Sect 8.3 - Graphs of Functions

Concept #1 Constant and Linear Functions

In this section, we will examine the graph of some elementary functions. Keep in mind when working with functions, y and f(x) are the same thing; we can use them interchangeably.
A linear equation in the form y = b, where b is a constant, is a horizontal line that passes through (0, b). If we replace y by f(x), then f(x) = b is called a constant function since the function value remains constant (the same) for every x-value. In general, a linear equation in two variables that is not a vertical line can be written in slope-intercept form, y = mx + b, where m is the slope of the line and (0, b) is the y-intercept. If we replace y by f(x), then f(x) = mx + b is a linear function.

Definition of a constant and a linear function.

Let m and b be real numbers such that $m \neq 0$, then

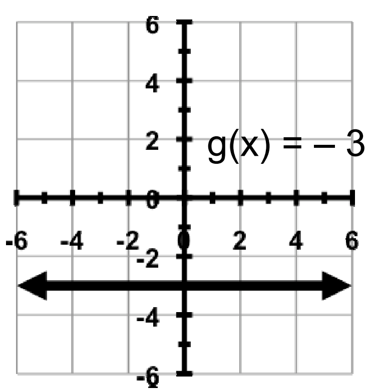
1) A **constant function** is a function that can be written in the form f(x) = b. The graph of this function is a horizontal line.
2) A **linear function** is a function that can be written in the form f(x) = mx + b. The graph of this function is a straight line.

<u>Sketch the graph of the following functions:</u>

Ex. 1a g(x) = – 3

Solution:

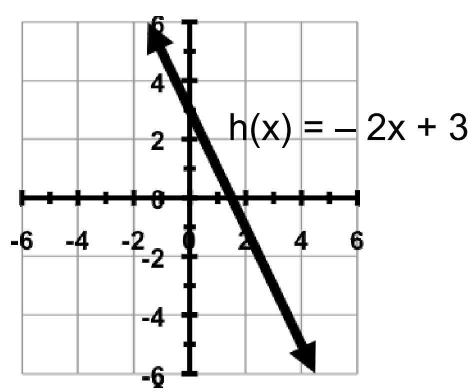
Since g(x) is a constant function, it's graph is a horizontal line passing through (0, – 3).



Ex. 1b h(x) = – 2x + 3

Solution:

$m = -2 = \frac{-2}{1}$ and the y-intercept is (0, 3). Plot the y-intercept and then from that point drop two two units and run one unit to the right. Repeat this process.



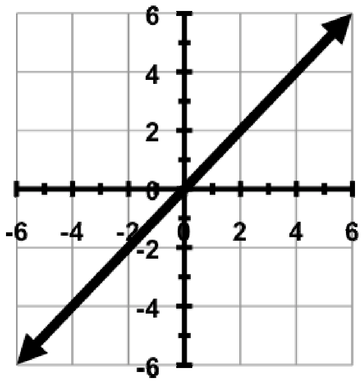
Concept #2 Graphs of Basic (or Parent) Functions

There are six basic functions that we will use in Algebra. It is important to know what their graphs are and their properties. The six functions are

1) $y = f(x) = x$ 2) $y = f(x) = x^2$ 3) $y = f(x) = x^3$

4) $y = f(x) = |x|$ 5) $y = f(x) = \sqrt{x}$ 6) $y = f(x) = \frac{1}{x}$

The first function is a linear equation with slope of 1 & a y-intercept of (0, 0).



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

To find the graphs of the other five functions, we will make a table of values and plot the points.

