Sect 1.2 - Order of Operations

Concept #1 Variables and Expressions

In Algebra, when there is a number that we do not know its value, we represent the number using a **variable** like the letter x. So, if we want to write three times an unknown number, we write 3•x or 3x. An **Algebraic Expression** is a collection of numbers, variables (letters), grouping symbols, and operations (addition "sum", subtraction "difference", multiplication "product", and division "quotient"). Some examples of algebraic expressions are:

Ex. 1a $9y - 6z + 2$

Ex. 1b $\frac{8x^2}{7y} - 3(x + 5)$

Ex. 2a $0.45x^2y - 3xy^2$

Ex. 2b $x^3 - 27y^3$

There are several ways we can represent operations in algebra:

Operation	**Algebraic Expression**	**Translation**
Addition	a + b	The sum of a and b.
Subtraction	a – b	The difference of a and b.
Multiplication	a•b, a(b), (a)b, (a)(b), ab (Note a x b is rarely used)	The product of a and b.
Division	$a \div b$, $\frac{a}{b}$, a/b, $b\overline{)a}$	The quotient of a and b.

Concept #2 Evaluating Algebraic Expressions

To evaluate an expression, replace the letters by the numbers given. Use a set of parenthesis around the number when you plug-in the number.

<u>Evaluate the following for x = 0.3 and y = 1.3:</u>

Ex. 3a $x + y$

Ex. 3b $5x - y$

Ex. 4a xy

Ex. 4b $\frac{x}{y}$

<u>Solution:</u>

3a) $x + y = (0.3) + (1.3) = 1.6$

3b) $5x - y = 5(0.3) - (1.3) = 1.5 - 1.3 = 0.2$

4a) $xy = (0.3)(1.3) = 0.39$

4b) $\frac{x}{y} = \frac{(0.3)}{(1.3)} = \frac{3}{13}$ (move the decimal point over one place)

Concept #3 Exponential Expressions

Exponential notation is a shortcut for repeated multiplication. Consider the follow:

$$5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5$$

Here, we are multiplying six factors of five. We will call the 5 our base and the 6 the exponent or power. We will rewrite this as 5 to the 6^{th} power:

$$5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 = 5^6$$

The number that is being multiplied is the base and the number of factors of that number is the power.

Definition

Let a be a real number and n be a natural number. Then

$$a^n = \underbrace{a \bullet a \bullet a \bullet a \bullet a \ldots \bullet a}_{\text{n factors of a}}$$ a is the base & n is the exponent or power.

a^n is read as "a to the nth power."
a^2 is read as "a squared" and a^3 is read as "a cubed."

Evaluate the following:

Ex. 5a 3^4

Ex. 5b $(0.2)^3$

Ex. 5c $\left(\frac{7}{3}\right)^2$

Ex. 5d 1^5

Solution:

a) $3^4 = 3 \bullet 3 \bullet 3 \bullet 3 = 81$

b) $(0.2)^3 = (0.2)(0.2)(0.2) = 0.008$

c) $\left(\frac{7}{3}\right)^2 = \left(\frac{7}{3}\right)\left(\frac{7}{3}\right) = \frac{49}{9}$

d) $1^5 = 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 = 1$

Concept #4 Square Roots

The square of a whole number is called a **perfect square**. So, 1, 4, 9, 16, 25 are perfect squares since $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, and $25 = 5^2$. We use the idea of perfect squares to simplify square roots. The square root of a number a asks what number times itself is equal to a. For example, the square root of 25 is 5 since 5 times 5 is 25.

The **square root** of a number a, denoted $\sqrt{a}$, is a number whose square is a. So, $\sqrt{25} = 5$. The symbol $\sqrt{}$ is called a radical sign.

