Section 2.3 – Simplifying Fractions to Lowest Terms

Objective b: Simplifying Fractions to Lowest Terms.

A fraction is **reduced**, **simplified**, or **in lowest terms** if one is the only common factor of the numerator and denominator.

Which of the following fractions are reduced to lowest terms:

Ex. 1 $\frac{7}{8}, \frac{15}{20}, \frac{6}{9}, \frac{25}{49}$

Solution:

The factors of 7 are 1, 7 and the factors of 8 are 1, 2, 4, 8. Since 1 is the only common factor, $\frac{7}{8}$ is reduced to lowest terms.

The factors of 15 are 1, 3, 5, 15 and the factors of 20 are 1, 2, 4, 5, 10, 20. Since 5 is the common factor, $\frac{15}{20}$ is not reduced to lowest terms. The factors of 6 are 1, 2, 3, 6 and the factors of 9 are 1, 3, 9. Since 3 is the common factor, $\frac{6}{9}$ is not reduced to lowest terms. The factors of 25 are 1, 5, 25 and the factors of 49 are 1, 7, 49. Since 1 is the only common factor, $\frac{25}{49}$ is reduced to lowest terms.

Recall that any non-zero number divided by itself is 1 (i.e., $\frac{3}{3} = 1$).

Also, since multiplication is commutative and associative, we can regroup and reorder the product of numbers. So, in order to reduce fractions, we find the prime factorization of both the numerator and the denominator and reorder the factors in both so that the common factors are aligned. Once aligned, they reduce to 1 since the same non-zero divided by itself is one. After we reduce the common factors, we will multiply the left over factors in the numerator and denominator to get our answer.

Simplify to lowest terms:

Ex. 2 $\frac{15}{20}$

Solution:

$15 = 3 \bullet 5$ and $20 = 2 \bullet 10 = 2 \bullet 2 \bullet 5$, so $\frac{15}{20} = \frac{3 \bullet 5}{2 \bullet 2 \bullet 5} = \frac{3 \bullet \cancel{5}}{2 \bullet 2 \bullet \cancel{5}} = \frac{3}{4}$.

Ex. 3 $\frac{72}{132}$

Solution:

$72 = 8\bullet 9 = 2\bullet 4\bullet 9 = 2\bullet 2\bullet 2\bullet 9 = 2\bullet 2\bullet 2\bullet 3\bullet 3$ and

$132 = 11\bullet 12 = 11\bullet 2\bullet 6 = 11\bullet 2\bullet 2\bullet 3 = 2\bullet 2\bullet 3\bullet 11$,

so, $\frac{72}{132} = \frac{2\bullet 2\bullet 2\bullet 3\bullet 3}{2\bullet 2\bullet 3\bullet 11} = \frac{\cancel{2}\bullet\cancel{2}\bullet\cancel{2}\bullet\cancel{3}\bullet 3}{\cancel{2}\bullet\cancel{2}\bullet\cancel{3}\bullet 11} = \frac{6}{11}$

Ex. 4 $\frac{60}{84}$

Solution:

$60 = 4\bullet 15 = 2\bullet 2\bullet 15 = 2\bullet 2\bullet 3\bullet 5$ and

$84 = 2\bullet 42 = 2\bullet 6\bullet 7 = 2\bullet 2\bullet 3\bullet 7$,

so, $\frac{60}{84} = \frac{2\bullet 2\bullet 3\bullet 5}{2\bullet 2\bullet 3\bullet 7} = \frac{\cancel{2}\bullet\cancel{2}\bullet\cancel{3}\bullet 5}{\cancel{2}\bullet\cancel{2}\bullet\cancel{3}\bullet 7} = \frac{5}{7}$.

There is nothing special about reducing numbers that are prime numbers; it works for any whole number greater than zero. In example four, we could have written 60 as $12\bullet 5$ and 84 as $12\bullet 7$ and then reduced the 12's: $\frac{60}{84} = \frac{12\bullet 5}{12\bullet 7} = \frac{\cancel{12}\bullet 5}{\cancel{12}\bullet 7} = \frac{5}{7}$. So, we can look for the largest common factor and reduce the problem that way.

Reduce to lowest terms:

Ex. 5 $\frac{40}{96}$

Solution:

$\frac{40}{96} = \frac{4\bullet 10}{4\bullet 24} = \frac{\cancel{2}\bullet 5}{\cancel{2}\bullet 12} = \frac{5}{12}$.

Ex. 6 $\frac{200}{300}$

Solution:

$\frac{200}{300} = \frac{2\bullet 100}{3\bullet 100} = \frac{2}{3}$.

Ex. 7 $\frac{405}{729}$

Solution:

$\frac{405}{729} = \frac{9\bullet 45}{9\bullet 81} = \frac{9\bullet 5}{9\bullet 9} = \frac{5}{9}$.

Ex. 8 $\frac{36}{20}$

Solution:

$\frac{36}{20} = \frac{4\bullet 9}{4\bullet 5} = 1\frac{4}{5}$.

Sometimes it is easier to write an improper fraction as a mixed number before reducing to lowest terms.

Ex. 9 $\frac{165}{22}$

Solution:

First, write as a mixed number:

$\frac{165}{22} = 7\frac{11}{22}$

Now, reduce:

$7\frac{11}{22} = 7\frac{1 \cdot \cancel{11}}{2 \cdot \cancel{11}} = 7\frac{1}{2}$.

$$\begin{array}{r|l} & 7 \\ \hline 22 & 165 \\ & -\,154 \\ \hline & 11 \end{array}$$

Ex. 10 $\frac{912}{452}$

Solution:

First, write as a mixed number:

$\frac{912}{452} = 2\frac{8}{452}$

Now, reduce:

$2\frac{8}{452} = 2\frac{\cancel{4} \cdot 2}{\cancel{4} \cdot 113} = 2\frac{2}{113}$.

$$\begin{array}{r|l} & 2 \\ \hline 452 & 912 \\ & -\,904 \\ \hline & 8 \end{array}$$

Objective a: Equivalent Fractions

Are the following fractions equal?

Ex. 11 $\frac{15}{55}$ and $\frac{33}{121}$

Solution:

Since $\frac{15}{55} = \frac{\cancel{5} \cdot 3}{\cancel{5} \cdot 11} = \frac{3}{11}$

and $\frac{33}{121} = \frac{\cancel{11} \cdot 3}{\cancel{11} \cdot 11} = \frac{3}{11}$,

they are equal. Yes.

Ex. 12 $\frac{7}{13}$ and $\frac{42}{91}$

Solution:

Since $\frac{7}{13} = \frac{7}{13}$ and

$\frac{42}{91} = \frac{\cancel{7} \cdot 6}{\cancel{7} \cdot 13} = \frac{6}{13}$,

they are not equal. No.

Objective c: Applications

Ex. 13 Before it can used, sixteen ounces of liquid fertilizer must be mixed with 64 ounces of water. What fraction of the final mixture is fertilizer?

Solution:

16 oz of fertilizer + 64 oz of water = 80 oz of the mixture.

16 oz out of 80 oz is fertilizer, thus are fractions is:

$\frac{16}{80} = \frac{8 \cdot 2}{8 \cdot 10} = \frac{2}{10} = \frac{2 \cdot 1}{2 \cdot 5} = \frac{1}{5}$

So, $\frac{1}{5}$ of the mixture is fertilizer.