Sect 1.5 - Multiplication of Whole Numbers and Area

Objective a: Understanding multiplication.

Multiplication is a shortcut for repeated addition. To see how this works, let's look at an example:

Perform the following operations:

Ex. 1 4 + 4 + 4 + 4 + 4 + 4

Solution:

4 + 4 + 4 + 4 + 4+ 4 = **8 + 4** + 4 + 4 + 4 = **12 + 4** + 4 + 4

= **16 + 4** + 4 = **20 + 4** = 24.

Notice that we added six 4's, so the shortcut is 6 times 4 = 24.

There are several different ways that multiplication can be written:

$6 \times 4 = 6 \bullet 4 = 6(4) = (6)4 = (6)(4)$

All of these mean 6 times 4. We will be using the dot notation more and more as the semester progresses. The reason for this is that in Algebra, letters are used for unknown numbers. Since x is a letter that is commonly used for an unknown number, it gets hard to tell the difference between the letter x used for an unknown number and × for multiplication. To avoid this confusion, we use • for multiplication in place of × in Algebra. When we say 6 • 4 = 24, we say that 6 and 4 are **factors** and 24 is the **product**.

Objective b: Understanding properties of multiplication

To illustrate some important properties involving multiplication, let's look at some examples.

Find the following:

Ex. 2a 9 • 5

Solution:

9 • 5 = 45

Ex. 2b 5 • 9

Solution:

5 • 9 = 45

Notice that both problems give us the same answer; it does not matter which order we write the numbers, the result is the same. This property of multiplication is called the **Commutative Property of Multiplication**.

Commutative Property of Multiplication

$a \bullet b = b \bullet a$

Find the following:

Ex. 3a $(8 \bullet 4) \bullet 5$

Solution:

$(8 \bullet 4) \bullet 5 = 32 \bullet 5 = 160.$

Ex. 3b $8 \bullet (4 \bullet 5)$

Solution:

$8 \bullet (4 \bullet 5) = 8 \bullet 20 = 160.$

Notice that both problems gave us the same answer. When multiplying three numbers together, it does not matter which two numbers you times together first, the first two or the last two, you will get the same result in the end. This is known as the **Associative Property of Multiplication.**

Associative Property of Multiplication

$(a \bullet b) \bullet c = a \bullet (b \bullet c)$

If you multiply any number by zero, you get zero. If you multiply any number by 1, you get the same number. These are known as the **Multiplication Property of Zero** and the **Multiplication Property of One** respectively.

Multiplication Property of Zero

$a \bullet 0 = 0 \bullet a = 0$

Multiplication Property of One

$a \bullet 1 = 1 \bullet a = a$

Find the following:

Ex. 4a $189 \bullet 0$

Ex. 4b $75 \bullet 1$

Solution:

a) $189 \bullet 0 = 0$ by the Multiplication Property of Zero.

b) $75 \bullet 1 = 75$ by the Multiplication Property of One.

The last property we want to look at is the distributive property. Let's look at an example that illustrates this property.

Find the following:

Ex. 5 $7(3 + 5)$

Solution:

Normally, we would add the three and the five together first and then multiply:

$7(\mathbf{3 + 5}) = 7(\mathbf{8}) = 56$

But, the distributive property says that we can multiply the 3 and 5 both by seven first and then add the results:

$\mathbf{7}(3 + 5) = \mathbf{7}\bullet 3 + \mathbf{7}\bullet 5 = 21 + 35 = 56.$

Notice that we get the same result either way. This is an extremely important property in mathematics. We will see how it is used in multiplication in a few minutes.

Distributive Property

a(b + c) = **a**•b + **a**•c

<u>Find the following using the distributive property:</u>

Ex. 6 5(4 + 7)

<u>Solution:</u>

5(4 + 7) = **5**•4 + **5**•7 = 20 + 35 = 55

Objective c: Multiplying by powers of 10.

<u>Find the following:</u>

Ex. 7a The product of 15 and 10.

<u>Solution:</u>

15 × 10 = 150. It is like taking 15 times 1 and putting a zero at the end.

Ex. 7b The product of 15 and 100.

<u>Solution:</u>

15 × 100 = 1500. It is like taking 15 times 1 and putting two zeros at the end.

Ex. 7c The product of 15 and 1000.

<u>Solution:</u>

15 × 1000 = 15,000. It is like taking 15 times 1 and putting three zeros at the end.

Ex. 7d The product of 15 and 100,000.

<u>Solution:</u>

15 × 100,000 = 1,500,000. It is like taking 15 times 1 and putting five zeros at the end.

Ex. 8a 6000 × 300

<u>Solution:</u>

Take 6 × 3 = 18 and add 3 + 2 = 5 zeros at the end of the problem: 18**00000** = 1,800,000.

Ex. 8b 456•75•0•56•2

Solution:

Any number times 0 is 0, so the answer is 0.

Ex. 8c 9,000,000 × 300,000

Solution:

Take 9 × 3 = 27 and add 6 + 5 = 11 zeros at the end of the problem: 27**00000000000** = 2,700,000,000,000.

