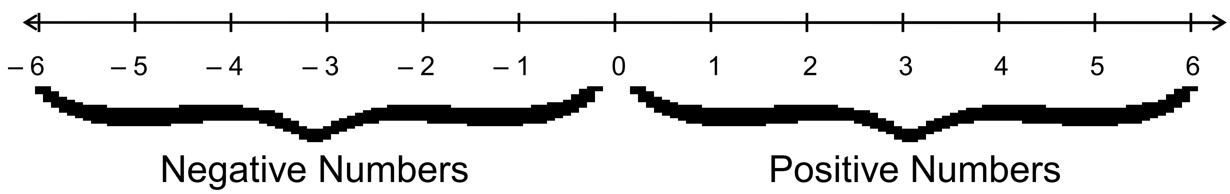
Sect 1.1 - Sets of Numbers and the Real Number Line

Concept #1: The Real Number line.

The real numbers include fractions, decimals, counting numbers, positive and negative numbers and zero. Each real number can be represented as a point on the real number line. The numbers that are greater than zero we will call the positive numbers and the numbers that are less than zero we will call the negative numbers. Typically, the positive real numbers are graphed as dots to the right of zero on the number line and the negative numbers are graphed as dots to the left of zero on the number line. Zero itself is consider neither positive or negative. Thus, our number line looks like:



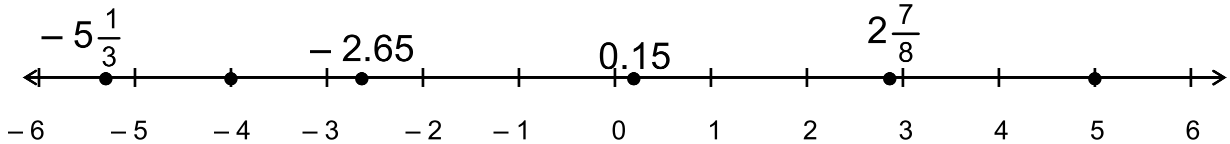
Concept #2: Plotting Points on the Number Line

<u>Plot each number on the real number line:</u>

Ex. 1 $2\frac{7}{8}$, – 2.65, 5, – $5\frac{1}{3}$, 0.15 and – 4

<u>Solution:</u>

$2\frac{7}{8}$ is between 2 and 3, but it is closer to 3 so we plot it close to 3. – 2.65 is between – 3 and – 2, but it is closer to – 3 so it is plotted slightly closer to – 3. Five is marked at 5. – $5\frac{1}{3}$ is between – 6 and – 5, but is closer to – 5, so we mark it accordingly. 0.15 is between 0 and 1, but much closer to 0, so it is marked near 0. Finally, – 4 is marked at – 4 .



Concept #3: Sets of Numbers

What precisely are the real numbers? If we look up “Real Numbers” in the dictionary, this is the definition that is given:

> “**re•al number** *n. Mathematics.*
> A number that is rational or irrational,” (American Heritage Electronic Dictionary © 1992 by Houghton Mifflin Company)

Clearly, this definition is not very helpful. To make sense of this definition, we need to discuss the set of numbers and their mean. We use “curly brackets” { } to denote a set. Thus, {4, 5, 6} represents the set containing the numbers 4, 5, and 6. We also use “three periods” ... to indicate that a pattern of numbers continues on forever. Thus, {4, 5, 6, 7, ...} is set containing the numbers 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on.

The counting numbers are essentially the numbers that we typically use to count items. The name we call this set is the **Natural Numbers**.

Natural Numbers: { 1, 2, 3, 4, 5, ... }

If we “add” zero to this set, we get the **Whole Numbers**.

Whole Numbers: { 0, 1, 2, 3, 4, 5, ... }

If we take the opposite of the Natural Numbers and “add” them to the Whole Numbers, we get the **Integers**.

Integers: { ... , – 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, ... }

Notice that every natural number is also a whole number, but not every whole number is a natural number. Similarly, every natural number and whole number is an integer, but not every integer is a whole number or a natural number.

