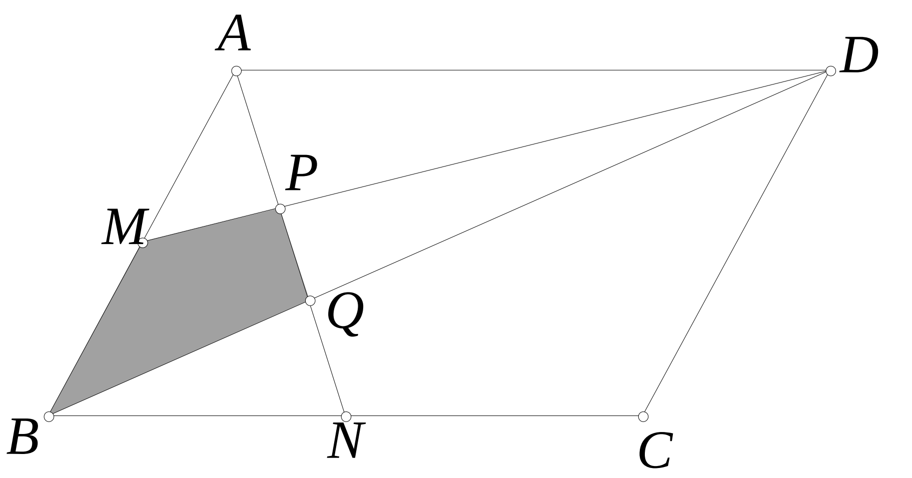
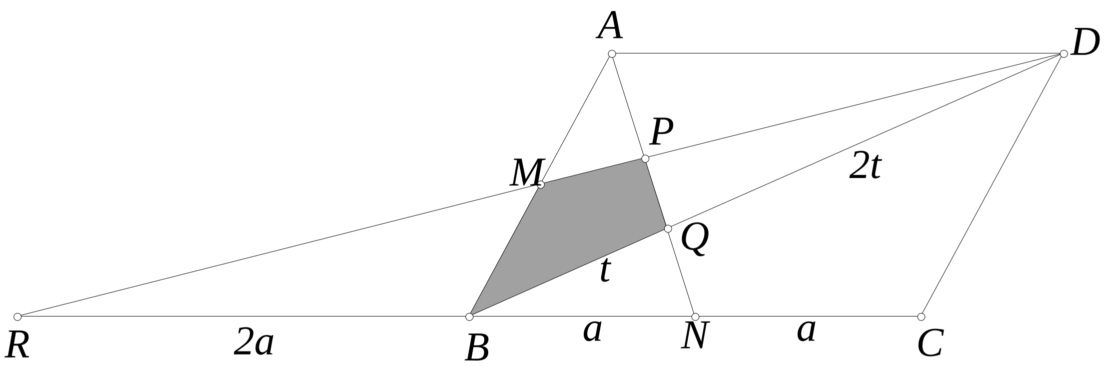
97-98 Individual	1	2	2	40	3	–12	4	3	5	$-\frac{1}{2}$
	6	466	7	19	8	3	9	2	10	744

97-98 Group	1	2	2	12	3	27	4	64	5	14
	6	14	7	$-\frac{1}{2}$	8	1	9	20	10	19

1998 Group 5

In the figure, the area of the parallelogram *ABCD* is 120, *M* and *N* are the mid-points of *AB* and *BC* respectively. Find the area of *BQPM*.





Produce *DM* and *CB* to meet at *R*. Let *BC* = 2*a*. Then *BN* = *NC* = *a* (mid-point)

$\Delta AQD \sim \Delta BQN$ (equiangular)

$\frac{BQ}{QD}=\frac{BN}{AD}$ (ratio of sides, ~Δ's)

$=\frac{1}{2}$ (*N* = mid-point, opp. sides of //-gram)

Area of ΔABD = $\frac{1}{2}\times 120 = 60$

Area of ΔAQD = $\frac{2}{3}\times \Delta \mathrm{ABD} = 40$

$$\frac{\text{Area of }\Delta\text{BQN}}{\text{Area of }\Delta\text{AQD}}=\left(\frac{BN}{AD}\right)^2=\frac{1}{4}$$

$\therefore$ Area of ΔBQN = $\frac{1}{4}\times 40$ = 10 ……..(1)

As M is the mid-point, $\Delta AMD \cong \Delta BMR$ (ASA)

$\Rightarrow$ RM = MD (corr. sides $\cong$ Δ's) ……………(2)

Also ΔAPD ~ ΔNPR (equiangular)

$\frac{DP}{PR}=\frac{AD}{NR}$ (ratio of sides, ~Δ's)

$=\frac{2a}{3a}=\frac{2}{3}$ (opp. sides of //-gram, corr. sides $\cong$ Δ's) ………..(3)

Combine (2) and (3)

$PD=\frac{2}{5}RD$; $MD=\frac{1}{2}RD$

$MP=MD-PD=\frac{1}{2}RD-\frac{2}{5}RD=\frac{1}{10}RD$

$\Rightarrow$ $\frac{MP}{PD}=\frac{\frac{1}{10}RD}{\frac{2}{5}RD}=\frac{1}{4}$ ……………..(4)

Area of ΔAMD = $\frac{1}{4}\times 120=30$

By (4):Area of ΔAMP = $\frac{1}{5}\times\text{Area of }\Delta\text{AMD}=\frac{1}{5}\times 30=6$ …………(5)

Area of ΔABN = $\frac{1}{4}\times 120=30$

$\therefore$ Area of $BQPM$ = Area of ΔABN –Area of ΔAMP –Area of ΔBQN

= 30 – 6 – 10 = 14 (by (1) and (5))