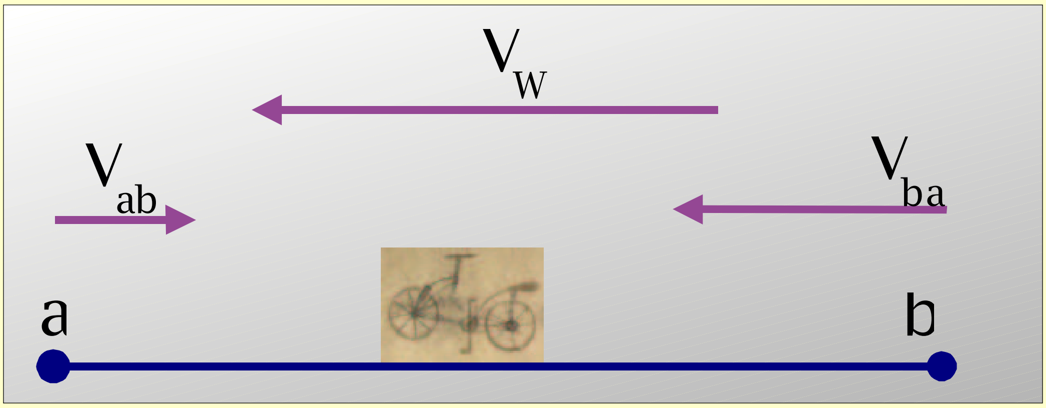
Influence of wind on the travel time

While one Holland vacation became acquainted with the national typical advantages for the cyclist. There would be for example the "fietspad". However also the national typical disadvantages did not remain saved to me. Among them the wind ranks; a completely unpleasant thing. One could be tried to talk itself merrily. Finally it gives a compensatory justice. If one drives one time against the wind, one has other time the wind in the back. Then as well known, al, holy help. But attention; that is not so simple. Nature is nonlinear.

Formally with formulas

I wanted not to let the affair be based on itself. Thus I made a formal description of the problem. The following model is the basis. It takes place a travel from point **a** to point **b** with the speed V_{ab} against the wind with the wind velocity V_W.



Subsequently, the same distance of **b** to **a** is driven, this time however with back wind and the speed V_{ba},. Both ways are thus driven with the same effort, with the same power

$$P = F\,V = \rho\,A\frac{c_w}{2}V^2 V$$

In it F is the air drag , A the cross section against the wind, c_W the air resistance factor and ρ the atmospheric density. The rolling friction is neglected here. The assumption of the same power for both ways (a→b,b→a) can be written as:

$$P_{ab} = P_{ba}$$

$$\rho A \frac{c_w}{2} (V_{ab} + V_w)^2 V_{ab} = \rho A \frac{c_w}{2} (V_{ba} - V_w)^2 V_{ba}$$