Classify each of the following as a Natural Number, Whole Number, and/or Integer.

Ex. 2 5, – 6, 0, 6, – 7, 0.8, $\frac{2}{3}$, – 238, 15, 456, – 9, and – 17

Solution:
5, 6, 15, and 456 are natural numbers.
5, 0, 6, 15, and 456 are whole numbers.
5, – 6, 0, 6, – 7, – 238, 15, 456, – 9, and – 17 are integers.

Note that 0.8 and $\frac{2}{3}$ do not belong to any of these sets.

If we “add” fractions and decimals that terminate or repeat to the integers, we get the **Rational Numbers**. This set is too complicated to write as a list, so we will use the fact that every integer can be written as a fraction by putting the integer over 1 and that any repeating decimal can be expressed as the ratio of two integers.

Rational Numbers: $\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers and } b \neq 0 \}$

This is read as: “ the set of all numbers of the form a over b such that a and b are integers and b is not zero.” So, every integer is a rational number, but not every rational number is an integer.

There are some numbers that cannot be expressed as the quotient of two integers. Numbers like $\pi = 3.14159265358979323846...$ and $\sqrt{2} = 1.4142135623730950488...$ are non-terminating, non-repeating decimals. Such numbers are called **Irrational Numbers**.

Irrational Numbers: {a | a is a non-terminating, non-repeating decimal}

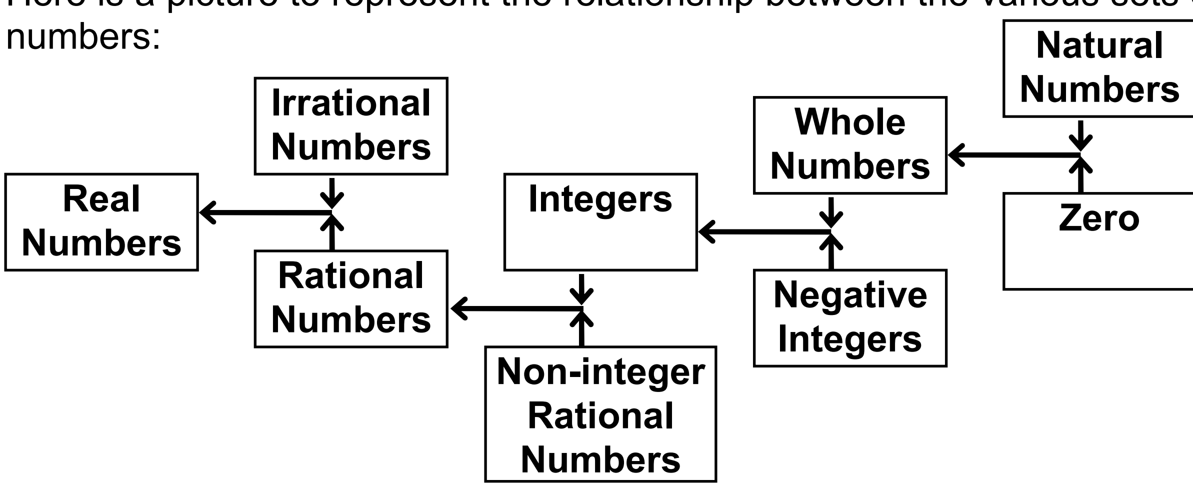
This is read as “the set of all numbers a such that a is a non-terminating, non-repeating decimal.” No rational number is an irrational numbers and no irrational number is a rational number.

If we put the set of rational numbers together the set of irrational numbers, we get the **Real Numbers**.

Real Numbers: {a | a is either a rational or an irrational number}

This is read as “the set of all numbers a such that a is a rational or an irrational number.”

Here is a picture to represent the relationship between the various sets of numbers:



Classify each of the following as a Natural Number, Whole Number, Integer, Rational Number, Irrational Number, and/or Real Number.

