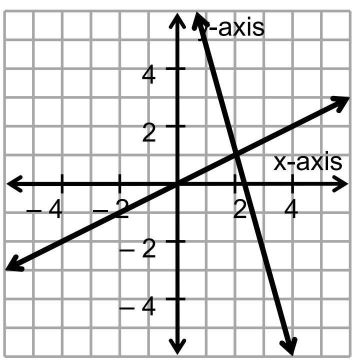
# Sect 4.1 - Solving Systems of Equations by Graphing

Concepts #1 & #2 Solutions to Systems of Linear Equations

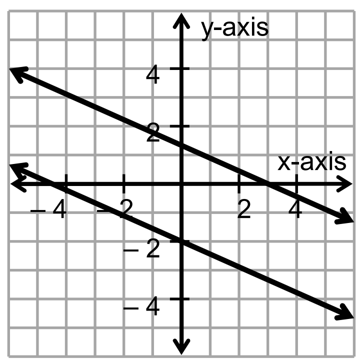
A **system of equations** consists of two or more equations. A solution to a system of equations is a point that satisfies all the equations in the system. In this chapter, our focus will be solving systems of linear equations. If we are solving a system of two linear equations, there are three possible types of systems we could have:

Case #1
Consistent System
The equations are independent.



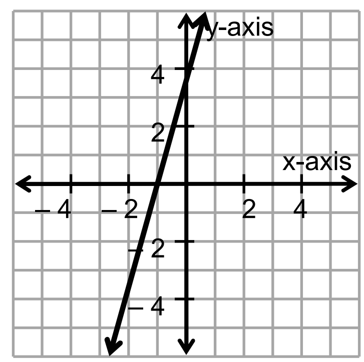

The solution is the where the lines intersect. In this case, the solution is (2, 1). The lines have different slopes.

Case #2
Inconsistent System
The equations are Independent.




The lines do not intersect, so there is no solution, { }. The lines are different, but the lines do have the same slope.

Case #3
Dependent System
The equations are dependent.




Every point on the line is a solution. We write the solution as: {(x, y) | ax + by = c} The lines are the same.

We will study four different ways for solving systems of linear equations in this chapter. The first way is graphing each equation on the same axes and seeing where they intersect. The second method is solving by substitution which will be studying in section 4.2. Here, we will solve one equation for one variable and substitute that expression for that variable in the other equation. The third method, in section 4.3, is solving by addition (elimination) where we add the equations together in such a way the one of the variable terms adds up to zero. Finally, in Section 9.4, we will solve systems of linear equations using Cramer's Rule.

Concept #3: Solving System of Linear Equations in Two Variables by Graphing.

To solve a system of equations by graphing, we will graph each equation on the same axes and see where they intersect. The point of intersection will be the solution to the system. For linear equations, we can graph them either using the slope and y-intercept or by point-plotting. The choice is your to make. We will use both techniques in this section so we can review them. Sometimes one is easier to apply than the other. Let's do example #1 by point-plotting and example #2 using the slope and y-intercept.

**Solving by graphing:**

Ex. 1 $x + 3y = 7$
$2x - 3y = -4$

Solution:

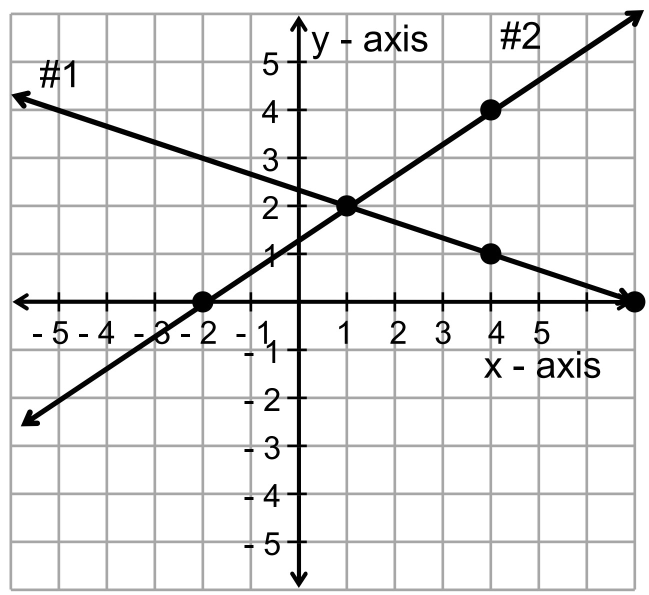
Let's make a table of values and graph each equation:

