Sect 7.6 - Rational Equations

Concept #1 Introduction to Rational Equations

Definition of a Rational Equation
A **Rational Equation** is equation that contains at least one rational expression.

Recall that when solving equations with fractions, we typically tried to clear the fractions before we proceed to solve the equation:

Solve:

Ex. 1 $\frac{4x-5}{9} - 4 = \frac{2x+5}{6}$

Solution:
First, clear fractions by multiplying both sides by 18:

$\frac{18}{1}\left(\frac{4x-5}{9} - 4\right) = \frac{18}{1}\left(\frac{2x+5}{6}\right)$ (distribute on the left side)

$\frac{18}{1}\left(\frac{4x-5}{9}\right) - 18(4) = \frac{18}{1}\left(\frac{2x+5}{6}\right)$ (reduce)

$\frac{2}{1}\left(\frac{4x-5}{1}\right) - 18(4) = \frac{3}{1}\left(\frac{2x+5}{1}\right)$

$2(4x - 5) - 18(4) = 3(2x + 5)$(distribute)

$8x - 10 - 72 = 6x + 15$ (combine like terms)

$8x - 82 = 6x + 15$

$2x - 82 = 15$

$2x = 97$

$\frac{2x}{2} = \frac{97}{2}$

$x = 48.5$

What allows us to clear fractions in equations is the multiplication property of equality which states we can multiply both sides of an equation by any **non-zero** number. We always choose the L.C.D. so as to "clear the fractions". We can do the same thing with rational equations. We just need to be careful not to multiply both sides by 0. We will accomplish by noting the values of the variables that make the L.C.D. equal to zero and restricting those values from our answer.

Concept #2 Solving Rational Equations

Solving a Rational Equation

1. Factor all the denominators and determine their L.C.D.

2. Restrict the values of the variables that make the L.C.D. equal to 0.

3. Multiply both sides of the rational equation by the L.C.D. and simplify.

4. Solve the resulting equation.

5. Check the answers against the restricted values in step #2. Any value that matches any of the restricted values must be excluding from the solution.

Solve the following:

Ex. 2 $\frac{2}{x+1} = \frac{3}{x-2}$

Solution:

(x + 1) and (x – 2) are prime.

The L.C.D. = (x + 1)(x – 2)

Our restrictions are x ≠ – 1 and x ≠ 2.

Now, multiply both sides of the equation by (x + 1)(x – 2):

$$\frac{2}{(x+1)} = \frac{3}{(x-2)}$$

$$\frac{2}{(x+1)} \bullet \frac{(x+1)(x-2)}{1} = \frac{3}{(x-2)} \bullet \frac{(x+1)(x-2)}{1}$$

$$\frac{2}{1} \bullet \frac{(x-2)}{1} = \frac{3}{1} \bullet \frac{(x+1)}{1}$$

2(x – 2) = 3(x + 1)

2x – 4 = 3x + 3

– 4 = x + 3

– 7 = x This does not match our restrictions x ≠ – 1 and x ≠ 2.

Thus, our solution is { – 7}.

Ex. 3 $\frac{x}{x+4} - \frac{4}{x-4} = \frac{x^2+16}{x^2-16}$

Solution:

(x + 4) and (x – 4) are prime.

x^2 – 16 = (x – 4)(x + 4)

The L.C.D. = (x – 4)(x + 4)

Our restrictions are x ≠ 4 and x ≠ – 4

Now, multiply both sides by (x – 4)(x + 4):

$$\frac{x}{x+4} - \frac{4}{x-4} = \frac{x^2+16}{(x-4)(x+4)}$$

$$\frac{x}{(x+4)} \bullet \frac{(x-4)(x+4)}{1} - \frac{4}{(x-4)} \bullet \frac{(x-4)(x+4)}{1} = \frac{(x^2+16)}{(x-4)(x+4)} \bullet \frac{(x-4)(x+4)}{1}$$

$$\frac{x}{1} \bullet \frac{(x-4)}{1} - \frac{4}{1} \bullet \frac{(x+4)}{1} = \frac{(x^2+16)}{1} \bullet \frac{1}{1}$$

$x(x - 4) - 4(x + 4) = (x^2 + 16)(1)$

$x^2 - 4x - 4x - 16 = x^2 + 16$

$x^2 - 8x - 16 = x^2 + 16$

$\underline{-x^2 \qquad\qquad = -x^2}$

$-8x - 16 = 16$

$-8x = 32$

$x = -4$ This does match our restriction $x \neq -4$ so this value must be exclude from our solution. Since there are no other values in our solution, there is no solution.