<u>Complete a table of values and plot the graphs of the following:</u>

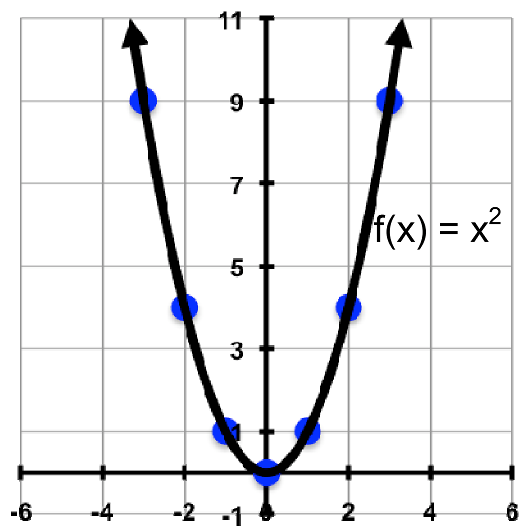
Ex. 2 $f(x) = x^2$ Ex. 3 $f(x) = x^3$ Ex. 4 $f(x) = |x|$

Ex. 5 $f(x) = \sqrt{x}$ Ex. 6 $f(x) = \frac{1}{x}$

<u>Solution:</u>

2) $f(x) = x^2$

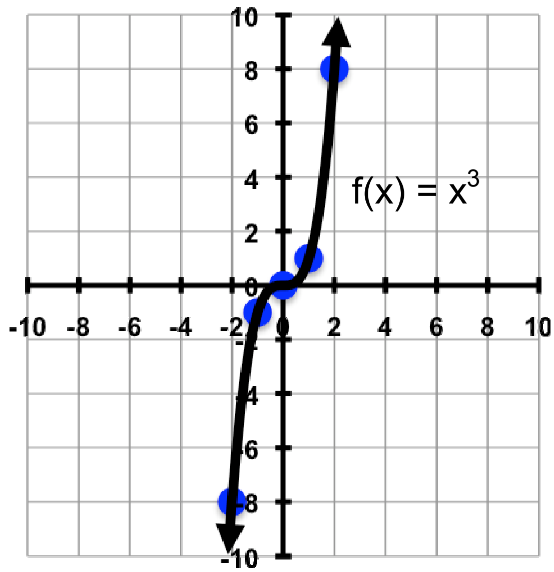
x	**$f(x) = x^2$**	**Points**
0	$(0)^2 = 0$	(0, 0)
1	$(1)^2 = 1$	(1, 1)
2	$(2)^2 = 4$	(2, 4)
3	$(3)^2 = 9$	(3, 9)
– 1	$(-1)^2 = 1$	(– 1, 1)
– 2	$(-2)^2 = 4$	(– 2, 4)
– 3	$(-3)^2 = 9$	(– 3, 9)



Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

3) f(x) = x^3

x	$f(x) = x^3$	Points
0	$(0)^3 = 0$	(0, 0)
1	$(1)^3 = 1$	(1, 1)
2	$(2)^3 = 8$	(2, 8)
3	$(3)^3 = 27$	(3, 27)
– 1	$(-1)^3 = -1$	(– 1, – 1)
– 2	$(-2)^3 = -8$	(– 2, – 8)
– 3	$(-3)^3 = -27$	(– 3, – 27)

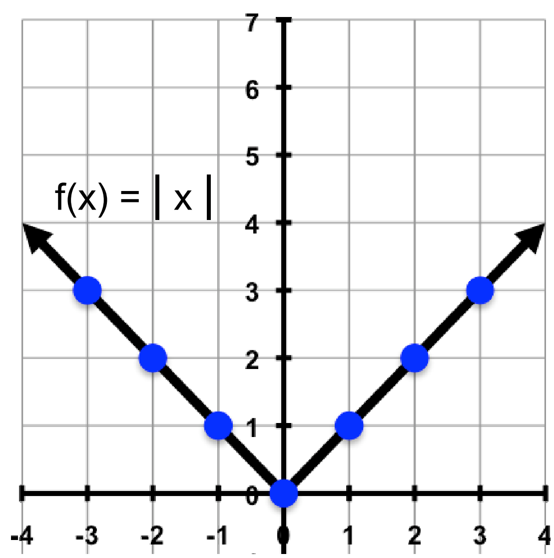


Notice that the points (3, 27) and (– 3, – 27) are not even on the graph.

