MATHEMATICS PRELIMINARY EXTENSION 1
ASSESSMENT TASK
TEST 2
SOLUTIONS

COURSE/LEVEL
NSW Secondary High School Year 11 Preliminary Extension Mathematics.

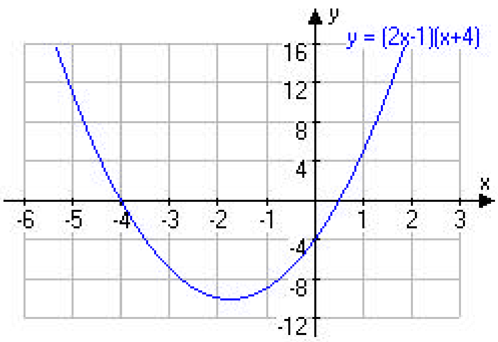
TOPICS
- Basic Arithmetic (Syllabus Reference: 1.1, 1.2)
- Algebra and Surds (Syllabus Reference: 1.3)
- Equations (Syllabus Reference: 1.4, 1.4E)
- Geometry 1 (Syllabus Reference: 2.1, 2.2, 2.3, 2.4)
- Functions and Graphs (Syllabus Reference: 4.1, 4.2, 4.3, 4.4, 6.4, 8.1, 8.2)
- Straight Line Graphs (Syllabus Reference: 6.1, 6.2, 6.3, 6.5, 6.6E, 6.7, 6.7E, 6.8)

QUESTION ONE

(a)

$$\begin{aligned}\sqrt{(x+2)^2+2x+5} &= \sqrt{x^2+4x+4+2x+5} \\ &= \sqrt{x^2+6x+9} \\ &= \sqrt{(x+3)^2} \\ &= |x+3|\end{aligned}$$

(b)



The parabola $y = (2x - 1)(x + 4)$ has x intercepts $x = -4$ and $x = \frac{1}{2}$.

From the graph, $(2x - 1)(x + 4) < 0 \Rightarrow -4 < x < \frac{1}{2}$.

(c)

$$\frac{2}{x^2 - x} + \frac{2}{x^2 - 3x + 2} = \frac{2}{x(x-1)} + \frac{2}{(x-1)(x-2)}$$

$$= \frac{2(x-2) + 2x}{x(x-1)(x-2)}$$

$$= \frac{2x - 4 + 2x}{x(x-1)(x-2)}$$

$$= \frac{4x - 4}{x(x-1)(x-2)}$$

$$= \frac{4(x-1)}{x(x-1)(x-2)}$$

$$= \frac{4}{x(x-2)}$$

(d) (i)

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+1)}{x-2}$$

$$= \lim_{x \to 2} x + 1$$

$$= 3$$

(ii)

$$\lim_{x \to \infty} \frac{2x+1}{3x^2 + x - 1} = \lim_{x \to \infty} \frac{2x+1}{3x^2 + x - 1} \div \frac{x^2}{x^2}$$

$$= \lim_{x \to \infty} \frac{\frac{2}{x} + \frac{1}{x^2}}{3 + \frac{1}{x} - \frac{1}{x^2}}$$

$$= \frac{0}{3}$$

$$= 0$$

(e) $f(x) = x^2 + 3x - 7$

(i) $f(x) = (-3)^2 + 3(-3) - 7 = -7$

(ii) $f(k+1) = (k+1)^2 + 3(k+1) - 7$

$= k^2 + 2k + 1 + 3k + 3 - 7$

$= k^2 + 5k - 3$

(ii)
$$f(x) = 3$$
$$x^2 + 3x - 7 = 3$$
$$x^2 + 3x - 10 = 0$$
$$(x + 5)(x - 2) = 0$$
$$x = -5, 2$$

QUESTION TWO

(a) Let $f(x) = \dfrac{(3-x)(3+x)}{x^2-1}$.

