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Suggested Solution (Matrices & Determinants -- Assignment 1)

1. (a) $4A+3B$

$= 4\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} + 3\begin{pmatrix} 4 & -1 \\ -2 & 6 \end{pmatrix}$

$= \begin{pmatrix} 8 & 4 \\ 12 & -8 \end{pmatrix} + \begin{pmatrix} 12 & -3 \\ -6 & 18 \end{pmatrix}$

$= \begin{pmatrix} 20 & 1 \\ 6 & 10 \end{pmatrix}$

(b) $B-2A$

$= \begin{pmatrix} 4 & -1 \\ -2 & 6 \end{pmatrix} - 2\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}$

$= \begin{pmatrix} 4 & -1 \\ -2 & 6 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 6 & -4 \end{pmatrix}$

$= \begin{pmatrix} 0 & -3 \\ -8 & 10 \end{pmatrix}$

(c) $2A^T$

$= 2\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}^T$

$= 2\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$

$= \begin{pmatrix} 4 & 6 \\ 2 & -4 \end{pmatrix}$

(d) $(2A)^T$

$= \left[2\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}\right]^T$

$= \begin{pmatrix} 4 & 2 \\ 6 & -4 \end{pmatrix}^T = \begin{pmatrix} 4 & 6 \\ 2 & -4 \end{pmatrix}$

2 (a) $A+B^T$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 0 & 4 \\ 7 & -1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & 7 \\ 3 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & 10 \\ 7 & 9 & -2 \end{pmatrix}$$

(b) $B-2A^T$

$$= \begin{pmatrix} -2 & 3 \\ 0 & 4 \\ 7 & -1 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{pmatrix}^T$$

$$= \begin{pmatrix} -2 & 3 \\ 0 & 4 \\ 7 & -1 \end{pmatrix} - 2\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -5 \\ -4 & -6 \\ 1 & 1 \end{pmatrix}$$

3 $\dfrac{2C+5B^T}{3}$

$$= \frac{1}{3}\left[2\begin{pmatrix} -1 & 3 \\ 0 & 4 \\ 1 & 6 \end{pmatrix} + 5\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{pmatrix}^T\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix} -2 & 6 \\ 0 & 8 \\ 2 & 12 \end{pmatrix} + 5\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & -1 \end{pmatrix}\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix} -2 & 6 \\ 0 & 8 \\ 2 & 12 \end{pmatrix} + \begin{pmatrix} 5 & 20 \\ 10 & 25 \\ 15 & -5 \end{pmatrix}\right]$$

$$= \frac{1}{3}\begin{pmatrix} 3 & 26 \\ 10 & 33 \\ 17 & 7 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & \frac{26}{3} \\ \frac{10}{3} & 11 \\ \frac{17}{3} & 7/3 \end{pmatrix}$$

4. (a) AA^T

$$= \begin{pmatrix} 2 & -1 & 7 \end{pmatrix}\begin{pmatrix} 2 & -1 & 7 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & -1 & 7 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 4+1+49 \end{pmatrix} = \begin{pmatrix} 54 \end{pmatrix}$$

(b) D^TB

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T\begin{pmatrix} 9 \\ 2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} 9 \\ 2 \\ 6 \end{pmatrix} \leftarrow \text{does not exist !}$$

(c) CDB

$$= \begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3+24 & 6+30 & 9+36 \\ -3+4 & -6+5 & -9+6 \end{pmatrix}\begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 27 & 36 & 45 \\ 1 & -1 & -3 \end{pmatrix}\begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 243+72 \\ 9-2 \end{pmatrix} = \begin{pmatrix} 315 \\ 7 \end{pmatrix}$$

(d) AD^TC

$$= \begin{pmatrix} 2 & -1 & 7 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T\begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 & 7 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+21 & 8-5+42 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 45 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 63-135 & 126+45 \end{pmatrix}$$

$$= \begin{pmatrix} -72 & 171 \end{pmatrix}$$

5. $$\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} 3x+y \\ 4x+2y \end{pmatrix} = \begin{pmatrix} 9 \\ 14 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 3x+y=9 \\ 4x+2y=14 \end{cases} \Rightarrow x=2,\ y=3$$