Domain: (– ∞, ∞) Range: (– ∞, ∞)

4) f(x) = | x |

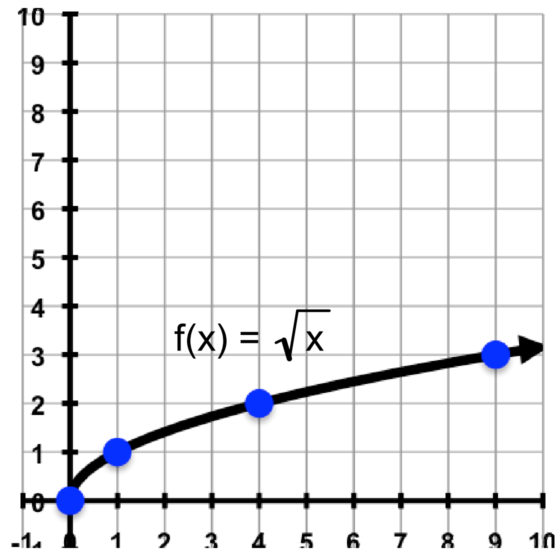
x	f(x) = | x |	Points
0	| 0 | = 0	(0, 0)
1	| 1 | = 1	(1, 1)
2	| 2 | = 2	(2, 2)
3	| 3 | = 3	(3, 3)
– 1	| – 1 | = 1	(– 1, 1)
– 2	| – 2 | = 2	(– 2, 2)
– 3	| – 3 | = 3	(– 3, 3)



Domain: (– ∞, ∞) Range: [0, ∞)

5) $f(x) = \sqrt{x}$

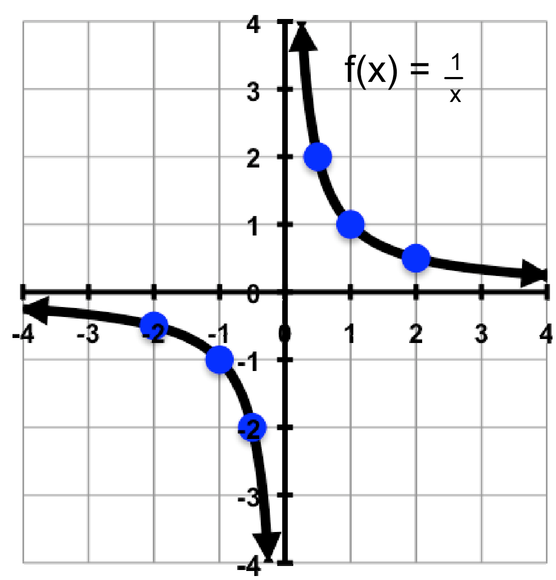
x	$f(x) = \sqrt{x}$	Points
0	$\sqrt{0} = 0$	(0, 0)
1	$\sqrt{1} = 1$	(1, 1)
4	$\sqrt{4} = 2$	(4, 2)
9	$\sqrt{9} = 3$	(9, 3)
– 1	$\sqrt{-1}$ is not a real #	No point
– 4	$\sqrt{-4}$ is not a real #	No point
– 9	$\sqrt{-9}$ = is not a real #	No point



Domain: [0, ∞) Range: [0, ∞)

6) $f(x) = \frac{1}{x}$

x	$f(x) = \frac{1}{x}$	Points
0	$\frac{1}{x}$ is undefined	No point
1	$\frac{1}{1} = 1$	(1, 1)
2	$\frac{1}{2} = \frac{1}{2}$	$(2, \frac{1}{2})$
$\frac{1}{2}$	$\frac{1}{\frac{1}{2}} = 2$	$(\frac{1}{2}, 2)$
– 1	$\frac{1}{-1} = -1$	(– 1, – 1)
– 2	$\frac{1}{-2} = -\frac{1}{2}$	$(-2, -\frac{1}{2})$
$-\frac{1}{2}$	$\frac{1}{-\frac{1}{2}} = -2$	$(-\frac{1}{2}, -2)$