(i) Domain: $x^2 - 1 \neq 0$

ie $x \neq \pm 1$

(ii) $f(-x) = \dfrac{(3-(-x))(3+(-x))}{(-x)^2-1}$

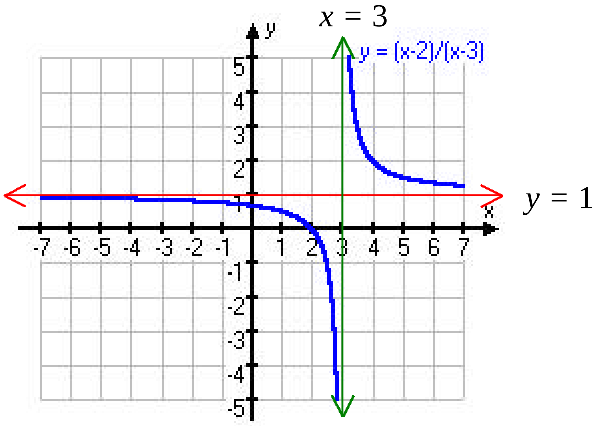
$= \dfrac{(3+x)(3-x)}{x^2-1}$

$= f(x)$

$\therefore$ $f(x)$ is even.

The graph of $y = f(x)$ is symmetric about the y–axis.

(b) $y = \dfrac{x-2}{x-3}$.



Working:

x intercept: When $y = 0$, $x = 2$.

y intercept: When $x = 0$, $y = \frac{2}{3}$.

$y \approx \frac{x}{x}$ for large x. Hence $y = 1$ is a horizontal asymptote.

Sign of y in "critical" regions:

For $x < 2$, $y > 0$.

For $2 < x < 3$, $y < 0$.

For $x > 3$, $y > 0$.

(c) Distance from point (0, 0) to line $3x + 4y + 5 = 0$:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|3\times 0 + 4\times 0 + 5|}{\sqrt{3^2 + 4^2}}$$

$$= 1,$$

which is equal to the radius of the circle. Hence the line is a tangent to the circle.

(d) $1 < y \le 4$.

(e) $2x + y + 3 = 0$, $y = mx + b$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

