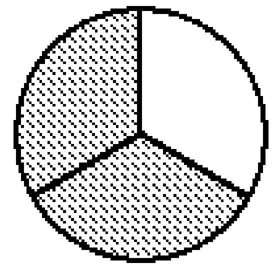
Section 2.1 – Introduction to Fractions and Mixed Numbers

Objective a: Definition of a Fraction.

Let's start with an example.



Ex. 1 Suppose a pie is cut into three pieces and one of the three pieces is consumed. This means there are two out of three pieces left. We can represent this as a fraction:

2 ← Number of pieces left
3 ← Number of pieces in the whole

So, there is $\frac{2}{3}$ of the pie left. Also, $\frac{1}{3}$ of the pie was consumed.

A **fraction** is a number written in the form $\frac{a}{b}$ where a and b are whole numbers and b cannot be 0. The top number is called the numerator and the bottom number is the denominator. The numerator is the number of pieces being considered and the denominator is the number of pieces it takes to make a whole.

Identify the Numerator and the Denominator of the following:

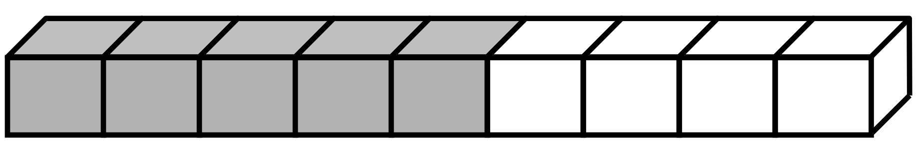
Ex. 2a $\frac{5}{9}$ Ex. 2b $\frac{11}{7}$ Ex. 2c $\frac{1}{5}$

Solution:

a) The numerator is 5 and the denominator is 9.
b) The numerator is 11 and the denominator is 7.
c) The numerator is 1 and the denominator is 5.

Represent the following as fractions:

Ex. 3



Solution:

Since 5 out of 9 blocks are shaded, we can say that $\frac{5}{9}$ of the blocks are shaded and $\frac{4}{9}$ of the blocks are not shaded.

Ex. 4 Wil was late to work two out of five times this week.

Solution:

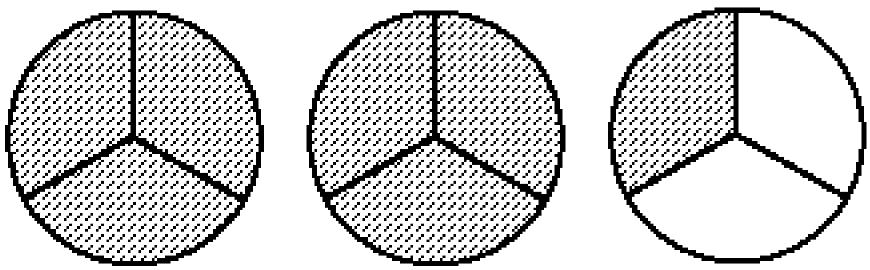
Since he was late two out of five times, we can say he was late $\frac{2}{5}$ of the time.

Ex. 5 In the 2008 baseball season, Roy Oswalt of the Houston Astros won 17 games and lost 10 games. If he pitched in a total of 32 games, what fraction of the games that he pitched did he win?

Solution:

Since he won 17 out of a total of 32 games he pitched, then he won $\frac{17}{32}$ of the games.

Ex. 6



Solution:

There are two ways to view this problem. Since three parts make a whole, we can say we have two wholes and then 1 out of 3 on the last circle. We can represent this as $2\frac{1}{3}$. The second way we can look at this is that three parts make a whole and there are seven parts. We can represent this as $\frac{7}{3}$. This means that $2\frac{1}{3} = \frac{7}{3}$.

Objective b: Proper and Improper Fractions.

A **proper fraction** is a fraction whose numerator is smaller than its denominator. It represents a quantity smaller than one.

An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator. It represents a quantity equal to or larger than one.

