Sect 4.4 – Problem Solving

Concept #1 Applications Involving Cost

Set-up a system of equations and solve the following:

Ex. 1 Maylina buys two packages of Skittles and three bottles of soda for \$5.15. Zoë buys four packages of Skittles and five bottles of soda for \$9.05. Find the cost of one package of Skittles and one bottle of soda.

Solution:

Let S = the cost of one package of Skittles

b = the cost of bottle of soda

For Maylina, she buys:

2 Skittles + 3 bottles of soda = \$5.15

1) $2S + 3b = 5.15$

For Zoë, she buys:

4 Skittles + 5 bottles of soda = \$9.05

2) $4S + 5b = 9.05$

So, our system is:

1) $2S + 3b = 5.15$

2) $4S + 5b = 9.05$

Let's multiply equation #1 by – 2 and add:

$$
\begin{array}{rl}
-2\bullet 1) & -4S - 6b = -10.30 \\
2) & \underline{4S + 5b = 9.05} \\
 & -b = -1.25
\end{array}
$$

So, $b = 1.25$

Substitute 1.25 for b in equation #1:

$2S + 3(1.25) = 5.15$

$2S + 3.75 = 5.15$

$2S = 1.40$

$S = 0.70$

So, one package of Skittles costs \$0.70 and one bottle of soda costs \$1.25.

Ex. 2 Theater tickets were sold for \$8.50 on the main floor and for \$5 in the balcony. The total revenue was \$3550 and 100 more main floor tickets were sold than balcony tickets. Find the number of each type sold.

Solution:

Let b = the number of balcony tickets sold

m = the number of main floor tickets sold

Since 100 more main floor tickets were sold then the number of floor tickets is 100 more than the number of balcony tickets:

1) $m = b + 100$

The revenue from the sale of main floor tickets is $8.50 times the number of main floor tickets sold or 8.5(m) while the revenue from the sale of balcony tickets is $5 times the number of balcony tickets sold or 5b. The total revenue is 2) $8.5m + 5b = 3550$.

Thus, our system is:

1) $m = b + 100$

2) $8.5m + 5b = 3550$

Now, replace m in equation #2 with b + 100:

$$8.5(b + 100) + 5b = 3550$$
$$8.5b + 850 + 5b = 3550$$
$$13.5b + 850 = 3550$$
$$13.5b = 2700$$
$$b = 200 \text{ tickets}$$

and $m = b + 100 = 300$ tickets

Thus, 200 balcony tickets and 300 main floor tickets were sold.

Concept #2 Applications Involving Principal and Interest

Ex. 3 Juanita decided to divide $8800 into two investments, one in bonds that ended up earning 8% annual interest and one in a mutual fund that ended up yielding 12% annual interest. If the interest from both was $924, how much did she invest in each?

Solution:

Let b = the amount invested in bonds.

m = the amount invested in a mutual fund

Since there was total $8800 invested, the amount invested in a mutual fund plus the amount invested in bonds was $8800:

1) $b + m = 8800$

The interest earned on the bonds was 8% times the amount invested in bonds or 0.08b and the interest earned on the mutual fund was 12% times the amount invested in the mutual fund or 0.12m. The total interest is 2) $0.08b + 0.12m = 924$.

So, our system is:

1) $b + m = 8800$

2) $0.08b + 0.12m = 924$.

Let's solve equation #1 for m:

$b + m = 8800 \quad \Rightarrow \quad m = 8800 - b$

Now, replace m is equation #2 by 8800 – b:

$0.08b + 0.12(8800 - b) = 924$

$0.08b + 1056 - 0.12b = 924$

$-0.04b + 1056 = 924$

$-0.04b = -132$

$b = 3300$

$m = 8800 - b = 5500$

So, \$3300 was invested in bonds and \$5500 in a mutual fund.

Concept #3 Applications Involving Mixtures

Ex. 4 A laboratory technician wishes to combine a 20% saline solution with a 60% saline solution to form 200 mL of a 44% solution. How much of each type should be used?

<u>Solution:</u>

Let t = number of mL of the 20% solution
and s = number of mL of the 60% solution

The total number of mL of the new solution equals the amount of 20% solution plus the amount of 60% solution which is t + s = 200. The amount of salt in the 20% solution is 0.2t, the amount of salt in the 60% solution is 0.6s, and amount of salt in the 44% solution is 0.44(200) = 88 mL. The salt in the 20% solution plus the salt in the 60% solution is equal to the salt in the 44% solution. Thus,

$0.2t + 0.6s = 88$

Our system of equations is:

1) $t + s = 200$

2) $0.2t + 0.6s = 88$

Let's multiply equation #1 by – 0.2 and add:

$$\begin{array}{lr} -0.2\bullet 1) & -0.2t - 0.2s = -40 \\ \underline{2)} & \underline{0.2t + 0.6s = 88} \\ & 0.4s = 48 \\ & s = 120 \end{array}$$

Substitute 120 in for s in eqn. #1:

1) $t + 120 = 200$

$t = 80$

80 mL of the 20% solution & 120 mL of 60% solution are needed.