6. In order to have the matrix $F = \begin{pmatrix} x & 3 \\ 2x & y \end{pmatrix}$ singular,

$|F|=0$, we have $xy-6x=0$

or $x(y-6)=0$

$\Rightarrow x=0$ or $y=6$

7. In order to have the matrix $A = \begin{pmatrix} 5-\lambda & -1 \\ 6 & -2-\lambda \end{pmatrix}$ to be singular,

$|A|=0$, we have $(5-\lambda)(-2-\lambda)+6=0$ or $\lambda^2-3\lambda-4=0$

$\therefore (\lambda-4)(\lambda+1)=0 \Rightarrow \lambda=4$ or -1

8. (a) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 3 & 2 \\ 2 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$

Expand in 1st column,

$= \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$

$= 1 + (2-1) = 2$

Expand in 3rd column,

$= 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$

$= 2(6-1) = 10$

(c) $\begin{pmatrix} 3 & -2 & -1 \\ 4 & 1 & 0 \\ 3 & 2 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$

Expand in the 3rd column,

$= -1 \times \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} - 1 \times \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix}$

$= -(8-3) - (3+8)$

$= -16$

Expand in the 1st row,

$= 2\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$

$= 2(4-2) - 2(2-4) + (1-4)$

$= 5$

9. (a) $\begin{pmatrix} 3 & 1 & -6 \\ 4 & 2 & 0 \\ 5 & 6 & -1 \end{pmatrix} \Rightarrow -2\begin{pmatrix} 3 & 1 & 6 \\ 2 & 1 & 0 \\ 5 & 6 & 1 \end{pmatrix} \xRightarrow{\text{row1} + (-6)\times\text{row3}} -2\begin{pmatrix} -27 & -35 & 0 \\ 2 & 1 & 0 \\ 5 & 6 & 1 \end{pmatrix}$

$\Rightarrow -2\begin{vmatrix} -27 & -35 \\ 2 & 1 \end{vmatrix} \Rightarrow -2(-27+70) = -86.$

(b) $\begin{pmatrix} 9 & 1 & 1 \\ 4 & 2 & 3 \\ 1 & 6 & 9 \end{pmatrix} \xRightarrow{\text{row2} + (-3)\text{row1}} \begin{pmatrix} 9 & 1 & 1 \\ -23 & -1 & 0 \\ 1 & 6 & 9 \end{pmatrix} \xRightarrow{\text{row3} + (-9)\text{row1}} \begin{pmatrix} 9 & 1 & 1 \\ -23 & -1 & 0 \\ -80 & -3 & 0 \end{pmatrix}$

$\Rightarrow \begin{vmatrix} -23 & -1 \\ -80 & -3 \end{vmatrix} \Rightarrow (69-80) = -11$

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(c) $\begin{pmatrix} 5 & 1 & 6 \\ 9 & 3 & 2 \\ 1 & -1 & 10 \end{pmatrix} \Rightarrow 2\begin{pmatrix} 5 & 1 & 3 \\ 9 & 3 & 1 \\ 1 & -1 & 5 \end{pmatrix} \xrightarrow{row2+(-9)row3} 2\begin{pmatrix} 5 & 1 & 3 \\ 0 & 12 & -44 \\ 1 & -1 & 5 \end{pmatrix}$

$\xRightarrow{row1+(-5)row3} 2\begin{pmatrix} 0 & 6 & -22 \\ 0 & 12 & -44 \\ 1 & -1 & 5 \end{pmatrix} \Rightarrow 2\begin{vmatrix} 6 & -22 \\ 12 & -44 \end{vmatrix} \Rightarrow 2(-264+264) = 0$

(d) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 5 & 6 & 8 \end{pmatrix} \xRightarrow{Column\,2+(-1)Col.1} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 5 & 1 & 8 \end{pmatrix} \Rightarrow -1\times\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1$

10. (a) $|A| = (18-16) = 2$

(b) $|B| = \begin{vmatrix} 9 & 24 \\ 6 & 18 \end{vmatrix} = (162-144) = 18$

12. (a) $A = \begin{pmatrix} -1 & 4 \\ 5 & 6 \end{pmatrix}$

$|A| = -6-20 = -26$ $\therefore$ $A^{-1} = \frac{-1}{26}\begin{pmatrix} 6 & -4 \\ -5 & -1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 6 & -4 \\ 2 & 3 \end{pmatrix}$