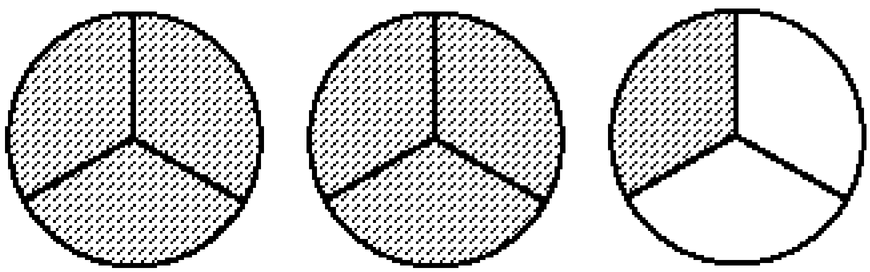
Identify each fraction as proper or improper fraction:

Ex. 7 $\frac{9}{2}, \frac{5}{7}, \frac{3}{4}, \frac{3}{3}, \frac{17}{4}, \frac{1}{7}, \frac{5}{9}, \frac{9}{8}$

Solution:
$\frac{5}{7}$, $\frac{3}{4}$, $\frac{1}{7}$, and $\frac{5}{9}$ are proper fractions.
$\frac{9}{2}$, $\frac{3}{3}$, $\frac{17}{4}$, and $\frac{9}{8}$ are improper fractions.

Objective c: Mixed Numbers.

A **mixed number** is a whole number plus a proper fraction.
Thus, $8\frac{5}{7}$, $3\frac{3}{4}$, $1\frac{1}{7}$, and $12\frac{5}{9}$ are examples of mixed numbers.
Recall Ex. 6:



|- 3 slices -| |- 3 slices -|

We say that $2\frac{1}{3} = \frac{7}{3}$. We can think of these as being two whole pies and $\frac{1}{3}$ of another one. If we take each of the whole pies and cut them into three slices, that will gives us $2 \bullet 3 = 6$ slices. Add the one slice from the third pie and you get 7 slices, which is the numerator of $\frac{7}{3}$. So, to convert from a mixed number to an improper fraction, take the whole number times the denominator and add the result to the numerator. Place this answer over the original denominator:

$$\text{Whole Number } \frac{\text{Numerator}}{\text{Denominator}} = \frac{(\text{Whole Number}) \times (\text{Denominator}) + \text{Numerator}}{\text{Denominator}}$$

<u>Write the following as improper fractions:</u>

Ex. 8 $13\frac{5}{7}$

Solution:
$13\frac{5}{7} = \frac{13 \bullet 7 + 5}{7} = \frac{96}{7}$.

Ex. 9 $66\frac{2}{3}$

<u>Solution:</u>

$66\frac{2}{3} = \frac{66\bullet 3+2}{3} = \frac{200}{3}$.

Ex. 10 $5\frac{19}{20}$

<u>Solution:</u>

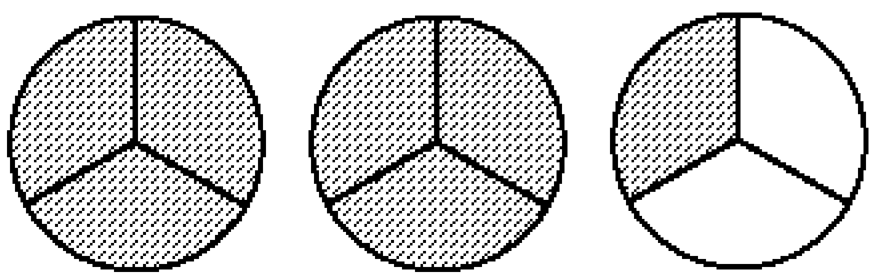
$5\frac{19}{20} = \frac{5\bullet 20+19}{20} = \frac{119}{20}$.

Ex. 11 $9\frac{5}{8}$

<u>Solution:</u>

$9\frac{5}{8} = \frac{9\bullet 8+5}{8} = \frac{77}{8}$.

Now, let's go in the other direction :



Here, we have 7 slices of pie. Since it takes three slices to make a whole, we can keep subtracting 3 until we run out of slices to make whole pies:

$$\begin{array}{rl} 7 & \\ \underline{-\,3} & \leftarrow 1^{st} \text{ full pie} \\ 4 & \\ \underline{-\,3} & \leftarrow 2^{nd} \text{ full pie} \\ 1 & \leftarrow \text{left-overs} \end{array}$$

We can make two whole pies and we will have one out of three left over. But division is a shortcut for repeated subtraction.