Ex. 5 A pharmacist wants to combine pure acid with 400 cm^3 of a 25% acid solution to increase the concentration to 40%. How much of the pure acid solution should she add?

Solution:

Let p = amount of pure acid she should add

t = the total amount of the mixture

The total amount t of the new solution will be the amount of pure acid solution, p, plus 400 cm^3 of the 25% solution or p + 400.

So, the amount of the new solution is: 1) t = p + 400

The amount of acid from the pure acid solution is 100% times the amount of the solution or 100%p or 1p. The amount of acid from the 25% solution is 25% times the amount of the solution or 25%(400) or 0.25(400). The amount of acid in the new solution is 40% times the amount of the solution or 40%t or 0.4t.

Thus, the amount of acid from the pure solution, 1p or p, plus the amount of acid from the 25% solution, 0.25(400), has to equal the amount of acid in the new solution, 0.4t:

$$p + 0.25(400) = 0.4t$$

2) $P + 100 = 0.4t$

Now, replace t in eqn. #2 by p + 400:

$$p + 100 = 0.4(p + 400)$$
$$p + 100 = 0.4p + 160$$
$$0.6p + 100 = 160$$
$$0.6p = 60$$
$$p = 100$$

She will need to add 100 cm^3 of pure acid.

Concept #4 Applications Involving Distance, Rate, and Time.

Ex. 6 Taos, New Mexico is about 800 miles from San Antonio, Texas. If John White leaves San Antonio at 7 am and drives nonstop to Taos, averaging 75 mph, when should he arrive at Taos? Assume he does not set his watch back an hour.

Solution:

Since d = rt, substitute 800 for d and 75 for r and solve:

$$d = rt$$

$$\frac{800}{75} = \frac{75t}{75} \quad \text{(divide both sides by 75)}$$

$$10\frac{50}{75} = t \text{ or } 10\frac{2}{3} \text{ hours.}$$

But $\frac{2}{3}$ of an hour = $\frac{2}{3}\bullet 60 = \frac{2}{3}\bullet\frac{60}{1} = \frac{2}{1}\bullet\frac{20}{1}$ = 40 minutes.
Now, add 10 hours and forty minutes to 7 am. He should arrive at 5:40 pm (San Antonio time).

Ex. 7 At 1 pm, a plane leaves St. Louis traveling north at 450 mph. At 2 pm, a second plane leaves St. Louis traveling south at 428 mph. At what time will the planes be 2645 miles apart?

Solution:

Let t_n = time the north bound plane is in the air.
t_s = time the south bound plane is in the air

The plane leaving at 2 pm (south bound) will have been in the air one hour less than the plane leaving at 1 pm (north bound):

1) $t_s = t_n - 1$

Since d = rt, the distance flown by the north bound plane is $450\bullet t_n$ and the distance flown by the south bound plane is $428\bullet t_s$. The planes are flying in opposite directions, so the sum of their distance will be 2645 miles: 2) $450t_n + 428t_s = 2645$

So, our system of equations is:

1) $t_s = t_n - 1$
2) $450t_n + 428t_s = 2645$

Replace t_s in eqn. #2 by $t_n - 1$:

$$450t_n + 428(t_n - 1) = 2645$$
$$450t_n + 428t_n - 428 = 2645$$
$$878t_n - 428 = 2645$$
$$878t_n = 3073$$
$$t_n = 3.5 \text{ hours}$$

The north bound is in the air for 3.5 hours. Since it left at 1 pm, add 3.5 hours to 1 pm. So, at 4:30 pm the planes are 2645 miles apart.

Ex. 8 In flying out of Chicago, a plane flew into a steady wind of 30 mph and took seven hours to reach its destination. On the return trip to Chicago with the wind, the plane covered the same distance in five hours. What was the plane's speed in still air and what was the distance traveled each way?

Solution:

Let p = plane’s speed in still air
and let d = the distance traveled one way.