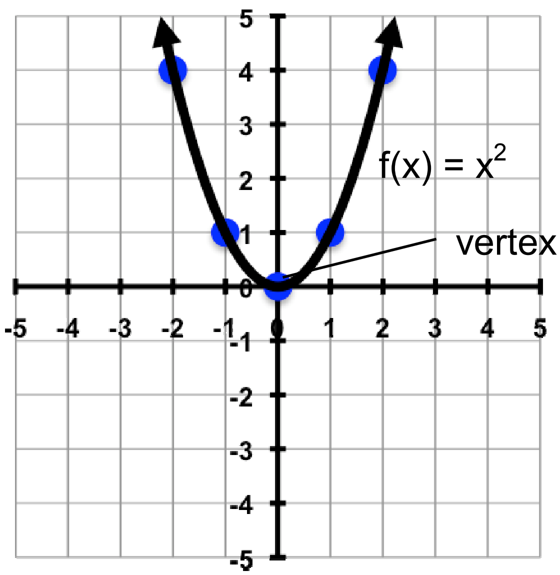
Domain: (– ∞, 0) U (0, ∞) Range: (– ∞, 0) U (0, ∞)

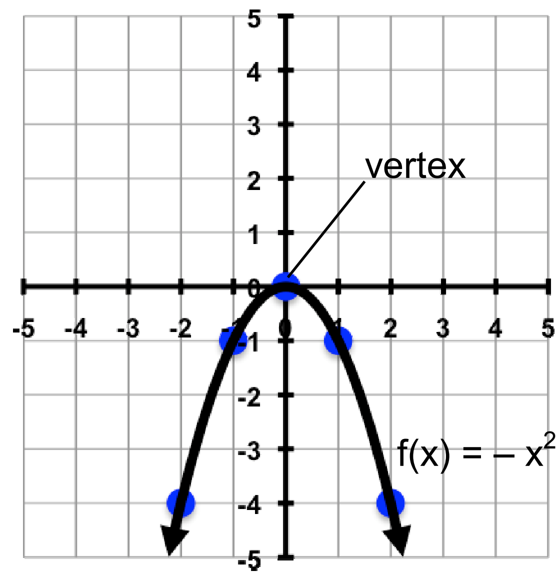
Concept #3 Definition of a Quadratic Function

Definition

A **Quadratic Function** is a function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The graph of a quadratic function is in the shape of **parabola**.

If the leading coefficient, a, in a quadratic function is positive, the parabola opens upward like a smile. The vertex on the parabola is the lowest point. If the leading coefficient, a, is negative, the parabola opens downward like a frown. The vertex on the parabola is then the highest point.





<u>Identify each function as a constant, linear, quadratic, or none of these:</u>

Ex. 7a $f(x) = 3x + 11$

Ex. 7b $g(y) = 9y^2 - 6y + 2$

Ex. 7c $r(w) = \frac{4}{13}w^3 - w^2$

Ex. 7d $p(x) = 8x^2 - 27$

Ex. 7e $h(x) = -4.6$

Ex. 7f $a(r) = \frac{r^2+4}{1-r^3}$

<u>Solution:</u>

a) The power of x is one, so this is a linear function.
b) The highest power of y is 2, so this is a quadratic function.
c) The highest power of w is 3, so this is none of these.
d) The power of x is 2, so this is a quadratic function.
e) Since h(x) equals a number, then h(x) is a constant function.
f) Since a(r) is a rational function, then a(r) is none of these.

Concept #4 x - and y - Intercepts

Definition
An **x-intercept** is a point (a, 0) where the graph intersects the x-axis.
A **y-intercept** is a point (0, b) where the graph intersects the y-axis.