$|A| = 18+8 = 26$ $\therefore$ $A^{-1} = \frac{1}{26}\begin{pmatrix} 3 & 4 \\ -2 & 6 \end{pmatrix}$

(c) $A = \begin{pmatrix} -3 & 5 \\ 0 & 1 \end{pmatrix}$

$|A| = -3$ $\therefore$ $A^{-1} = \frac{-1}{3}\begin{pmatrix} 1 & -5 \\ 0 & -3 \end{pmatrix}$

(d) $A = \frac{1}{2}\begin{pmatrix} -7 & -2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -7/2 & -1 \\ 2 & 3/2 \end{pmatrix}$

$|A| = \frac{1}{4}(-21+8) = \frac{-13}{4}$ $\therefore$ $A^{-1} = \frac{-4}{13}\begin{pmatrix} 3/2 & 1 \\ -2 & -7/2 \end{pmatrix}$

$= \frac{2}{13}\begin{pmatrix} -3 & -2 \\ 4 & 7 \end{pmatrix}$

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13\. (a) (i) $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$, then $|A| = 2$ (from question 8(a))

To find the Adj(A), first find $A^T = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$. Then replace each element of A^T by its cofactor, we have

$a_{11} = (0+1) = 1$, $a_{12} = -(0+1) = -1$, $a_{13} = (2-1) = 1$

$a_{21} = -(0-1) = 1$, $a_{22} = (0-1) = -1$, $a_{23} = -(1-0) = -1$

$a_{31} = (0-1) = -1$, $a_{32} = -(-1-2) = 3$, $a_{33} = (1-0) = 1$

$$\therefore A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ -1 & 3 & 1 \end{pmatrix}$$

(b) Using the similar procedures as in (a),

$$A^{-1} = \frac{1}{10}\begin{pmatrix} 0 & 6 & -2 \\ 0 & -2 & 4 \\ 5 & 6 & -7 \end{pmatrix}$$

(c)

$$A^{-1} = \frac{1}{16}\begin{pmatrix} 1 & 4 & -1 \\ -4 & 0 & 4 \\ -5 & 12 & -11 \end{pmatrix}$$

(d)

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 & 2 \\ 2 & 2 & -3 \\ -3 & 2 & 2 \end{pmatrix}$$

14\. (a) $sI - A$

$$= s\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} s & -1 \\ 3 & s+2 \end{pmatrix}$$

(b)

$$|sI-A| = s(s+2) + 3$$
$$= s^2+2s+3$$

$$\therefore (sI-A)^{-1} = \frac{1}{s^2+2s+3}\begin{pmatrix} s+2 & 1 \\ -3 & s \end{pmatrix}$$

15 (a) $\begin{pmatrix} 1 & 2 & 18 \\ 0 & 1 & 7 \end{pmatrix} \Rightarrow \begin{cases} x+2y=18 \\ y=7 \end{cases} \Rightarrow \begin{array}{l} x=4 \\ y=7. \end{array}$

(b) $\begin{pmatrix} 1 & -2 & 10 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x-2y=10 \\ 0=0 \end{cases}$

As there is only one equation with two variables, we let $y=t$ (a free variable), then $x-2t=10$ or $x=10+2t$

$\therefore$ the solution is $x=10+2t$, $y=t$

(c) $\begin{pmatrix} 1 & 5 & 6 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{cases} x+5y=6 \\ 0=1 \end{cases} \Rightarrow$ no solution

(d) $\begin{pmatrix} 1 & -2 & -1 & 5 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \Rightarrow \begin{cases} x-2y-z=5 \\ y+3z=1 \\ z=1 \end{cases} \Rightarrow x=2,\ y=-2,\ z=1$

(e) $\begin{pmatrix} 1 & 3 & 4 & 2 \\ 0 & 1 & -2 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x+3y+4z=2 \\ y-2z=6 \\ 0=0 \end{cases}$