Simplify the following:

Ex. 6a $\sqrt{81}$ Ex. 6b $\sqrt{121}$

Ex. 6c $\sqrt{0}$ Ex. 6d $\sqrt{900}$

Ex. 6e $\sqrt{\frac{1}{36}}$ Ex. 6f $\sqrt{\frac{49}{144}}$

Solution:

a) $\sqrt{81} = 9$. b) $\sqrt{121} = 11$.

c) $\sqrt{0} = 0$. d) $\sqrt{900} = 30$.

e) $\sqrt{\frac{1}{36}} = \frac{1}{6}$. f) $\sqrt{\frac{49}{144}} = \frac{7}{12}$.

Concept # 5 Order of Operations

If you ever have done some cooking, you know how important it is to follow the directions to a recipe. An angel food cake will not come out right if you just mix all the ingredients and bake it in a pan. Without separating the egg whites from the egg yolks and whipping the eggs whites and so forth, you will end up with a mess. The same is true in mathematics; if you just mix the operations up without following the order of operations, you will have a mess. Unlike directions for making a cake that differ from recipe to recipe, the order of operations always stays the same. The order of operations are:

Order of Operations

1) Parentheses - Simplify the expression inside of Parentheses or grouping symbol: ( ), [ ], { }, $| \ |$, $\sqrt{\ }$
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

A common phrase people like to use is:

Please
Excuse
My **D**ear
Aunt **S**ally

(Be careful with the My Dear and the Aunt Sally part. Multiplication does not precede Division and Division does not precede multiplication; they are done as they appear from left to right. The same is true for addition and subtraction.)

Simplify the following:

Ex. 7 $45 \div 3^2 \bullet \sqrt{25} + 3 \bullet 7^2$

Solution:

Since there are no parentheses, we start with step #2, exponents:

$45 \div 3^2 \bullet \sqrt{25} + 3 \bullet 7^2$ (#2-exponents)

$= 45 \div 9 \bullet 5 + 3 \bullet 49$ (#3-division)

$= 5 \bullet 5 + 3 \bullet 49$ (#3-multiplication)

$= 25 + 3 \bullet 49$ (#3-multiplication)

$= 25 + 147$ (#4-addition)

$= 172$

Ex. 8 $\left(\frac{21}{4} \bullet \frac{1}{6} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$

Solution:

$\left(\frac{21}{4} \bullet \frac{1}{6} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$ (reduce inside the parenthesis)

$= \left(\frac{3 \bullet 7}{4} \bullet \frac{1}{3 \bullet 2} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$

$= \left(\frac{7}{4} \bullet \frac{1}{2} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$ (#1-parentheses, #3-multiplication)

$= \left(\frac{7}{8} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$ (#1-parentheses, #4-subtraction)

$= \left(\frac{6}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} = \left(\frac{2 \bullet 3}{2 \bullet 4}\right)^2 \div \frac{11}{24} - \frac{3}{11}$ (reduce)

$= \left(\frac{3}{4}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$ (#2-exponents)

$= \frac{9}{16} \div \frac{11}{24} - \frac{3}{11}$ (invert and change operation to •)

$= \frac{9}{16} \bullet \frac{24}{11} - \frac{3}{11} = \frac{9}{2 \bullet 8} \bullet \frac{3 \bullet 8}{11} - \frac{3}{11}$ (reduce)

$= \frac{9}{2 \bullet 8} \bullet \frac{3 \bullet 8}{11} - \frac{3}{11}$ (#3-multiplication)

$= \frac{9}{2} \bullet \frac{3}{11} - \frac{3}{11}$ (#3-multiplication)

$= \frac{27}{22} - \frac{3}{11}$ (L.C.D. = 22)

$= \frac{27}{22} - \frac{3 \bullet 2}{11 \bullet 2} = \frac{27}{22} - \frac{6}{22}$ (#4-subtraction)

$= \frac{21}{22}$.