1) $x + 3y = 7$

| x | y |
|---|---|
| 7 | 0 |
| 4 | 1 |
| 1 | 2 |

2) $2x - 3y = -4$

| x | y |
|---|---|
| – 2 | 0 |
| 1 | 2 |
| 4 | 4 |




The lines intersect at (1, 2) so the solution is (1, 2). This is a consistent system and the equations are independent.

You can always check the answer by plugging in the solution into each equation and see if it satisfies all the equations in the system:

| | |
|---|---|
| $x + 3y = 7$ | $2x - 3y = -4$ |
| $(1) + 3(2) = 7$ | $2(1) - 3(2) = -4$ |
| $1 + 6 = 7$ | $2 - 6 = -4$ |
| $7 = 7$ true | $-4 = -4$ true |

So, the solution checks.

Ex. 2 $y = \frac{2}{3}x - 1$

$x + 2y = 5$

<u>Solution:</u>

The first equation is already solved for y.

1) $y = \frac{2}{3}x - 1$

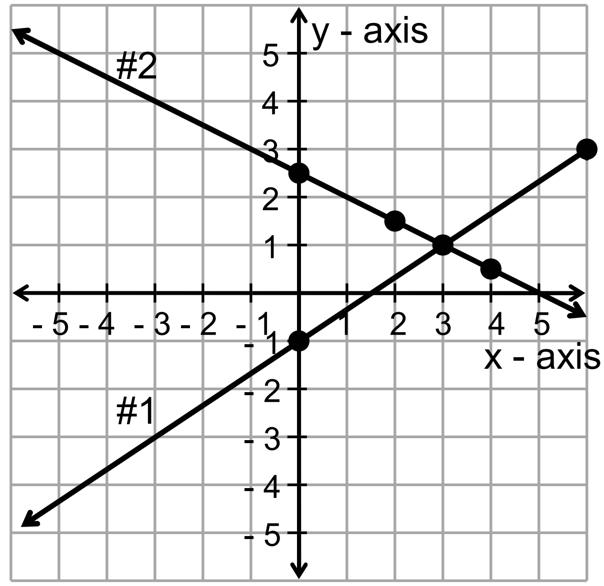
$m = \frac{2}{3}$, y-int: (0, – 1)

To solve the second equation for y, subtract x from both sides and divided both sides by 2:

2) $x + 2y = 5$

$y = -\frac{1}{2}x + \frac{5}{2}$

$m = -\frac{1}{2}$, y-int: $(0, \frac{5}{2})$




The lines intersect at (3, 1) so the solution is (3, 1). This is a consistent system and the equations are independent.

Ex. 3 $4x = 2 - 2y$

$y = -2x + 3$

<u>Solution:</u>

Make a table for equation #1.

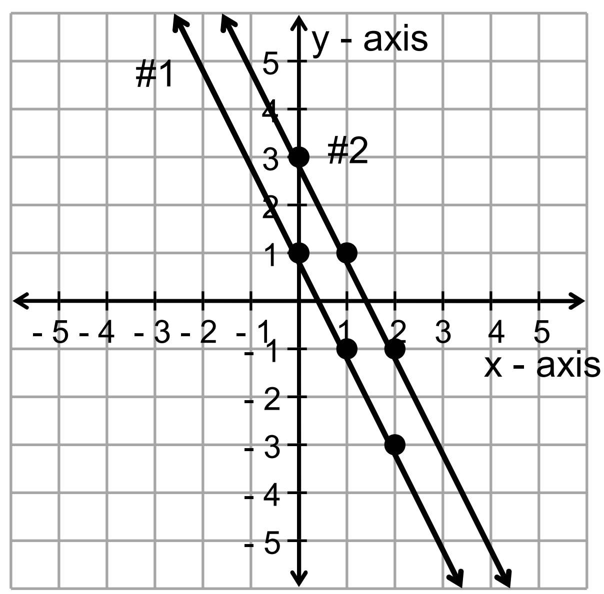
1) $4x = 2 - 2y$

| x | y |
|---|---|
| 0 | 1 |
| 1 | – 1 |
| 2 | – 3 |

The second equation is already solved for y.