$$\Rightarrow (V_{ab} + V_w)^2 V_{ab} = (V_{ba} - V_w)^2 V_{ba}$$

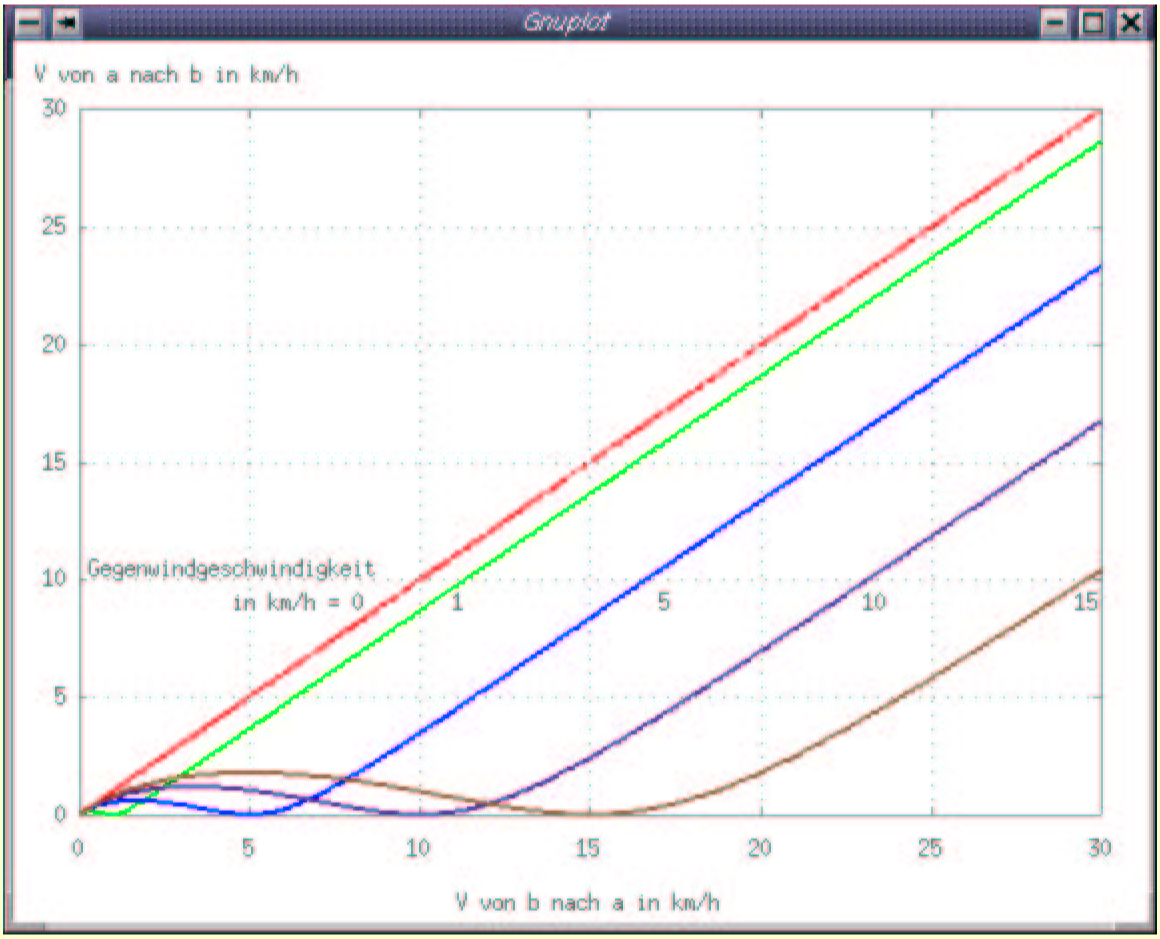
If one resolves this cubic equation to V_{ab} , the speed against the wind V_{ab} can be calculated with an assumed wind velocity and a driving speed with the wind V_{ba}.

$$\frac{1}{6}\left(8 V_w^3 + 108 V_{ba}^3 - 216 V_{ba}^2 V_w + 108 V_{ba} V_w^2 + 12\sqrt{-312 V_w^3 V_{ba}^3 + 57 V_w^4 V_{ba}^2 + 12 V_w^5 V_{ba} + 81 V_{ba}^6 - 324 V_{ba}^5 V_w + 486 V_{ba}^4 V_w^2}\right)^{\left(\frac{1}{3}\right)} + \frac{2}{3} V_w^2 \bigg/ \left(8 V_w^3 + 108 V_{ba}^3 - 216 V_{ba}^2 V_w + 108 V_{ba} V_w^2 + 12\sqrt{-312 V_w^3 V_{ba}^3 + 57 V_w^4 V_{ba}^2 + 12 V_w^5 V_{ba} + 81 V_{ba}^6 - 324 V_{ba}^5 V_w + 486 V_{ba}^4 V_w^2}\right)^{\left(\frac{1}{3}\right)} - \frac{2}{3} V_w$$