Ex. 9 $(9.1)^2 - (2.4 - |-1.2| \div 3) + (5-2)^2$

Solution:

$(9.1)^2 - (2.4 - |-1.2| \div 3) + (5-2)^2$ (#1-paren., #1-Abs. Val)

$= (9.1)^2 - (2.4 - 1.2 \div 3) + (5-2)^2$ (#1-parentheses, #3-division)

$= (9.1)^2 - (2.4 - 0.4) + (5-2)^2$ (#1-parentheses, #4-subtr.)

$= (9.1)^2 - 2 + (3)^2$ (#2-exponents)

$= 82.81 - 2 + 9$ (#4-subtraction)

$= 80.81 + 9$ (#4-addition)

$= 89.81$

Ex. 10 $75 - 3\sqrt{100-(16-7)(4)}$

Solution:

$75 - 3\sqrt{100-(16-7)(4)}$ (#1-radical, #1-parenth., #4-subt.)

$= 75 - 3\sqrt{100-(9)(4)}$ (#1-radical, #3-multiplication)

$= 75 - 3\sqrt{100-36}$ (#1-radical, #4-subtraction)

$= 75 - 3\sqrt{64}$ (#2-exponents)

$= 75 - 3(8)$ (#3-multiplication)

$= 75 - 24$ (#4-subtraction)

$= 51$

Concept #6 Translation

Here are some key phrases and their translation

Addition

the total of 11 and x	(11 + x)
8 added to y	y + 8
7 more than f	f + 7
4 increased by w	4 + w
the sum of 8 and h	(8 + h)
r plus s	r + s
13 greater than t	t + 13
exceeds r by 6	r + 6

Subtraction

6 minus x	6 – x
4 less y	4 – y
4 less than y	y – 4
the difference between 8 and x	(8 – x)

y decreased by 5	y – 5
x subtract 5	x – 5
x subtracted from 5	5 – x
q reduced by 43	q – 43

Multiplication

6 times d	6d
the product of 11 and r	(11r)
$\frac{2}{5}$ of J	$\frac{2}{5}$ J
V multiplied by $\frac{5}{6}$	$\frac{5}{6}$ V
twice R	2R
triple w	3w
double q	2q

Division

7 divided into x	$x \div 7 = \frac{x}{7}$
7 divided by x	$7 \div x = \frac{7}{x}$
the quotient of 5 and y	$(5 \div y) = \frac{5}{y}$
the ratio of 4 to 11	$4 \div 11 = \frac{4}{11}$
y split into 7 equal parts	$y \div 7 = \frac{y}{7}$
x over 4	$\frac{x}{4}$
9 per y	$9 \div y = \frac{9}{y}$

Note: Subtraction and division are not commutative. You need to learn the correct order with the vocabulary word.

<u>a) Translate and then b) evaluate for a = 36, b = 2, and c = 72:</u>

Ex. 11 The quotient of the square root of a and b.

<u>Solution:</u>

a) The square root of a means $\sqrt{a}$. The quotient means write the first quantity divided by the second, so the quotient of $\sqrt{a}$ and b is $\sqrt{a} \div b$.

b) $\sqrt{a} \div b = \sqrt{(36)} \div (2) = 6 \div 2 = 3$

Ex. 12 Twice the sum of a and c.

Solution:

a) The sum of a and c means [a + c] and twice the sum means to multiply the sum by 2, so the answer will be $2\bullet[a + c] = 2[a + c]$.

b) $2[a + c] = 2[(36) + (72)] = 2[108] = 216$

Ex. 13 Triple the absolute value of the difference between c and a, exceeded by b.

Solution:

a) We need to break this one down into little pieces.

The difference between c and a: $(c – a)$

The absolute value of (c – a): $|c – a|$

Triple $|c – a|$: $3|c – a|$

$3|c – a|$ exceeded by b: $3|c – a| + b$

b) $3|c – a| + b = 3|(72) – (36)| + 2 = 3|36| + 2$

$= 3(36) + 2 = 108 + 2 = 110$