 3, x = \frac{3}{10}$

$c = 2$ (There are 2 solutions.)

G4.4 Let x and y be real numbers and define the operation * as $x*y = px^y + q + 1$. It is given that $1*2 = 869$ and $2*3 = 883$. If $2*9 = d$, find d.

$\begin{cases}p+q+1=869\\8p+q+1=883\end{cases}$

(2) – (1): $7p = 14$

$p = 2, q = 866 \Rightarrow d = 2\times2^9 + 866 + 1 = 1891$

Group Event 5

G5.1 If a is a positive multiple of 5, which gives remainder 1 when divided by 3, find the smallest possible value of a.

$a = 5k = 3m + 1$

The smallest possible $a = 10$.

G5.2 If $x^3 + 6x^2 + 12x + 17 = (x + 2)^3 + b$, find b.

$(x + 2)^3 + b = x^3 + 6x^2 + 12x + 8 + b$; $b = 9$

G5.3 If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c.

$c = 10x + y$, where $0 < x < 10$, $0 \le y < 10$.

$x + y = 10$

$xy = 25$

Solving these two equations gives $x = y = 5$; $c = 55$

G5.4 Let $S_1, S_2, \ldots, S_{10}$ be the first ten terms of an A.P., which consists of positive integers.

If $S_1 + S_2 + \ldots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \ldots + (S_3 - S_1) = d$, find d.

Let the general term be $S_n = a + (n - 1)t$

$\frac{10}{2}[2a + (10-1)t] = 55 \Rightarrow 2a + 9t = 11$

$\because a, t$ are positive integers, $a = 1$, $t = 1$

$d = (S_{10}-S_8)+(S_9-S_7)+\ldots+(S_3 - S_1) = [a + 9t - (a + 7t)] + [a + 8t - (a + 6t)] + \ldots + (a+2t-a)$

$d = 2t + 2t + 2t + 2t + 2t + 2t + 2t + 2t = 16t = 16$

Spare Group

GS1 $ABCD$ is a parallelogram and E is the midpoint of CD. If the ratio of the area of the triangle ADE to the area of the parallelogram $ABCD$ is $1 : a$, find a.

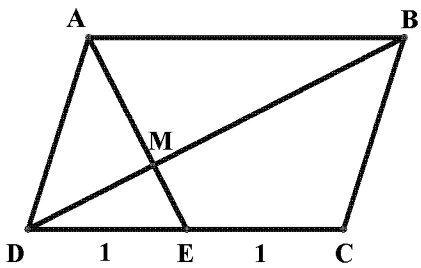
$1 : a = 1 : 4$; $a = 4$

GS2 $ABCD$ is a parallelogram and E is the midpoint of CD. AE and BD meet at M. If $DM : MB = 1 : k$, find k.

It is easy to show that $\Delta ABM \sim \Delta EDM$ (equiangular)

$DM : MB = DE : AB = 1 : 2$

$k = 2$



GS3 If the square root of 5 is approximately 2.236, the square root of 80 with the same precision is d. Find d.

$\sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5} = 4\times 2.236 = 8.944$

GS4 A square is changed into a rectangle by increasing its length by 20% and decreasing its width by 20%. If the ratio of the area of the rectangle to the area of the square is $1 : r$, find r.

Let the side of the square be x

Ratio of areas $= 1.2x\cdot 0.8x : x^2 = 0.96 : 1 = 1 : \frac{25}{24}$; $r = \frac{25}{24}$.