Finding x- and y-intercepts
Step 1. Find the x-intercept(s) by setting f(x) = 0 and solving for x.
Step 2. Find the y-intercept(s) by finding f(0).

<u>a) Find the x-intercept, b) the y-intercept, and c) sketch the graph:</u>

Ex. 8 $f(x) = -\frac{2}{3}x + 3$

Solution:

a) To find the x-intercept, f(x) = 0 and solve:

$0 = -\frac{2}{3}x + 3$

$-3 = -\frac{2}{3}x$

$-\frac{3}{2}\left(-3\right) = -\frac{3}{2}\left(-\frac{2}{3}x\right)$

$4.5 = x$

So, the x-intercept is (4.5, 0).

b) To find the y-intercept, evaluate f(0):

$f(0) = -\frac{2}{3}(0) + 3$

$= 0 + 3$

$= 3$

So, the y-intercept is (0, 3).

c) Before we graph, let us find a third point:
Let x = 6:

$f(6) = -\frac{2}{3}(6) + 3$

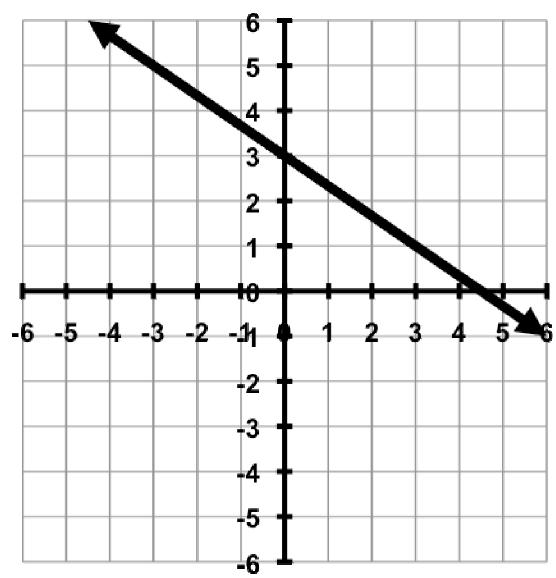
$= -4 + 3$

$= -1$

Thus, our table looks like:

x	y
4.5	0
0	3
6	– 1

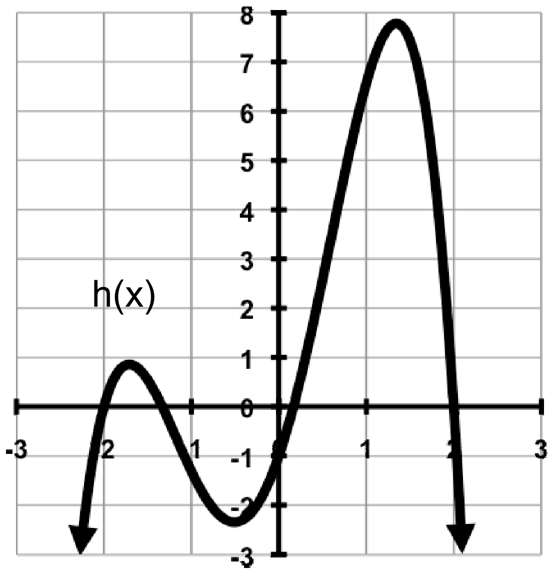
Now, graph the line.



Note, we could also have graphed the line using the slope & y-intercept.

<u>Given the graph of the functions below, find the x-intercepts and the y-intercepts:</u>

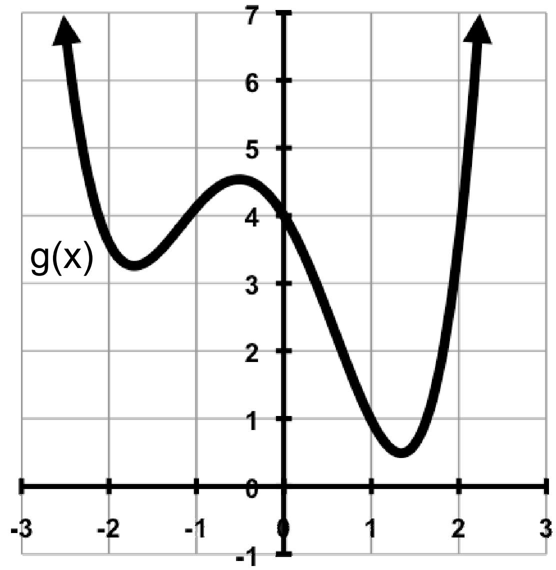
Ex. 9



Solution:

The graph crosses the x-axis at x = – 2, x ≈ – 1.3, x ≈ 0.15 and x = 2. So, the x-intercepts are (– 2, 0), (– 1.3, 0), (0.15, 0), and (2, 0). The graph crosses the y-axis at h(x) = – 1. So, the y-intercept is (0, – 1).

Ex. 10

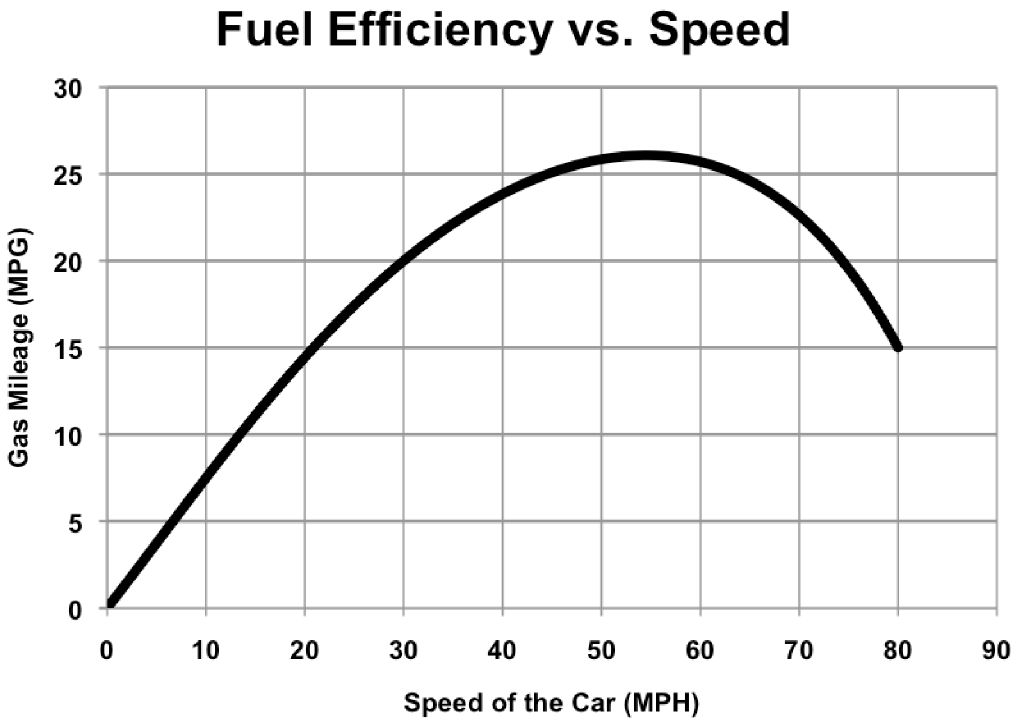


Solution:

The graph does not cross the x-axis which means the equation g(x) = 0 has no solution. Hence, there are no x-intercepts. The graph crosses the y-axis at g(x) = 4. So, the y-intercept is (0, 4).

