Sect 7.7 - Applications of Rational Expressions and Proportions

Concept #1 Solving Proportions

Definition of a Proportion

A **proportion** is a statement that two ratios or two rates are equal.

If b ≠ 0 and d ≠ 0, then $\frac{a}{b} = \frac{c}{d}$ is a proportion.

To solve a proportion, we multiply both sides by the L.C.D.

Solve:

Ex. 1 $\frac{n}{8} = \frac{7}{11}$

Solution:

$\frac{n}{8} = \frac{7}{11}$ (LCD = 88)

$\frac{n}{8} \bullet \frac{88}{1} = \frac{7}{11} \bullet \frac{88}{1}$

$n \bullet 11 = 7 \bullet 8$

$11n = 56$

$n = \frac{56}{11}$ or $5\frac{1}{11}$ or $5.0\overline{9}$.

Ex. 2 $\frac{7-3x}{4} = \frac{2x+3}{9}$

Solution:

$\frac{7-3x}{4} = \frac{2x+3}{9}$ (LCD = 36)

$\frac{7-3x}{4} \bullet \frac{36}{1} = \frac{2x+3}{9} \bullet \frac{36}{1}$

$(7 - 3x) \bullet 9 = (2x + 3) \bullet 4$

$63 - 27x = 8x + 12$

$63 - 35x = 12$

$-35x = -51$

$x = \frac{51}{35}$

Concept #2 Applications of Proportions

Solve the following using a proportion:

Ex. 3 If 11.5 ounce jar of peanut butter contains 80.5 grams of protein, how many grams of protein does 28 ounce jar of peanut butter contain?

Solution:

Let p = the number of grams of protein in a 28 ounce jar.

Let's write the amount of protein over the number of ounces:

$\frac{\text{protein}}{\text{ounces}}$: $\frac{80.5}{11.5} = \frac{p}{28}$ (L.C.D. = 322)

$\frac{80.5}{11.5} \bullet \frac{322}{1} = \frac{p}{28} \bullet \frac{322}{1}$

$80.5 \bullet 28 = p \bullet 11.5$

$2254 = 11.5p$

$196 = p$

A 28 ounce jar contains 196 grams of protein.

Ex. 4 On a blueprint, 3 inches corresponds to 4 feet in real life. If a wall measures 10.5 inches on the blueprint, how long is it?

Solution:

Let L = the length of the wall in real life.

Let's write blueprint over real life:

$$\frac{\text{blueprint}}{\text{real life}}: \quad \frac{3}{4} = \frac{10.5}{L} \quad \text{(L.C.D. = 4L)}$$

$$\frac{3}{4} \bullet \frac{4L}{1} = \frac{10.5}{L} \bullet \frac{4L}{1}$$

$$3 \bullet L = 10.5 \bullet 4$$

$$3L = 42$$

$$L = 14$$

The wall is 14 feet long.

Ex. 5 At a particular college, there are 15 female students to every 13 male students. If there is 35,000 students enrolled at the college, how many of the students are female?

Solution:

Since there are 15 female students to every 13 male students, then there are 15 female students to every 15 + 13 = 28 students.

Let F = the number of female students at the college

Let's write the number of female over the total:

$$\frac{\text{Females}}{\text{Total}}: \quad \frac{15}{28} = \frac{F}{35000} \quad \text{(L.C.D. = 980,000)}$$

$$\frac{15}{28} \bullet \frac{980000}{1} = \frac{F}{35000} \bullet \frac{980000}{1}$$

$$15 \bullet 35000 = F \bullet 28$$

$$525000 = 28F$$

$$18750 = F$$

There are 18, 750 female students enrolled at the college.