Our answer is { }.

Ex. 4 $\dfrac{3a-5}{a^2+4a+3} + \dfrac{2a+2}{a+3} = \dfrac{a-1}{a+1}$

Solution:

$a^2 + 4a + 3 = (a + 3)(a + 1)$.

$a + 3$ and $a + 1$ are prime.

Our L.C.D. = $(a + 3)(a + 1)$.

Our restrictions are $a \neq -3$ and $a \neq -1$.

Now, multiply both sides by $(a + 3)(a + 1)$:

$$\frac{3a-5}{a^2+4a+3} + \frac{2a+2}{a+3} = \frac{a-1}{a+1}$$

$$\frac{(3a-5)}{(a+3)(a+1)} \bullet \frac{(a+3)(a+1)}{1} + \frac{(2a+2)}{(a+3)} \bullet \frac{(a+3)(a+1)}{1} = \frac{(a-1)}{(a+1)} \bullet \frac{(a+3)(a+1)}{1}$$

$$\frac{(3a-5)}{1} \bullet \frac{1}{1} + \frac{(2a+2)}{1} \bullet \frac{(a+1)}{1} = \frac{(a-1)}{1} \bullet \frac{(a+3)}{1}$$

$(3a - 5)(1) + (2a + 2)(a + 1) = (a - 1)(a + 3)$

$3a - 5 + 2a^2 + 4a + 2 = a^2 + 2a - 3$

$2a^2 + 7a - 3 = a^2 + 2a - 3$

$\underline{-a^2 - 2a + 3 = -a^2 - 2a + 3}$

$a^2 + 5a = 0$

$a(a + 5) = 0$

$a = 0$ or $a = -5$

These do not match our restrictions $a \neq -3$ and $a \neq -1$.

Thus, our solution is { – 5, 0}.

Ex. 5 $\frac{3c}{2c-1} - \frac{1}{c+2} = \frac{5}{2c^2+3c-2}$

Solution:

$2c - 1$ and $c + 2$ are prime and $2c^2 + 3c - 2 = (2c - 1)(c + 2)$

Thus, L.C.D. = $(2c - 1)(c + 2)$

Our restrictions are $c \neq 0.5$ and $c \neq -2$.

Now, multiply both sides by $(2c - 1)(c + 2)$:

$$\frac{3c}{2c-1} - \frac{1}{c+2} = \frac{5}{2c^2+3c-2}$$

$$\frac{3c}{(2c-1)} \bullet \frac{(2c-1)(c+2)}{1} - \frac{1}{(c+2)} \bullet \frac{(2c-1)(c+2)}{1} = \frac{5}{(2c-1)(c+2)} \bullet \frac{(2c-1)(c+2)}{1}$$

$$\frac{3c}{1} \bullet \frac{(c+2)}{1} - \frac{1}{1} \bullet \frac{(2c-1)}{1} = \frac{5}{1} \bullet \frac{1}{1}$$

$3c(c + 2) - 1(2c - 1) = 5$

$3c^2 + 6c - 2c + 1 = 5$

$3c^2 + 4c + 1 = 5$

$3c^2 + 4c - 4 = 0$

$(3c - 2)(c + 2) = 0$

$c = \frac{2}{3}$ and $c = -2$

But $c = -2$ matches our restriction that $c \neq -2$ and it must be excluded from the solution. Thus, $c = \frac{2}{3}$ is the only value left.

Hence, our solution is $\{\frac{2}{3}\}$.

Ex. 6 $\frac{4}{3x^2+7x+2} + \frac{5}{2x^2+9x+10} = \frac{9}{6x^2+17x+5}$

Solution:

$3x^2 + 7x + 2 = (3x + 1)(x + 2)$

$2x^2 + 9x + 10 = (x + 2)(2x + 5)$

$6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

So, L.C.D. = $(3x + 1)(x + 2)(2x + 5)$

Our restrictions are $x \neq -\frac{1}{3}$, $x \neq -2$, and $x \neq -2.5$.