or

$$\sqrt{5} = \left|\frac{m+2}{1-2m}\right|$$

$\therefore$ $\frac{m+2}{1-2m} = \sqrt{5}$ or $\frac{m+2}{1-2m} = -\sqrt{5}$

$$m + 2 = \sqrt{5}(1-2m)$$

$$m + 2 = \sqrt{5} - 2\sqrt{5}m$$

$$2\sqrt{5}m + m = \sqrt{5} - 2$$

$$m = \frac{\sqrt{5}-2}{2\sqrt{5}+1}$$

$$m + 2 = -\sqrt{5}(1-2m)$$

$$m + 2 = 2\sqrt{5}m - \sqrt{5}$$

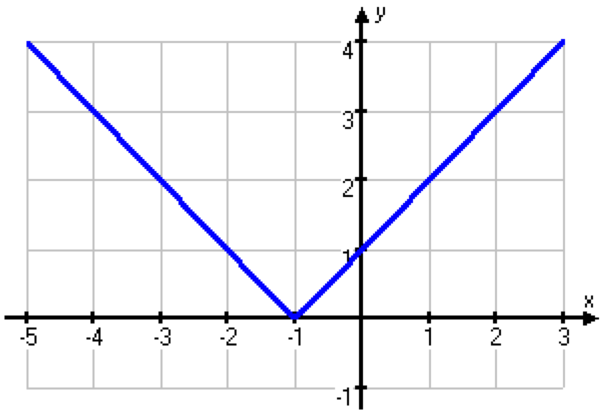
$$2\sqrt{5}m - m = \sqrt{5} + 2$$

$$m = \frac{\sqrt{5}+2}{2\sqrt{5}-1}$$

ie $m = \frac{\sqrt{5}-2}{2\sqrt{5}+1}$ or $m = \frac{\sqrt{5}+2}{2\sqrt{5}-1}$

QUESTION THREE

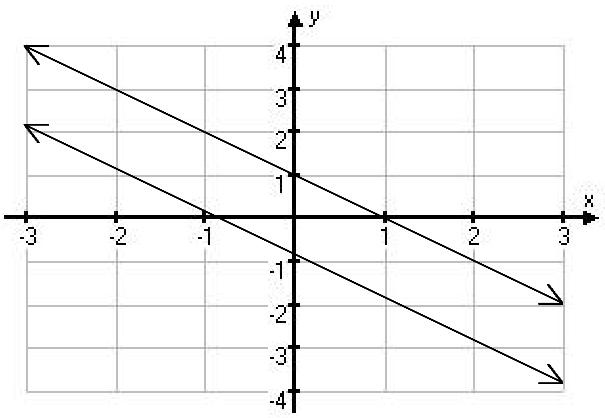
(a) (i) $y = |x+1|$.



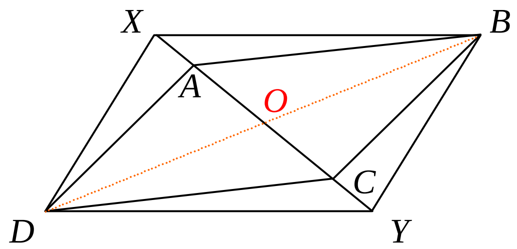
(ii) $|x+y| = 1$.

$x + y = 1$ or $x + y = -1$

ie $y = 1 - x$ or $y = -x - 1$.



(b)



Construct diagonal *BD* bisected meeting *AC* at *O*.

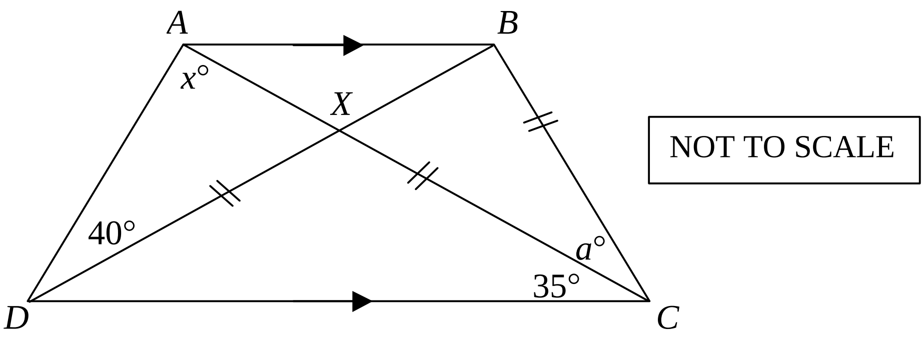
OB = *OD* and *OA* = *OC* (diagonals of ||gram *ABCD* bisect each other)

But *OA* + *AX* = *OC* + *CY* (*AX* = *CY* as given)

∴ *OX* = *OY*

∴ *XBYD* is a parallelogram (diagonals bisect each other)

(c)



(i) $\angle BDC = 35°$ (equal base angles of isos ΔDXC)

$\angle ABD = 35°$ (alt $\angle$s, $AB \parallel DC$)

$\angle CAB = 35°$ (alt $\angle$s, $AB \parallel DC$)

$\therefore \Delta AXB$ is isosceles (base angles equal)

(ii) $\Delta AXD \equiv \Delta BXC$

Reasons:

In ΔAXD and ΔBXC,

$AX = BX$	(equal sides of isos ΔDXC)	(S)
$\angle AXD = \angle BXC$	(vert. opp $\angle$s)	(A)
$DX = CX$	(given)	(S)

$\therefore \Delta AXD \equiv \Delta BXC$ (SAS)

(iii) $a = 40$ (corr $\angle$s in cong Δs)

(iv) $\angle ADC = 40° + 35° = 75°$ (adjacent angles)

$\therefore x° + 75° + 35° = 180°$ ($\angle$sum of ΔADC)

$\therefore x° = 70°$