Similar Triangles

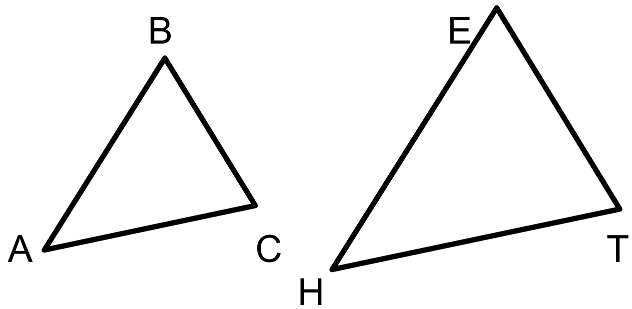
Two triangles are **<u>similar</u>** if they have the same shape, but not necessarily the same size. The notation for writing that triangle ABC is similar to triangle HET is Δ**ABC ~** Δ**HET**. The ordering of the letters shows the corresponding vertices. In this case, $\angle$A corresponds to $\angle$H, $\angle$B corresponds to $\angle$E and $\angle$C corresponds to $\angle$T.

In similar triangles, the corresponding angles are equal. Thus, if ΔABC ~ ΔHET, then m∠A = m∠H, m∠B = m∠E, and m∠C = m∠T. The corresponding sides are not equal. However, the ratios of corresponding sides are equal since one triangle is in proportion to the other triangle.

Thus, if ΔABC ~ ΔHET, then

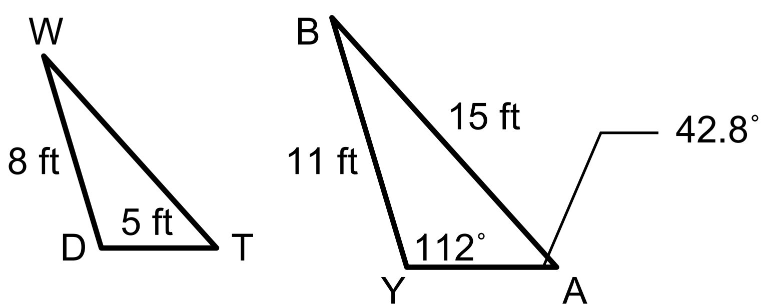
$$\frac{AB}{HE} = \frac{BC}{ET} = \frac{AC}{HT}$$

We can use proportions to then find the missing sides in a pair of similar triangles. In setting up a proportion, we always start with a pair of corresponding sides that of which we know both values. Let's try some examples.



<u>Find the missing sides and angles of the following:</u>

Ex. 6 ΔDWT ~ ΔYBA



<u>Solution:</u>

The pair of corresponding sides we will start with are DW and YB. We write the length from the smaller triangle over the length from the bigger triangle. So, our proportions are:

$\frac{DW}{BY} = \frac{WT}{BA}$ and $\frac{DW}{BY} = \frac{DT}{YA}$

$\frac{8}{11} = \frac{WT}{15}$ and $\frac{8}{11} = \frac{5}{YA}$. Now, cross multiply and solve:

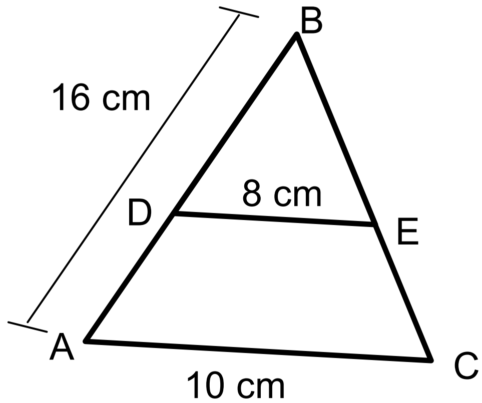
$\frac{8}{11} = \frac{WT}{15}$	$\frac{8}{11} = \frac{5}{YA}$.
$11WT = 8 \bullet 15$	$8YA = 11 \bullet 5$
$\frac{11WT}{11} = \frac{120}{11}$	$\frac{8YA}{8} = \frac{55}{8}$
$WT = 10\frac{10}{11}$ ft	YA = 6.875 ft.

Since m∠D = m∠Y and m∠T = m∠A, then m∠D = 112˚ and m∠T = 42.8˚. Also, m∠W = m∠B = 180˚ – 112˚ – 42.8˚ = 25.2˚.