Since there are two equations with three variables, we let $z=t$ (a free variable), then

$$\begin{cases} x+3y+4t=2 & \text{— (1)} \\ y-2t=6 & \text{— (2)} \end{cases}$$

From (2) $\quad y=6+2t \quad$ — (3)

(3) → (1) $\quad x+3(6+2t)+4t=2$

$x+18+6t+4t=2$

$\therefore \quad x=-10t-16.$

$\therefore \quad x=-10t-16,\ y=2t+6,\ z=t.$

(f) $\begin{pmatrix} 1 & 3 & 6 & 25 \\ 0 & 0 & 1 & 3 \end{pmatrix} \Rightarrow \begin{cases} x+3y+6z=25 \\ z=3 \end{cases}$

$\therefore$ we let $y=t$, then $x+3t+6(3)=25$

$\therefore \quad x=7-3t$

$\therefore \quad x=7-3t,\ y=t,\ z=3$

16 (a)
$$\begin{cases} x+2y+z=6 \\ 3x-4y+2z=1 \\ 7x-6y+5z=8 \end{cases}$$

Consider the augmented matrix

$$\begin{pmatrix} 1 & 2 & 1 & 6 \\ 3 & -4 & 2 & 1 \\ 7 & -6 & 5 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 1 & 6 \\ 0 & -10 & -1 & -17 \\ 0 & -20 & -2 & -34 \end{pmatrix} \quad \begin{matrix} row2 + (-3)row1 \\ row3 + (-7)row1 \end{matrix}$$

$$= \begin{pmatrix} 1 & 2 & 1 & 6 \\ 0 & 10 & 1 & 17 \\ 0 & 10 & 1 & 17 \end{pmatrix} \quad \begin{matrix} row2 \div (-1) \\ row3 \div (-2) \end{matrix}$$

$$= \begin{pmatrix} 1 & 2 & 1 & 6 \\ 0 & 10 & 1 & 17 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad row3 + (-1)row2$$

As there are two equations with three variables, we let $z=t$, then

$$\begin{cases} x+2y+t=6 & —① \\ 10y+t=17 & —② \end{cases}$$

From ②

$$y=\frac{17-t}{10} \quad —③$$

③ → ①

$$x+\frac{2(17-t)}{10}+t=6$$

$$\therefore \quad x=\frac{13-4t}{5}$$

16(a) $x + 2y + z = 6$

$3x - 4y + 2z = 1$

$7x - 6y + 5z = 8$

By Cramer's rule

$$x = \frac{\begin{vmatrix} 6 & 2 & 1 \\ 1 & -4 & 2 \\ 8 & -6 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 2 \\ 7 & -6 & 5 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 6 & 1 \\ 3 & 1 & 2 \\ 7 & 8 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 2 \\ 7 & -6 & 5 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} 1 & 2 & 6 \\ 3 & -4 & 1 \\ 7 & -6 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 2 \\ 7 & -6 & 5 \end{vmatrix}}$$

$$= \frac{0}{0} \qquad y = \frac{0}{0} \qquad z = \frac{0}{0}$$

Since the nominator and the denominator are both zeros, in that case, many solutions exist. We have to use Gaussian Elimination to obtain the solution.

16(b) $2x - y + z = -2$

$x + 2y + 3z = -1$

$2x + 2y - z = 8$

By Cramer's rule

$$x = \frac{\begin{vmatrix} -2 & -1 & 1 \\ -8 & 2 & 3 \\ 8 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 2 & -2 & 1 \\ 1 & -8 & 3 \\ 2 & 8 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} 2 & -1 & -2 \\ 1 & 2 & -8 \\ 2 & 2 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{vmatrix}}$$

$$x = \frac{-25}{-25}, \qquad y = \frac{-50}{-25} \qquad z = \frac{50}{-25}$$

$\therefore\ x = 1,\ y = 2,\ z = -2$

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Using Gaussian Elimination, then consider the augmented matrix

$$\begin{pmatrix} 2 & -1 & 1 & -2 \\ 1 & 2 & 3 & -1 \\ 2 & 2 & -1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & 1 & -2 \\ 2 & 2 & -1 & 8 \end{pmatrix} \quad \text{interchange row1 \& row2}$$

$$= \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -5 & 0 \\ 0 & -2 & -7 & 10 \end{pmatrix} \quad \begin{matrix} \text{row2} = \text{row2} - (2 \times \text{row1}) \\ \text{row3} = \text{row3} - (2 \times \text{row1}) \end{matrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & -7 & 10 \end{pmatrix} \quad \text{row2}/-5$$

$$= \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -5 & 10 \end{pmatrix} \quad \text{row3} = \text{row3} + (2 \times \text{row2})$$

$$= \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix} \quad \text{row3}/-5$$

$$\therefore \begin{aligned} x + 2y + 3z &= -1 \\ y + z &= 0 \\ z &= -2 \end{aligned}$$