Against the wind, the plane's net speed was the plane's speed in still air (p) minus the wind speed (30) which is p – 30. The time it took was 7 hours. Since distance equal rate times time, then

$d = (p - 30)(7)$

$d = 7p - 210$

With the wind, the plane's net speed was the plane's speed in still air (p) plus the wind speed (30) which is p + 30. The time it took was 5 hours. Since distance equal rate times time, then

$d = (p + 30)(5)$

$d = 5p + 150$

Thus, our system is:

1) $d = 7p - 210$

2) $d = 5p + 150$

Replace d by 7p – 210 in eqn. #2:

2) $7p - 210 = 5p + 150$ (solve)

$2p - 210 = 150$

$2p = 360$

$p = 180$

Substitute 180 for p in eqn. #2

2) $d = 5(180) + 150 = 900 + 150 = 1050$

Therefore, the distance each way was 1050 miles.

Concept #5 Miscellaneous Mixture Applications

Ex. 9 A Grocer sold 450 pounds of ground beef in one day. Two grades of beef, regular at \$3.30 per pound and lean at \$4.20 per pound, were sold. If the total sales were \$1719, how much of each was sold?

<u>Solution:</u>

Let r = the number of pounds of regular ground beef sold and let L = the number of pounds of lean ground beef sold.

The total weight of the meat sold was r + L = 450. The amount of money received from the regular meat was 3.30r and the amount of money received from the lean meat was 4.20L. Thus, the total money received is 3.30r + 4.20L = 1719. Thus, our system is:

1) $r + L = 450$

2) $3.3r + 4.2L = 1719$

Let's solve eqn. #1 for L:

1) $r + L = 450$

$L = 450 - r$

Now, replace L in eqn. #2 with 450 – r:

2) $3.3r + 4.2(450 - r) = 1719$
$3.3r + 1890 - 4.2r = 1719$
$-0.9r + 1890 = 1719$
$-0.9r = -171$
$r = 190$

Since L = 450 – r, then L = 450 – 190 = 260

Thus, 260 pounds of lean meat and 190 pounds of regular meat were sold.

Ex. 10 A newspaper stand has 180 coins at the end of the day, all dimes and quarters. If the value of the coins is $36, how many quarters were in the stand?

Solution:

Let q = the number of quarters in the stand.
d = the number of dimes in the stand

Since there are a total 180 coins in the stand, the number of quarters plus the number of dimes has to be equal to the total number of coins in the stand. 1) $q + d = 180$

The amount of money in quarters is $0.25 times the number of quarters or 0.25q while the amount of money in dimes is $0.10 times the number of dimes or 0.10d. The total amount in dimes and quarters then is 2) $0.25q + 0.10d = 36$

Our system is:

1) $q + d = 180$
2) $0.25q + 0.10d = 36$

Multiply eqn. #1 by – 0.10 and add:

– 0.1•1) $-0.1q - 0.1d = -18$
2) $\underline{0.25q + 0.10d = 36}$
$q = 120$ quarters

There were 120 quarters in the stand.

Ex. 11 Ronco is trying to decide if they should introduce the Ronco All-In-One Potato Peeler/Masher. They estimate that $15,450 will be need for new equipment to produce the product and it will cost $15 to produce each Peeler/Masher. They also estimate that the revenue from each Peeler/Masher will be $25.

a) Determine the revenue equation.
b) Determine the cost equation.
c) Find the break-even point.

Solution:

Let p = the number of Peeler/Mashers expected to be sold.

a) If each Peeler/Masher will sell for $25, then the price times the number expected to be sold will equal the revenue equation:
$R = 25p$

b) The cost equation is equal to a fixed cost of $15,450 plus $15 times the expected number of Peeler/Masher produced and sold:
$C = 15p + 15450$

c) Our system of equations is:
1) $R = 25p$
2) $C = 15p + 15450$
The break-even point is where the revenue equals the cost.
So, since C = R = 25p, replace C in eqn. #2 by 25p:
2) $25p = 15p + 15450$
$10p = 15450$
$p = 1545$
So, Ronco has to produce and sell 1545 Peeler/Mashers to break-even.

Ex. 12 With the current, Juanita can row 11.7 mph. Against the current, she can row 7.1 mph. What is her rowing speed in still water and the current speed?

Solution:

Let r = rowing speed in still water
and let c = current speed.
With the current, the total speed is the rowing speed plus the current speed, so $r + c = 11.7$.
Against the currents, the total speed is the rowing speed minus the current speed, so $r - c = 7.1$. Our system of equations is:
1) $r + c = 11.7$ (since the c-terms are opposite, add)
2) $r - c = 7.1$
$2r = 18.8$
$r = 9.4$
Replace r by 9.4 in eqn. #1 to find c:
1) $9.4 + c = 11.7$
$c = 2.3$
So, her rowing speed in still water is 9.4 mph and the current speed is 2.3 mph.