Now, multiply both sides by $(3x + 1)(x + 2)(2x + 5)$:

$$\frac{4}{3x^2+7x+2} + \frac{5}{2x^2+9x+10} = \frac{9}{6x^2+17x+5}$$

$$\frac{4}{(3x+1)(x+2)} \bullet \frac{(3x+1)(x+2)(2x+5)}{1} + \frac{5}{(x+2)(2x+5)} \bullet \frac{(3x+1)(x+2)(2x+5)}{1}$$

$$= \frac{9}{(3x+1)(2x+5)} \bullet \frac{(3x+1)(x+2)(2x+5)}{1}$$

$\frac{4}{1} \bullet \frac{(2x+5)}{1} + \frac{5}{1} \bullet \frac{(3x+1)}{1} = \frac{9}{1} \bullet \frac{(x+2)}{1}$

4(2x + 5) + 5(3x + 1) = 9(x + 2)
8x + 20 + 15x + 5 = 9x + 18
23x + 25 = 9x + 18
14x + 25 = 18
14x = – 7
x = – 0.5

This does not match our restrictions $x \neq -\frac{1}{3}$, $x \neq -2$, and $x \neq -2.5$.
Hence, our solution is { – 0.5}.

Concept #3 Solving Literal Equations that are Rational Equations

We can use the same principles to solve a Literal Equations for a specific variable. With a literal equation, we do not worry about the restrictions.

Solve the following equation for the indicated variable:

Ex. 7 $P = \frac{nrT}{V}$ for V

Solution:
The L.C.D. = V. Multiply both sides by V:

$P = \frac{nrT}{V}$

$P \bullet V = \frac{nrT}{V} \bullet \frac{V}{1}$

$\frac{PV}{P} = \frac{nrT}{P}$ (divide both sides P)

$V = \frac{nrT}{P}$

Ex. 8 $n = \frac{2P}{P-m}$ for m

Solution:
The L.C.D. = (P – m). Multiply both sides by (P – m).

$n = \frac{2P}{P-m}$

$n(P-m) = \frac{2P}{(P-m)} \bullet \frac{(P-m)}{1}$

n(P – m) = 2P
nP – nm = 2P
nP – nm = 2P

$$\begin{array}{l} \underline{-nP \quad\quad = -nP} \\ -nm = 2P - nP \end{array}$$

$$\frac{-nm}{-n} = \frac{2P - nP}{-n}$$

$$m = \frac{2P - nP}{-n} = \frac{-nP + 2P}{-n} = \frac{-P(n-2)}{-n} = \frac{P(n-2)}{n}$$

Hence, $m = \frac{P(n-2)}{n}$.

Ex. 9 $F = \frac{r+c}{r-c}$ for c

Solution:

The L.C.D. = (r – c). Multiply both sides by (r – c).

$F = \frac{r+c}{r-c}$

$F(r-c) = \frac{r+c}{r-c} \cdot \frac{(r-c)}{1}$

$F(r-c) = r + c$

$Fr - Fc = r + c$

Since we are solving for c, we need to get the terms with c in them on one side of the equation and the terms without c in them on the opposite side of the equation:

$$\begin{array}{l} Fr - Fc = r + c \\ \underline{-r + Fc = -r + Fc} \\ Fr - r = c + Fc \end{array}$$

Factor out c from both terms on the right side:

$$Fr - r = (1 + F)c$$

$$\frac{Fr - r}{1 + F} = \frac{(1 + F)c}{(1 + F)}$$

$$c = \frac{Fr - r}{1 + F} = \frac{r(F-1)}{F+1}$$

Hence, $c = \frac{r(F-1)}{F+1}$.

Ex. 10 $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1

Solution:

The L.C.D. = RR_1R_2. Multiply both sides by RR_1R_2.

$$\frac{1}{R} \bullet \frac{RR_1R_2}{1} = \frac{1}{R_1} \bullet \frac{RR_1R_2}{1} + \frac{1}{R_2} \bullet \frac{RR_1R_2}{1}$$

$$\frac{1}{1} \bullet \frac{R_1R_2}{1} = \frac{1}{1} \bullet \frac{RR_2}{1} + \frac{1}{1} \bullet \frac{RR_1}{1}$$

$$R_1R_2 = RR_2 + RR_1$$

$$\underline{-RR_1 = \qquad -RR_1}$$

$$R_1R_2 - RR_1 = RR_2$$

Factor out R_1.

$$(R_2 - R)R_1 = RR_2$$

$$R_1 = \frac{RR_2}{R_2 - R}$$