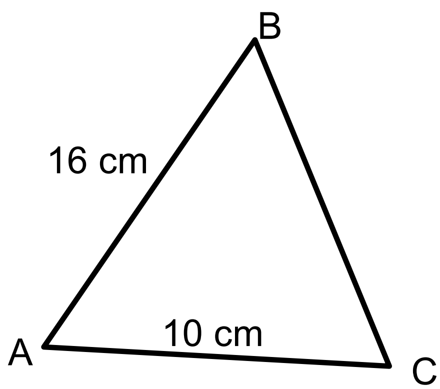
Ex. 7 On ΔABC, there is a point D on $\overline{AB}$ and a point E on $\overline{BC}$ such that $\overline{DE} \parallel \overline{AC}$. If AB = 16 cm, AC = 10 cm, and DE = 8 cm, find BD.

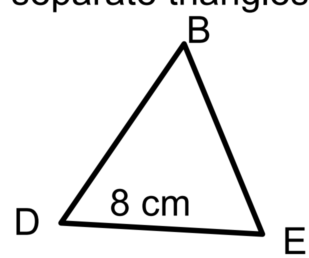
<u>Solution:</u>

To see how we would use similar triangles to solve this problem, let's begin by drawing a diagram:



Since $\overline{DE} \parallel \overline{AC}$, then $\overline{AB}$ is a transversal. This means that $\angle A$ and $\angle D$ are corresponding angles and therefore, m $\angle A$ = m $\angle D$. Similarly, $\overline{BC}$ is a transversal. This implies that $\angle C$ and $\angle E$ are corresponding angles and so, m $\angle C$ = m $\angle E$. Now, redraw the figure as two separate triangles.





Note that m $\angle B$ = m $\angle B$. Since the measures of the corresponding angles are equal, then $\Delta ABC \sim \Delta DBE$. We know the values of AC and DE and we also have the value for AB. Since DB is the missing side and the triangles are similar, we can set-up a proportion to solve for DB.

$\frac{AC}{DE} = \frac{AB}{DB}$ (Substitute)

$\frac{10}{8} = \frac{16}{DB}$ (Cross Multiply)

$10\ DB = 8 \cdot 16$

$10\ DB = 128$ (Divide by 10)

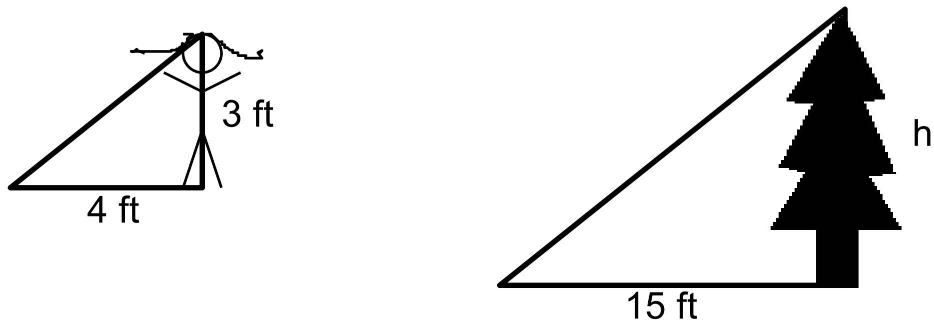
$DB = 12.8$ cm

Suppose the question had asked for AD instead of DB. In that case, we would work the problem exactly the same way and still find DB. Since AD = AB – DB, then AD = 16 cm – 12.8 cm = 3.2 cm.

Ex. 8 A tree casts a 15-foot shadow at the same time a child 3 feet tall casts a 4-foot shadow. How tall is the tree?