Objective d: Multiplying Whole Numbers.

In multiplying two whole numbers, we can stack the numbers vertically being careful to align the digits according to their place values. We can think of the bottom number in expanded form and use the distributive property to generate our rows of partial products. Then, add the results. Here are some key words that imply multiplication:

6 times 9	6•9
the product of 11 and 6	(11•6)
$\frac{2}{5}$ of 5	$\left(\frac{2}{5}\right)(5)$
8 multiplied by $\frac{5}{6}$	$8\bullet\frac{5}{6}$
twice 6	2(6)
triple 5	3(5)
double 7	2•7

Perform the Indicated Operation. Check the answer on a calculator:

Ex. 9 Find the product of 684 and 73.

Solution:

Think of 684 × 73 as **684** × (70 + 3) = **684** × 70 + **684** × 3 and multiply:

21	4 × 3 = 12, write down 2, carry the 1.
684	8 × 3 = 24, 24 + 1 = 25, write down 5, carry the 2.
× 3	6 × 3 = 18, 18 + 2 = 20, write down 20
2052	(1st partial product)
	Write down a zero.
52	4 × 7 = 28, write down 8, carry the 2.
684	8 × 7 = 56, 56 + 2 = 58, write down 8, carry the 5.
× 70	6 × 7 = 42, 42 + 5 = 47, write down 47
47880	(2nd partial product)

Now add the two partial products:

$$\begin{array}{rl} & \overset{}{684} \\ \times & \underline{73} \\ & \mathbf{\mathit{1}} \\ & 2052 \quad \leftarrow 684 \times 3 \\ + & \underline{47880} \quad \leftarrow 684 \times 70 \\ & 49{,}932 \end{array}$$

So, the answer is 49,932.

Ex. 10 2174 multiplied by 5034

Solution:

Write 5034 in expanded form, use the distribute and multiply:

21	$4 \times 4 = 16$, write down 6, carry the 1.
2174	$7 \times 4 = 28$, $28 + 1 = 29$, write down 9, carry the 2.
$\times$ 4	$1 \times 4 = 4$, $4 + 2 = 6$, write down 6.
8696	$2 \times 4 = 8$, write down 8.
	Write down zero
21	$4 \times 3 = 12$, write down 2, carry the 1.
2174	$7 \times 3 = 21$, $21 + 1 = 22$, write down 2, carry the 2.
$\times$ 30	$1 \times 3 = 3$, $3 + 2 = 5$, write down 5.
65220	$2 \times 3 = 6$, write down 6.
	Write down three zeros
32	$4 \times 5 = 20$, write down 0, carry the 2.
2174	$7 \times 5 = 35$, $35 + 2 = 37$, write down 7, carry the 3.
$\times$ 5000	$1 \times 5 = 5$, $5 + 3 = 8$, write down 8.
10870000	$2 \times 5 = 10$, write down 10.

Now add the partial products:

$$\begin{array}{rr} & 2174 \\ \times & \underline{5034} \\ & \mathit{1} \\ & \mathit{1}\,8696 \\ & \mathit{1}\,65220 \\ + & \underline{10870000} \\ & 10943916 \end{array}$$

So, the answer is 10,943,916.

Objective d: Estimating Products by Rounding.

Solve the following:

Ex. 11 The average price for a ticket to a Spurs game is $74. If the AT&T center can hold 18,797 people, estimate the maximum revenue during a basketball game by rounding the capacity to the nearest ten-thousand and the ticket price to the nearest ten.

Solution:

18,797	→	20,000	$2 \times 7 = 14$
$\times$ 74	→	$\times$ 70	Add five zeros:
		1,400,000	

Objective e: Finding the area of a rectangle.

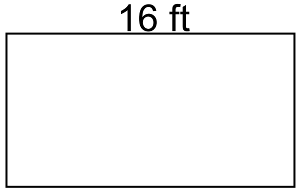
The area of a object is the amount of region inside the object. For rectangles, the area of a rectangle in equal to the length times the width.

Find the area of a rectangle:

Ex. 12

16 ft

9 ft



Solution

$A = 9 \bullet 16 = 9(10 + 6)$

$= 9 \bullet 10 + 9 \bullet 6 = 90 + 54 = 144$

So, the area is 144 square feet.

Ex. 13 In a small neighborhood in San Antonio, land is appraised at $2 per square foot. If Juan has a rectangle vacant lot measuring 360 feet by 120 feet, how much is his land valued at?

Solution:

First we need to find the area of Juan's lot by multiplying 360 by 120:

360	
$\times$ 120	
7200	Write down 2 zeros, $36 \times 2 = 72$, write down 72.
36000	Write down 3 zeros, $36 \times 1 = 36$, write down 36.
43200	Add the two rows

The area of the lot is 43,200 square feet (1 Acre = 43,560 sq. ft) which is little less than one acre. The value of the land is appraised at $2 per square foot, so to find the value of the lot, we multiply 43,200 by 2:

$43,200 \times 2 = 86,400$.

So, the land is valued at $86,400.