# TRIGONOMETRY – SUMMARY

**COURSE/LEVEL**

NSW Secondary High School Year 11 Preliminary Mathematics Extension.

**TOPIC**

Trigonometry. (Syllabus Ref: 5)

## FUNDAMENTAL FORMULAE

RATIOS. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$.

SYMMETRY. $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = -\cos\theta$, $\tan(-\theta) = -\tan\theta$

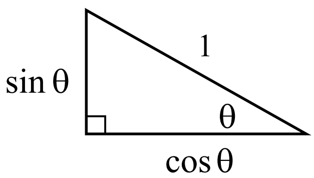
COMPLEMENTARY ANGLES.

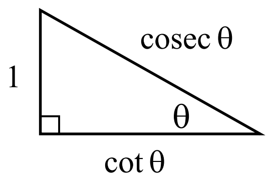
$\cos\theta = \sin(90° - \theta)$, $\cos\theta = \sin(90° - \theta)$,

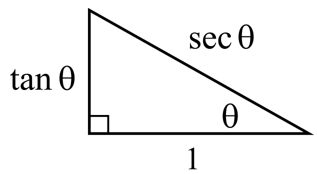
$\cot\theta = \tan(90° - \theta)$, $\tan\theta = \cot(90° - \theta)$,

$\operatorname{cosec}\theta = \sec(90° - \theta)$, $\sec\theta = \operatorname{cosec}(90° - \theta)$.

PYTHAGOREAN IDENTITIES.




$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

## SUMS AND DIFFERENCES OF ANGLES

- $\left.\begin{array}{l}\sin(A+B) = \sin A\cos B + \cos A\sin B \\ \sin(A-B) = \sin A\cos B - \cos A\sin B\end{array}\right\}$
- $\left.\begin{array}{l}\cos(A+B) = \cos A\cos B - \sin A\sin B \\ \cos(A-B) = \cos A\cos B + \sin A\sin B\end{array}\right\}$
- $\left.\begin{array}{l}\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B} \\ \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}\end{array}\right\}$

## DOUBLE ANGLES

- $\cos 2\theta = \cos^2\theta - \sin^2\theta$

  $= 1 - 2\sin^2\theta$ (using $\cos^2\theta = 1 - \sin^2\theta$)

  $= 2\cos^2\theta - 1$ (using $\sin^2\theta = 1 - \cos^2\theta$)
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

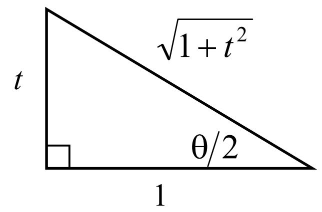
## ACUTE ANGLE BETWEEN TWO INTERSECTING STRAIGHT LINES

- $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$, where $m_1$ and $m_2$ are the gradients of the two lines.

## HALF ANGLE FORMULAE OR *t*-FORMULAE

- If $t = \tan\frac{\theta}{2}$ then

$$\sin\theta = \frac{2t}{1+t^2} \qquad \cos\theta = \frac{1-t^2}{1+t^2} \qquad \tan\theta = \frac{2t}{1-t^2}$$




## SOLVING $a\sin\theta + b\cos\theta = c$:

- **METHOD A: USING *t*-FORMULAE:**

(i) Substitute $\sin\theta = \dfrac{2t}{1+t^2}$ and $\cos\theta = \dfrac{1-t^2}{1+t^2}$.

(ii) Simplify and then solve the resulting *quadratic equation* for *t*.

(iii) Substitute $t = \tan\frac{\theta}{2}$ and solve for $\theta$.

- **METHOD B: USING AUXILIARY ANGLE METHOD**

(i) Express $a\sin\theta + b\cos\theta = c$ in the form $r\cos(\theta - \alpha) = c$, where $r > 0$.

(ii) Expand $r\cos(\theta - \alpha)$.

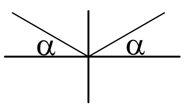
(iii) Equate coefficients to find $r$ and $\alpha$.

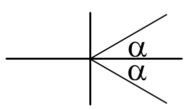
(iv) With $r$ and $\alpha$ known solve $r\cos(\theta - \alpha) = c$

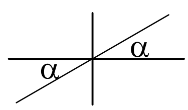
Note: Any of the forms $r\sin(\theta - \alpha)$ or $r\sin(\theta + \alpha)$ or $r\cos(\theta + \alpha)$ can be used.

## GENERAL SOLUTIONS

| $\sin\theta = \sin\alpha$ | $\cos\theta = \cos\alpha$ | $\tan\theta = \tan\alpha$ |
|---|---|---|
| $\theta = 180^\circ n + (-1)^n\alpha$ | $\theta = 360^\circ n \pm \alpha$ | $\theta = 180^\circ n + \alpha$ |




$n$ is any integer ($n = 0, \pm 1, \pm 2, \ldots$)