So, to convert an improper fraction to a mixed number, divide the numerator by the denominator. The quotient will be the whole number and the remainder will be the numerator of the proper fraction:

$$\begin{array}{r} \text{Whole \#} \\ \text{Denominator}\,\overline{)\,\text{Numerator}} \\ \underline{-\ \ldots\ldots\ldots\ldots} \\ \text{Remainder} \end{array} \quad \rightarrow \quad \text{Whole \#}\ \frac{\text{Remainder}}{\text{Denominator}}$$

<u>Write the following as a mixed number if possible:</u>

Ex. 12 $\frac{9}{5}$

Solution:

$$\begin{array}{r|l} & 1 \\ \hline 5 & 9 \\ & -5 \\ \hline & 4 \end{array}$$

So, the answer is $1\frac{4}{5}$.

Ex. 13 $\frac{77}{9}$

Solution:

$$\begin{array}{r|l} & 8 \\ \hline 9 & 77 \\ & -72 \\ \hline & 5 \end{array}$$

So, the answer is $8\frac{5}{9}$.

Ex. 14 $\frac{100}{7}$

Solution:

$$\begin{array}{r|l} & 14 \\ \hline 7 & 100 \\ & -7 \\ \hline & 30 \\ & -28 \\ \hline & 2 \end{array}$$

So, the answer is $14\frac{2}{7}$.

Ex. 15 $\frac{132}{11}$

Solution:

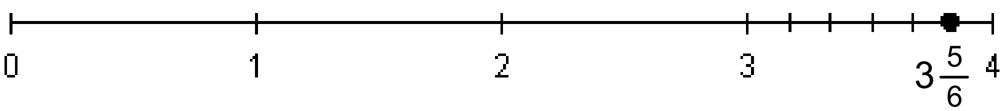
$$\begin{array}{r|l} & 12 \\ \hline 11 & 132 \\ & -11 \\ \hline & 22 \\ & -22 \\ \hline & 0 \end{array}$$

So, the answer is 12.

Objective d: Graphing Fractions on the Number Line.

To graph a proper fraction on the number line, divide the distance between 0 and 1 into a number of equal parts equal to the denominator. Then, starting at zero, count the number of parts over equal to the numerator. Thus, we can graph $\frac{3}{5}$ by dividing the distance between 0 and 1 into five equal parts and then starting at 0, we can count over 3 parts:

If we have a mixed number, we do the same thing except we start at the whole number part instead of 0. Hence, we can graph $3\frac{5}{6}$ by dividing the distance between 3 and 4 into six equal parts and then starting at 3, we can count over 5 parts:

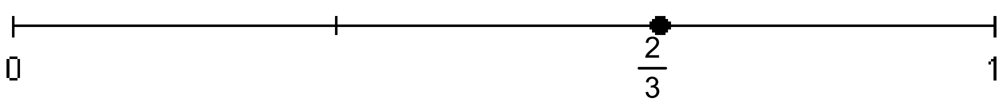


<u>Graph the following on a number line:</u>

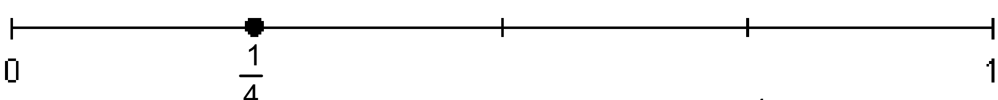
Ex. 16a $\frac{2}{3}$ Ex. 16b $\frac{1}{4}$ Ex. 16c $\frac{34}{5}$

<u>Solution:</u>

a) Divide the distance between 0 and 1 into three equal parts. Now, start at 0 and count over 2 parts:



b) Divide the distance between 0 and 1 into four equal parts. Now, start at 0 and count over 1 part:



c) Since 34 ÷ 5 = 6 with a remainder 4 or $6\frac{4}{5}$, divide the distance between 6 and 7 into five equal parts. Now, start at 6 and count over 4 parts: