

## EDUC 250 Mathematical Analysis Homework I

Due: 14th September, 2004. Hand in before the lecture starts at 9:00 a.m.

Let  $\mathbb{F}$  be a field,  $a, b, c, d \in \mathbb{F}$ . Prove the followings:

1.  $-(-a) = a$ .
2.  $-(a - b) = b - a$ .
3.  $a \cdot (-b) = -(a \cdot b) = (-a) \cdot b$ .
4.  $(-a) \cdot (-b) = a \cdot b$ .
5.  $a/a = 1$  for all  $a \in \mathbb{F} \setminus \{0\}$ .
6.  $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$ .
7.  $(\frac{a}{b}) \cdot (\frac{c}{d}) = \frac{ac}{bd}$  for all  $b, d \in \mathbb{F} \setminus \{0\}$ .
8.  $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ .
9.  $\frac{a}{c} + \frac{b}{c} = \frac{(a + b)}{c}$ .
10.  $\frac{(-a)}{b} = \frac{a}{-b} = -\left(\frac{a}{b}\right)$ .
11.  $\frac{(a/b)}{(c/d)} = \frac{a \cdot d}{b \cdot c} = \left(\frac{a}{b}\right) \cdot \left(\frac{d}{c}\right)$ , for all  $b, c \in \mathbb{F} \setminus \{0\}$ .
12.  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ , for all  $b, d \in \mathbb{F} \setminus \{0\}$ .

## EDUC 250 Mathematical Analysis Homework II

Due: 15th September, 2004. Hand in before the lecture starts at 9:00 a.m.

1. Suppose that  $a, b$  are in an ordered field  $\mathbb{F}$ . If  $a > b$  and  $c > d$ , prove that  
(i)  $a + c > b + d$ ; (ii)  $a - b > d - c$ ; (iii)  $-b + c > -a + d$ ; (iv)  $-b - d > -a - c$ .
2. Suppose that  $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$  are elements in a ring  $\mathbb{F}$ , which is not necessarily commutative. Use the mathematical induction to prove that

$$\left(\sum_{i=1}^m a_i\right) \cdot \left(\sum_{j=1}^n b_j\right) = \sum_{i=1}^m \sum_{j=1}^n a_i b_j.$$