Solution:

We begin by drawing a picture:



These two triangles are similar triangles so we will take the ratio the lengths of the shadows and set it equal to the ratio of the heights:

$$\frac{\text{Length of the child's shadow}}{\text{Length of the tree's shadow}} = \frac{\text{Child's height}}{\text{Tree's height}}$$

$\frac{4}{15} = \frac{3}{h}$ Cross multiply and solve:

$4h = 15 \bullet 3$

$\frac{4h}{4} = \frac{45}{4}$

$h = 11.25$ ft

The tree is 11.25 feet tall.

Concept #3 Distance, Rate, and Time Applications

<u>Set-up and solve the following:</u>

Ex. 9 Leroy drove 330 miles in the same amount of time as Juanita drove 400 miles. If Juanita was driving 14 miles per hour faster than Leroy, how fast were they driving?

Solution:

Let L = Leroy's Speed

Juanita was driving 14 mph faster than Leroy, so:

L + 14 = Juanita's speed.

Since the travel times were the same for both, we can use the equation d = rt and solve for t: d = rt (divide by r)

$$t = \frac{d}{r}$$

For Leroy, his speed was L and the distance was 330 miles, so we can express his time as: $t = \frac{330}{L}$

For Juanita, her speed was L + 14 and the distance was 400 miles, so we can express her time as: $t = \frac{400}{L+14}$

Their times were equal so, $\frac{330}{L} = \frac{400}{L+14}$

Now, solve for L: $\frac{330}{L} = \frac{400}{L+14}$

330(L + 14) = 400L
330L + 4620 = 400L
4620 = 70L
66 = L and L + 14 = 80

So, Leroy was driving at 66 mph and Juanita was driving at 80 mph.

Ex. 10 Flying out of Chicago, a plane flies with a 25-mph wind and travels 1800 miles. On the return trip, the plane flies against the wind and it takes half of an hour longer to cover the same distance. Find the speed of the plane in still air.

Let P = the plane's speed in still.

With the wind, the plane's net speed is P + 25. Since $t = \frac{d}{r}$, then the plane's time with the wind is $t_w = \frac{1800}{P+25}$.

Against the wind, the plane's net speed is P – 25. Since $t = \frac{d}{r}$, then the plane's time against the wind is $t_a = \frac{1800}{P-25}$.

The time against the wind, t_a, is $\frac{1}{2}$ hour longer than the time with the wind t_w: $t_a = t_w + \frac{1}{2}$ (replace t_a by $\frac{1800}{P-25}$ and t_w by $\frac{1800}{P+25}$)

$$\frac{1800}{P-25} = \frac{1800}{P+25} + \frac{1}{2}$$

The L.C.D. = 2(P – 25)(P + 25). Multiply both sides by the L.C.D.:

$$\frac{1800}{(P-25)} \bullet \frac{2(P-25)(P+25)}{1} = \frac{1800}{(P+25)} \bullet \frac{2(P-25)(P+25)}{1} + \frac{1}{2} \bullet \frac{2(P-25)(P+25)}{1}$$

$$\frac{1800}{1} \bullet \frac{2(P+25)}{1} = \frac{1800}{1} \bullet \frac{2(P-25)}{1} + \frac{1}{1} \bullet \frac{(P-25)(P+25)}{1}$$

$$3600(P + 25) = 3600(P - 25) + (P - 25)(P + 25)$$
$$3600P + 90000 = 3600P - 90000 + P^2 - 625$$
$$3600P + 90000 = P^2 + 3600P - 90625$$
$$\underline{-3600P - 90000 = \quad -3600P - 90000}$$
$$0 = P^2 - 180625$$

$P^2 - 180625 = 0$
$(P - 425)(P + 425) = 0$
$P = 425$ or $P = -425$ (reject, the plane cannot fly backwards)
So, the plane's speed in still air is 425 mph.