Concept #5 Determining Intervals of Increasing, Decreasing, or Constant Behavior

If we were to plot the miles per gallon a car was achieving versus its speed, we would find that the faster the car travels, the better gas mileage it would get until the car's speed was near 55 mph. Once the car's speed exceeded that point, the gas mileage would start to decrease as the car started going faster. The optimal speed range would be between 50 and 60 mph. That is where the car is most fuel-efficient.
Let's take a closer look at the graph:



Notice that as the speed increases from 0 mph to about 52 mph, the fuel efficiency increases. Thus, the function is **increasing** on the interval (0, 52). When the speed is between 52 mph and about 57 mph, the fuel efficiency stays constant. Thus, the function is **constant** on the interval (52, 57). Finally, when the speed exceeds 57 mph, the fuel efficiency decreases. Hence, the function is **decreasing** on the interval (57, 80).

Intervals over which a Function is Increasing, Decreasing, or Constant

Let I be an open interval in the domain of the function f. Then,

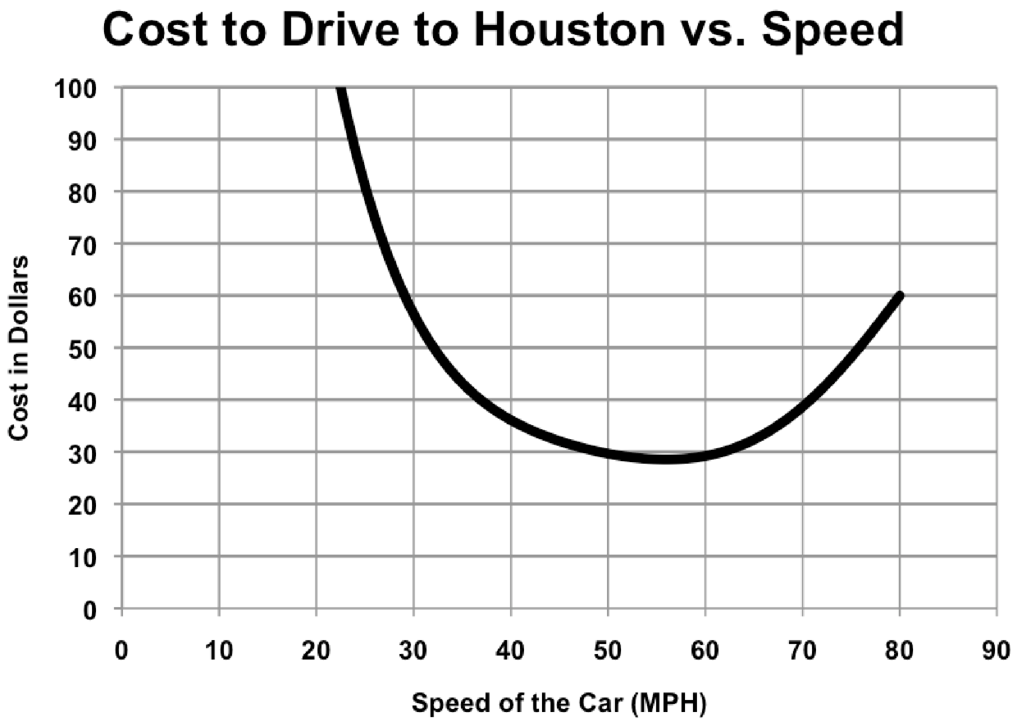
1) f is **increasing** on I if f(a) < f(b) for all a < b in I.
(a function is increasing on an interval if it rises or goes uphill as you move from left to right. ➚

2) f is **decreasing** on I if f(a) > f(b) for all a < b in I.
(a function is decreasing on an interval if it falls or goes downhill as you move from left to right. ➘

3) f is **constant** on I if f(a) = f(b) for all a and b in I.
a function is constant on an interval if it stays level or flat as you move from left to right. ➔

<u>Determine where the following function is a) increasing, b) decreasing, or c) constant:</u>

Ex. 11



Solution:

a) The function rises as we move from about 58 mph to 80 mph, thus the function is increasing on (58, 80).

b) The function falls as we move from 0 mph to about 53 mph, thus the function is decreasing on (0, 53).

c) The function stays level as we move from about 53 mph to about 58 mph, thus the function is constant on (53, 58).