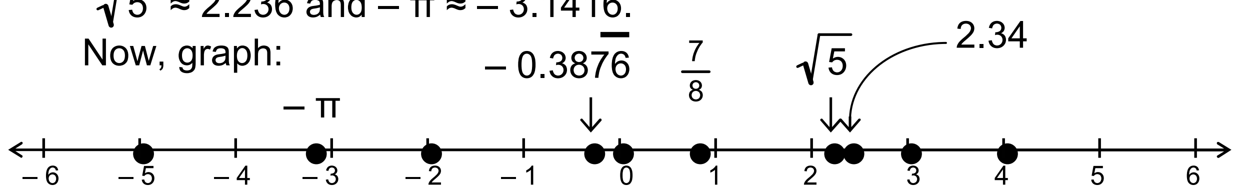
Ex. 3 $\{\sqrt{5}, 0, \frac{7}{8}, -0.38\overline{76}, -2, 4, 2.34, -\pi, 3, -5\}$

Solution:
4 and 3 are Natural Numbers.
0, 4, and 3 are Whole Numbers.
0, – 2, 4, 3, and – 5 are integers.
$0, \frac{7}{8}, -0.38\overline{76}, -2, 4, 2.34\ 3,$ and -5 are Rational Numbers.
$\sqrt{5}$ and $-\pi$ are Irrational Numbers.
$\sqrt{5}, 0, \frac{7}{8}, -0.38\overline{76}, -2, 4, 2.34, -\pi, 3,$ and -5 are Real Numbers.

Graph each number on a number line:

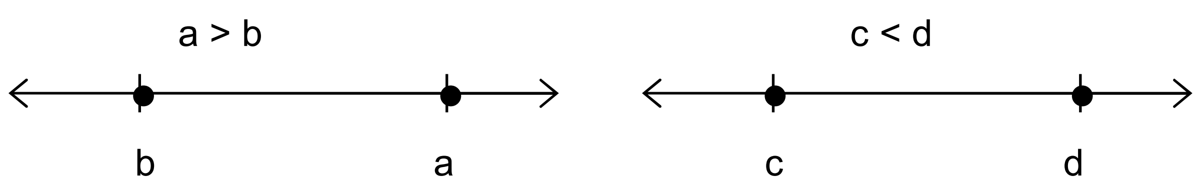
Ex. 4 $\{\sqrt{5}, 0, \frac{7}{8}, -0.38\overline{76}, -2, 4, 2.34, -\pi, 3, -5\}$

Solution:
First, find some approximations for the irrational numbers:
$\sqrt{5} \approx 2.236$ and $-\pi \approx -3.1416$.
Now, graph:



Concept #4: Inequalities

In Mathematics, if we want to say that "5 is greater than 3," we write 5 > 3. Notice that 5 lies to the right of 3 on the number line. Thus, a > b if and only if a lies to the right of b on the number line. Similarly, if we want to say that "2 is less than 6," we write 2 < 6. Notice that 2 lies to the left of 6 on the number line. Thus, c < d if and only if c lies to the left of d on the number line.



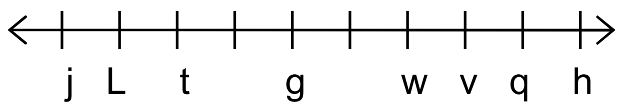
The symbols > and < are called inequality signs. The statements 5 > 3 and 2 < 6 are called inequalities. Think of the inequality sign as a mouth that wants to eat the larger number.

Listed below are other types of symbols

Inequality	Translation	Example
a > b	a is greater than b.	6 > 4
c < d	c is less than d.	3 < 8
e ≥ f	e is greater than or equal to f.	8 ≥ 7
g ≤ h	g is less than or equal to h.	9 ≤ 9
j = k	j is equal to k.	0.6 = 3/5
m ≠ n	m is not equal to n.	11 ≠ 7
r ≈ s	r is approximately equal to s	3.98 ≈ 4

Compare using >, <, or = :

Ex. 5



a) g h
b) t j
c) v w
d) j h
e) L v
f) q t

Solution:

a) Since g is to the left of h, then g < h.
b) Since t is to the right of j, then t > j.
c) Since v is to the right of w, then v > w.
d) Since j is to the left of h, then j < h.
e) Since L is to the left of v, then L < v.
f) Since q is to the right of t, then q > t.