Concept #4 Work Applications

Suppose a person can paint a room in six hours. This means that the person can paint $\frac{1}{6}$ of the room in one hour. So, the person's rate equals $\frac{1}{\text{time to complete the job}}$. If a second person can paint the same room in five hours, then that person can paint $\frac{1}{5}$ of the room in one hour. Now suppose that the two people decide to work together. If t is the total time it takes for the two to finish the job, then the first person's rate, $\frac{1}{6}$, times the time t would represent the amount of the room he or she painted. Thus, the first person painted $\frac{1}{6}t$ of the room. Similarly, for the second person, he or she would have painted $\frac{1}{5}t$ of the room. Since the two people were painting one room, then

$$\frac{1}{6}t + \frac{1}{5}t = 1$$

Multiplying both sides by the L.C.D. of 30 yields:

$$30\left(\frac{1}{6}t\right) + 30\left(\frac{1}{5}t\right) = 30(1)$$
$$5t + 6t = 30$$
$$11t = 30$$
$$t = \frac{30}{11} \text{ or } 2\frac{8}{11} \text{ hours}$$

It will take the two people $2\frac{8}{11}$ hours to paint the room.

All work or job problems have a similar format in terms of how we set up the equation and solve it.

<u>Solve:</u>

Ex. 11 One printer can print 1000 copies of a manuscript in 16 hours. A second printer do the same job in 12 hours, Working together, how long will it take the two printers to complete the job?

<u>Solution:</u>
Let t = the time for the two printers to do the job working together. Since first printer can do the job in 16 hours, its rate is $\frac{1}{16}$ of the job per hour. If it works for t hours, the portion of the job it completes is $\frac{1}{16}$t. The second printer can do the job in 12 hours, so its rate is $\frac{1}{12}$ of the job per hour. If it works for t hours, the portion of the job it completes is $\frac{1}{12}$t. Working together, the printers complete one job, so:

$$\frac{1}{16}t + \frac{1}{12}t = 1 \qquad \text{(multiply by the L.C.D. of 48)}$$
$$48\left(\frac{1}{16}t\right) + 48\left(\frac{1}{12}t\right) = 48(1)$$
$$3t + 4t = 48$$
$$7t = 48$$
$$t = \frac{48}{7} \text{ or } 6\frac{6}{7}$$

It will take the printers $6\frac{6}{7}$ hours to complete the job.

Ex. 12 Working together, Carol and Joyce can make a dress in four hours. If Joyce works alone, it will take her six hours longer to make the dress than it would take Carol to make the same dress working alone. How long does it take each one to make the dress working alone?

<u>Solution:</u>
Let c = Carol's time to make the dress working alone. Joyce takes six hours longer to make the same dress, so her time is c + 6. Carol's time to complete the dress is c hours, so her rate is $\frac{1}{c}$ of a dress per hour. If she works for four hours, the portion of the dress she makes is $\frac{1}{c}(4)$ or $\frac{4}{c}$. Joyce's time to complete the dress is c + 6 hours, so her rate is $\frac{1}{c+6}$ of a dress per hour. If she works for four hours, the portion of the dress she makes is $\frac{1}{c+6}$ (4) or $\frac{4}{c+6}$. Since they are making one dress, then

$$\frac{4}{c} + \frac{4}{c+6} = 1 \qquad \text{(multiply by the L.C.D. of c(c + 6))}$$
$$\frac{4}{c} \bullet \frac{c(c+6)}{1} + \frac{4}{c+6} \bullet \frac{c(c+6)}{1} = 1 \bullet c(c+6)$$
$$\frac{4}{1} \bullet \frac{(c+6)}{1} + \frac{4}{1} \bullet \frac{c}{1} = c(c+6)$$

$4(c + 6) + 4c = c(c + 6)$

$4(c + 6) + 4c = c(c + 6)$

$4c + 24 + 4c = c^2 + 6c$

$8c + 24 = c^2 + 6c$

$\underline{-8c - 24 = -8c - 24}$

$0 = c^2 - 2c - 24$

$0 = (c - 6)(c + 4)$

$c = 6$ or $c = -4$

Reject,
time cannot be negative.

$c = 6$ and $c + 6 = 12$

It takes Carol 6 hours to make the dress by herself and it takes Joyce 12 hours to make the dress by herself.