With the well−known speeds, the time needed for the travel way results. This somewhat unmanageable solution for V_{ab} can be interpreted difficult, however simply plotted. For those, which want to plot other cases, gnuplot syntax is in the appendix:

Figurativly with pictures

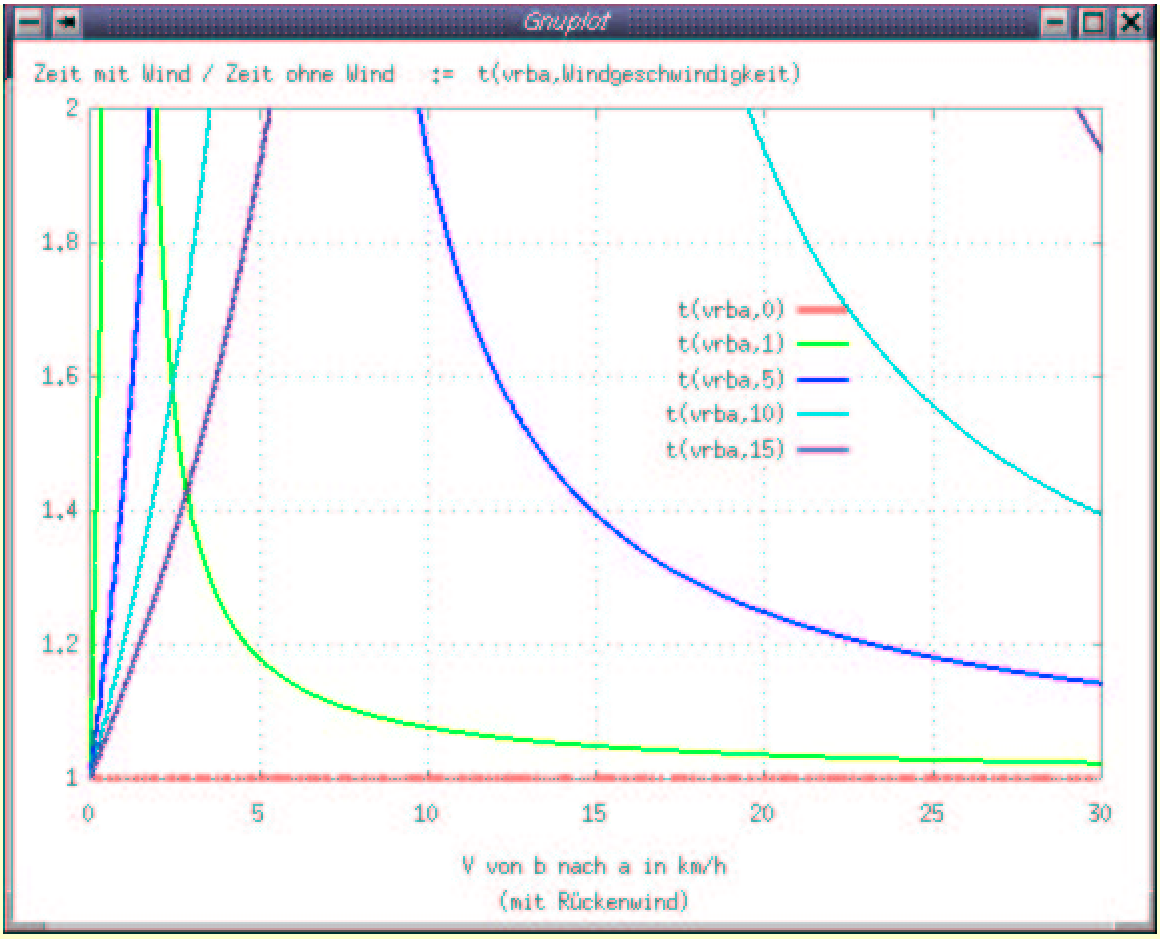
The following chart shows the dependency of the speed against the wind of the speed with the wind and the wind velocity $V_{ab} = V_{ab}(V_{ba}, V_W)$.