$\therefore$ $x = 1$, $y = 2$, $z = -2$

16(c)

$$2x + y - z = -3$$
$$3x + 2y + z = 6$$
$$x + y + z = 8$$

By Cramer's rule

$$x = \frac{\begin{vmatrix} -3 & 1 & -1 \\ 6 & 2 & 1 \\ 8 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 2 & -3 & -1 \\ 3 & 6 & 1 \\ 1 & 8 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} 2 & 1 & -3 \\ 3 & 2 & 6 \\ 1 & 1 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}}$$

$$x = \frac{-3}{0}, \quad y = \frac{5}{0}, \quad z = \frac{-1}{0}$$

As the denominator is zero while the nominator is non-zero, in that case, the system of equation has no solution!

16(c) Using Gaussian Elimination, the augmented matrix is

$$\begin{pmatrix} 2 & 1 & -1 & -3 \\ 3 & 2 & 1 & 6 \\ 1 & 1 & 2 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 2 & 8 \\ 3 & 2 & 1 & 6 \\ 2 & 1 & -1 & -3 \end{pmatrix}$$ interchange row1 and row3

$$= \begin{pmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & -5 & -18 \\ 0 & -1 & -5 & -19 \end{pmatrix}$$ row2 = row2 − (3×row1), row3 = row3 − (2×row1)

$$= \begin{pmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 5 & 18 \\ 0 & 1 & 5 & 19 \end{pmatrix}$$ $\frac{\text{row2}}{-1}$, $\frac{\text{row3}}{-1}$

$$= \begin{pmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 5 & 18 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ row3 = row3 − row2

From the 3rd row, we have $0x + 0y + 0z = 1$, which is impossible. ∴ the system has no solution!

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6(a) $2x+y-z=-3$

$3x+2y+z=6$

$x+y+2z=9$

By Cramer's rule, we get $x=\frac{0}{0}$, $y=\frac{0}{0}$, $z=\frac{0}{0}$. Again, the system has many solutions. Now using Gaussian Elimination, consider the augmented matrix,

$$\begin{pmatrix} 2 & 1 & -1 & -3 \\ 3 & 2 & 1 & 6 \\ 1 & 1 & 2 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 2 & 9 \\ 3 & 2 & 1 & 6 \\ 2 & 1 & -1 & -3 \end{pmatrix}$$ interchange row1 and row3

$$= \begin{pmatrix} 1 & 1 & 2 & 9 \\ 0 & -1 & -5 & -21 \\ 0 & -1 & -5 & -21 \end{pmatrix}$$ row2 = row2 − (3 × row1)
row3 = row3 − (2 × row1)

$$= \begin{pmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 5 & 21 \\ 0 & 1 & 5 & 21 \end{pmatrix}$$ $\frac{\text{row2}}{-1}$, $\frac{\text{row3}}{-1}$

$$= \begin{pmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 5 & 21 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ row3 = row3 − row2

As there are two equations with three variables, we let $z=t$,

then $\begin{cases} x+y+2t=9 & \text{--- ①} \\ \quad\; y+5t=21 & \text{--- ②} \end{cases}$