$y = -2x + 3$

$m = -2 = \frac{-2}{1}$; y-int: (0, 3)




The lines are parallel so they do not intersect. There is no solution. The system is inconsistent and the equations are independent.

Ex. 4 $\quad 3x + y = 4$

$0.25y + 0.75x = 1$

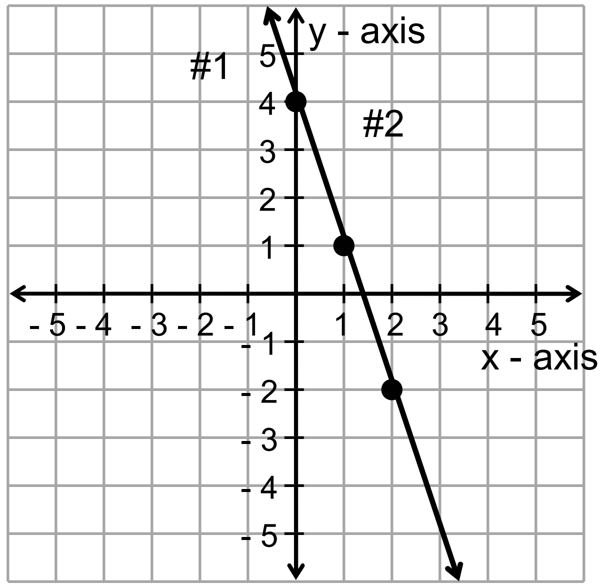
Solution:

Solve the first equation for y

1) $\quad 3x + y = 4$

$y = -3x + 4$

$m = -3 = \frac{-3}{1}$, y-int: (0, 4)




Make a table for equation #2

2) $\quad 0.25y + 0.75x = 1$

| x | y |
|---|---|
| 0 | 4 |
| 1 | 1 |
| 2 | – 2 |

These two lines are the same line. Every point on the line is a solution. So, the solution is { (x, y)| y = – 3x + 4}. The system is dependent and the equations are dependent.

Ex. 5 $\quad 6x - 2y = 4$

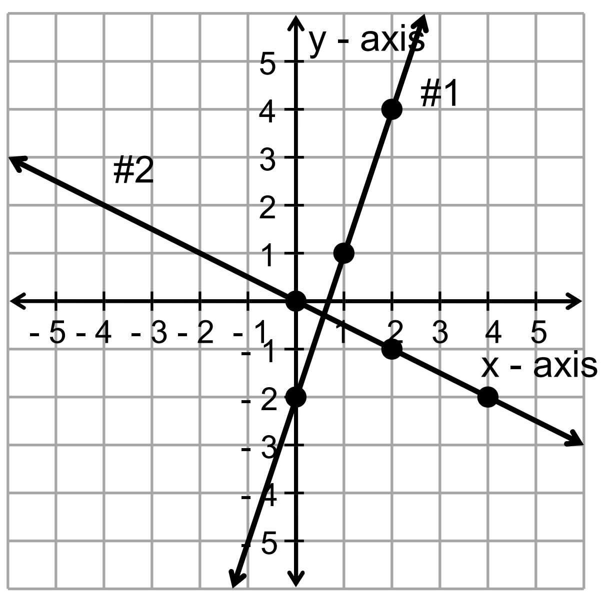
$y = -\frac{1}{2}x$

Solution:

Make a table for equation #1.

1) $\quad 6x - 2y = 4$

| x | y |
|---|---|
| 0 | – 2 |
| 1 | 1 |
| 2 | 4 |




The second equation is already solved for y.

$y = -\frac{1}{2}x$

$m = -\frac{1}{2} = \frac{-1}{2}$; y-int: (0, 0)

The lines do intersect, but it is impossible to read what the exact solution is. This is the major drawback to solving system by graphing. The points have to have integral coordinates. In the next section, we will find the exact solution to this system using substitution.