The red curve shows the trivial case without wind; then the speeds for a→b and b→a are the same. The speed against the wind disappears in two cases.

- If one drives exactly with the wind with the wind velocity, one does not need a drive power; that is to be managed against the wind only with the speed of 0.
- If one does not drive with the wind, thus with the speed 0, one needs no drive power likewise; that is to be managed against the wind again only with speed 0.

These cases are surely not very practice relevant; it becomes more interesting if the driving speed is clearly larger as the wind velocity. If one drives for example with the wind with 30km/h, one reaches only approx. 23km/h against the wind with a wind velocity of 5km/h while driving with same power. The following chart shows how this affects the total travel time. There the relation of the total travel time for a→b and b→a with wind to the travel time without wind is shown.



If the wind velocity is V_W= 0, the time relation is 1 (red curve). If one can be almost driven by the wind, one needs only very few power. With this small power one advances against the wind only very slowly. Then the total travel time becomes very large. In other words; one approaches a singularity. Now a somewhat more realistic case is assumed: Wind velocity V_W= 5km/h and driving speed with the wind V_{ba}= 25km/h. Then the time relation amounts to approx. 1.2. Then about 20% more time is necessary compared with a travel without wind.

Result

One sees thus whith constant driving power only a part of the time delay becomes balanced with back wind on the return with the wind. If one wants to drive nevertheless without time delay, this would be possible only at higher power and higher total energy consumption. (*this, by the way substantially simpler calculation, is left to the reader*) Is there any practical application of these formulas and physical understanding? The main laws of thermodynamics cannot be outwitted with it; thus you won't drive faster with this knowledge. If you have some problems while cycling on a windy day, you know however now at least why.

Appendix

Gnuplot Syntax:

V_{ab}:

```
vrba(vrba,vw)= vw*−2.0/3.0+(vw**3*1.0/27.0+vrba**3*1.0/2.0−vw*vrba*vrba+1.0/2.0
         *vw*vw*vrba+sqrt(vrba**6*1.0/4.0−vw*vrba**5+1.0/27.0*vw**5*vrba+3.0/2.0
         *vw*vw*vrba**4−26.0/27.0*vw**3*vrba**3+19.0/108.0*vw**4*vrba*vrba))**(1.0/3.0)
         +(vw**3*1.0/27.0+vrba**3*1.0/2.0−vw*vrba*vrba+1.0/2.0*vw*vw*vrba
         −sqrt(vrba**6*1.0/4.0−vw*vrba**5+1.0/27.0*vw**5*vrba+3.0/2.0
         *vw*vw*vrba**4−26.0/27.0*vw**3*vrba**3+19.0/108.0*vw**4*vrba*vrba))**(1.0/3.0)
```

time relation:

```
t(vrba,vw)=(1+vrba/f(vrba,vw))/2.0
           vrab(vrba,vw)= vw*−2.0/3.0+(vw**3*1.0/27.0+vrba**3*1.0/2.0
           −vw*vrba*vrba+1.0/2.0*vw*vw*vrba+sqrt(vrba**6*1.0/4.0−vw*vrba**5
           +1.0/27.0*vw**5*vrba+3.0/2.0*vw*vw*vrba**4−26.0/27.0*vw**3*vrba**3
           +19.0/108.0*vw**4*vrba*vrba))**(1.0/3.0)+(vw**3*1.0/27.0+vrba**3
           *1.0/2.0−vw*vrba*vrba+1.0/2.0*vw*vw*vrba−sqrt(vrba**6*1.0/4.0
           −vw*vrba**5+1.0/27.0*vw**5*vrba+3.0/2.0*vw*vw*vrba**4−26.0/27.0
           *vw**3*vrba**3+19.0/108.0*vw**4*vrba*vrba))**(1.0/3.0)
```

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