From ② $\quad y=21-5t$ --- ③

③ → ① $\quad x+(21-5t)+2t=9$

$\therefore\ x=3t-12$

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16(e)

$$3x-2y+4z=27$$
$$2x+5y+3z=-13$$
$$7x+2y-6z=-1$$

By Cramer's rule

$$x=\frac{\begin{vmatrix}27 & -2 & 4\\ -13 & 5 & 3\\ -1 & 2 & -6\end{vmatrix}}{\begin{vmatrix}3 & -2 & 4\\ 2 & 5 & 3\\ 7 & 2 & -6\end{vmatrix}},\quad y=\frac{\begin{vmatrix}3 & 27 & 4\\ 2 & -13 & 3\\ 7 & -1 & -6\end{vmatrix}}{\begin{vmatrix}3 & -2 & 4\\ 2 & 5 & 3\\ 7 & 2 & -6\end{vmatrix}},\quad z=\frac{\begin{vmatrix}3 & -2 & 27\\ 2 & 5 & -13\\ 7 & 2 & -1\end{vmatrix}}{\begin{vmatrix}3 & -2 & 4\\ 2 & 5 & 3\\ 7 & 2 & -6\end{vmatrix}}$$

$$x=\frac{-894}{-298},\quad y=\frac{1490}{-298},\quad z=\frac{-596}{-298}$$

$\therefore\ x=3 \qquad y=-5 \qquad z=2$

Using Gaussian Elimination, consider the augmented matrix.

$$\begin{pmatrix}3 & -2 & 4 & 27\\ 2 & 5 & 3 & -13\\ 7 & 2 & -6 & -1\end{pmatrix}$$

$$=\begin{pmatrix}1 & -7 & 1 & 40\\ 2 & 5 & 3 & -13\\ 7 & 2 & -6 & -1\end{pmatrix}$$ row1 = row1 − row2

$$=\begin{pmatrix}1 & -7 & 1 & 40\\ 0 & 19 & 1 & -93\\ 0 & 51 & -13 & -281\end{pmatrix}$$ row2 = row2 − (2×row1)
row3 = row3 − (7×row1)

$$=\begin{pmatrix}1 & -7 & 1 & 40\\ 0 & 19 & 1 & -93\\ 0 & 0 & -298 & -596\end{pmatrix}$$ row3 = (19×row3) − (51×row2)

$$=\begin{pmatrix}1 & -7 & 1 & 40\\ 0 & 19 & 1 & -93\\ 0 & 0 & 1 & 2\end{pmatrix}.$$

$\therefore\ z=2,\ y=-5,\ x=3.$

16 (f)

$$2x-3y-5z=14$$
$$3x+7y+8z=15$$
$$-x-7y+5z=9$$

By Cramer's rule

$$x=\frac{\begin{vmatrix}14&-3&-5\\15&7&8\\9&-7&5\end{vmatrix}}{\begin{vmatrix}2&-3&-5\\3&7&8\\-1&-7&5\end{vmatrix}},\quad y=\frac{\begin{vmatrix}2&14&-5\\3&15&8\\-1&9&5\end{vmatrix}}{\begin{vmatrix}2&-3&-5\\3&7&8\\-1&-7&5\end{vmatrix}},\quad z=\frac{\begin{vmatrix}2&-3&14\\3&7&15\\-1&-7&9\end{vmatrix}}{\begin{vmatrix}2&-3&-5\\3&7&8\\-1&-7&5\end{vmatrix}}$$

$$x=\frac{2123}{321},\quad y=\frac{-526}{321},\quad z=\frac{266}{321}$$

Using Gaussian Elimination, consider the augmented matrix.

$$\begin{pmatrix}2&-3&-5&14\\3&7&8&15\\-1&-7&5&9\end{pmatrix}$$

$$=\begin{pmatrix}1&7&-5&-9\\3&7&8&15\\2&-3&-5&14\end{pmatrix}$$ interchange row1 + row3, $\frac{\text{row1}}{-1}$

$$=\begin{pmatrix}1&7&-5&-9\\0&-14&23&42\\0&-17&5&32\end{pmatrix}$$ row2 = row2 − (3×row1)
row3 = row3 − (2×row1)

$$=\begin{pmatrix}1&7&-5&-9\\0&14&-23&-42\\0&17&-5&-32\end{pmatrix}$$ row2×(−1), row3×(−1)

$$=\begin{pmatrix}1&7&-5&-9\\0&14&-23&-42\\0&0&321&266\end{pmatrix}$$ row3 = 14×row3 − 17×row2

From the 3rd row, $z=\frac{266}{321}$, then by substitution, we get
$y=\frac{-526}{321}$ + $x=\frac{2123}{321}$