Ex. 6a 8 11

Ex. 6b – 8 – 11

Ex. 6c $\sqrt{5}$ 2.236

Ex. 6d $-\frac{5}{8}$ – 0.63

Ex. 6e $-\frac{2}{3}$ 0.12

Solution:

a) Since 8 is smaller than 11, then 8 < 11.
b) Since – 8 is to the right of – 11, then – 8 > – 11.
c) Since $\sqrt{5}$ = 2.23606... is to the right of 2.236, then $\sqrt{5}$ > 2.236.
d) $\frac{5}{8}$ = 5 ÷ 8 = 0.625. Since 0.625 < 0.630 = 0.63, then – 0.625 > – 0.63. So, $-\frac{5}{8}$ > – 0.63.
e) Since a – # is always less than a + #, then $-\frac{2}{3}$ < 0.12.

Concept #5: The Opposite of a Real Number

The **Opposite or Additive Inverse** of a number is a number that is the same distance from 0, but on the opposite side of zero. The opposite of 3 is – 3 and the opposite of – 2 is 2. Mathematically, if we want to say the opposite of 3, we write: – (3) and if we want to say the opposite of – 2, we write: – (– 2).

<u>Evaluate the following:</u>

Ex. 7a) – (– 8)

Ex. 7b) $-\left(\frac{2}{3}\right)$

<u>Solution:</u>

a) The opposite of – 8 is 8, so – (– 8) = 8.

b) The opposite of $\frac{2}{3}$ is $-\frac{2}{3}$, so $-\left(\frac{2}{3}\right) = -\frac{2}{3}$.

Concept #6: The Absolute Value of a Real Number

The **Absolute Value** of a number is the distance that number is from zero. The absolute value is denoted with two vertical lines so if we want to say the absolute value of a number, we write: | the number |.

Note: | – # | = + #, but | + # | = + #.

<u>Evaluate the following:</u>

Ex. 8a) $\left|\frac{2}{9}\right|$

Ex. 8b $|-7.8|$

Ex. 8c) $-|3|$

Ex. 8d $-|-6.1|$

Ex. 8e) – (8)

Ex. 8f $-\left(-\frac{5}{9}\right)$

<u>Solution:</u>

a) The absolute value of $\frac{2}{9}$ is $\frac{2}{9}$, so $\left|\frac{2}{9}\right| = \frac{2}{9}$.

b) The absolute value of – 7.8 is 7.8, so $|-7.8| = 7.8$.

c) The absolute value of 3 is 3. The opposite of 3 is – 3, so $-|3| = -3$.

d) The absolute value of – 6.1 is 6.1. The opposite of 6.1 is – 6.1, so $-|-6.1| = -6.1$.

e) The opposite of 8 is – 8, so – (8) = – 8.

f) The opposite of $-\frac{5}{9}$ is $\frac{5}{9}$, so $-\left(-\frac{5}{9}\right) = \frac{5}{9}$.

Remember: Additive Inverses or Opposites - we switch signs.
Absolute Value - we make the number inside the two lines positive.

Compare using >, <, or =:

Ex. 9a $-(-5.3) \qquad -|-5.3|$

Solution:

Since $-(-5.3) = 5.3$ and

$-|-5.3| = -(5.3) = -5.3$

and $5.3 > -5.3$, then

$-(-5.3) \quad > \quad -|-5.3|$.

Ex. 9b $-\left(\frac{8}{13}\right) \qquad -\left|\frac{8}{13}\right|$

Solution:

Since $-\left(\frac{8}{13}\right) = -\frac{8}{13}$ and

$-\left|\frac{8}{13}\right| = -\left(\frac{8}{13}\right) = -\frac{8}{13}$, and

$-\frac{8}{13} = -\frac{8}{13}$, then

$-\left(\frac{8}{13}\right) \quad = \quad -\left|\frac{8}{13}\right|$