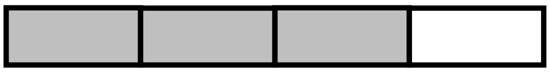
Section 2.4 – Multiplication of Fractions and Applications

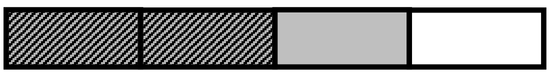
Objective a: Multiplication of Fractions.

Ex. 1 Suppose you buy a candy bar and eat one quarter of it. This would leave you with three quarters ($\frac{3}{4}$) of the candy bar left (shaded in gray):



$\frac{3}{4}$

Later, you give a friend one of the three remaining pieces, leaving two of the three ($\frac{2}{3}$) remaining pieces left (shaded in gray with black diagonals):



$\frac{2}{3}$ of $\frac{3}{4}$

As compared to the original candy bar, there is half of the original candy bar remaining. This says that $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{1}{2}$. But the word "of" means multiplication in this context, so, $\frac{2}{3} \bullet \frac{3}{4} = \frac{1}{2}$. If we try the natural thing of multiplying the numerators together and then multiplying the denominators together, we get: $\frac{2}{3} \bullet \frac{3}{4} = \frac{6}{12} = \frac{\cancel{6} \bullet 1}{\cancel{6} \bullet 2} = \frac{1}{2}$.

Notice, we get the same answer. So, when multiplying fractions, we need to multiply the numerators and then multiply the denominators. Also, since multiplication is both commutative and associative, we can actually reduce before we multiply. To see how this works, let's reexamine this problem:

$$\frac{2}{3} \bullet \frac{3}{4} = \frac{1 \bullet 2}{1 \bullet 3} \bullet \frac{3 \bullet 1}{2 \bullet 2} = \frac{1 \bullet \cancel{2}}{1 \bullet 3} \bullet \frac{3 \bullet 1}{2 \bullet \cancel{2}} = \frac{1}{1 \bullet \cancel{3}} \bullet \frac{\cancel{3} \bullet 1}{2} = \frac{1}{1} \bullet \frac{1}{2} = \frac{1}{2}.$$

Many times it is easier to reduce before you multiply, so we will try to reduce as much as possible before multiplying.

Simplify the following:

Ex. 2 $\frac{22}{45} \bullet \frac{5}{11}$

Solution:

$$\frac{22}{45} \bullet \frac{5}{11} = \frac{2 \bullet \cancel{11}}{9 \bullet \cancel{5}} \bullet \frac{\cancel{5} \bullet 1}{\cancel{11} \bullet 1} = \frac{2}{9}.$$

Ex. 3 $\frac{39}{70} \bullet \frac{14}{45} \bullet \frac{10}{13}$

Solution:

$$\frac{39}{70} \bullet \frac{14}{45} \bullet \frac{10}{13} = \frac{\cancel{3}\bullet 13}{\cancel{7}\bullet 10} \bullet \frac{2\bullet \cancel{7}}{\cancel{3}\bullet 15} \bullet \frac{10}{13} = \frac{1\bullet \cancel{13}}{1\bullet \cancel{10}} \bullet \frac{2}{15} \bullet \frac{1\bullet \cancel{10}}{1\bullet \cancel{13}} = \frac{2}{15}.$$

Ex. 4 $\frac{65}{34} \bullet \frac{35}{26} \bullet \frac{51}{25}$

Solution:

$$\frac{65}{34} \bullet \frac{35}{26} \bullet \frac{51}{25} = \frac{5\bullet \cancel{13}}{34} \bullet \frac{\cancel{5}\bullet 7}{2\bullet \cancel{13}} \bullet \frac{51}{\cancel{5}\bullet 5} = \frac{1\bullet \cancel{5}}{2\bullet \cancel{17}} \bullet \frac{7}{2} \bullet \frac{3\bullet \cancel{17}}{1\bullet \cancel{5}} = \frac{21}{4}.$$

Now, write the answer as a mixed number:

$$\begin{array}{r} 5 \\ 4\,\overline{)\,21} \\ \underline{-20} \\ 1 \end{array}$$

So, the answer is $5\frac{1}{4}$.

Ex. 5 $\frac{2}{3} \bullet 24$

Solution:

When multiplying fractions and whole numbers, rewrite the whole number as a fraction by putting it over 1:

$$\frac{2}{3} \bullet 24 = \frac{2}{3} \bullet \frac{24}{1} = \frac{2}{1\bullet \cancel{3}} \bullet \frac{\cancel{3}\bullet 8}{1} = \frac{16}{1} = 16.$$

Ex. 6 $15 \bullet \frac{3}{59}$

Solution:

When multiplying fractions and whole numbers, rewrite the whole number as a fraction by putting it over 1:

$$15 \bullet \frac{3}{59} = \frac{15}{1} \bullet \frac{3}{59} = \frac{45}{59}.$$

Objective b: Fractions and the Order of Operations.

Recall the order of operations from chapter one:

Order of Operations

1) Parentheses - Do operations inside of Parentheses ( ), [ ], { }, $|\ |$
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

Simplify:

Ex. 7 $7 + \frac{16}{9} \bullet \left(\frac{3}{2}\right)^3$

Solution:

$7 + \frac{16}{9} \bullet \left(\frac{3}{2}\right)^3$ (#2 – Exponents)

$= 7 + \frac{16}{9} \bullet \left(\frac{3}{2} \bullet \frac{3}{2} \bullet \frac{3}{2}\right)$

$= 7 + \frac{16}{9} \bullet \left(\frac{27}{8}\right)$ (#3 – Multiply)

$= 7 + \frac{8 \bullet 2}{9 \bullet 1} \bullet \frac{9 \bullet 3}{8 \bullet 1}$ (Reduce)

$= 7 + \frac{2}{1} \bullet \frac{3}{1}$

$= 7 + 6$ (#4 – Add)

$= 13$

Ex. 8 $\left(3 \bullet \frac{1}{12} \bullet \frac{2}{5}\right)^4$

Solution:

$\left(3 \bullet \frac{1}{12} \bullet \frac{2}{5}\right)^4$ (Write 3 over 1)

$= \left(\frac{3}{1} \bullet \frac{1}{12} \bullet \frac{2}{5}\right)^4$ (#1 – Parenthesis, #3 – Multiply)

$= \left(\frac{3 \bullet 1}{1} \bullet \frac{1}{3 \bullet 2 \bullet 2} \bullet \frac{2 \bullet 1}{5}\right)^4$ (Reduce)

$= \left(\frac{1}{1} \bullet \frac{1}{2} \bullet \frac{1}{5}\right)^4$

$= \left(\frac{1}{10}\right)^4$ (#2 – Exponents)

$= \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

$= \frac{1}{10{,}000}$

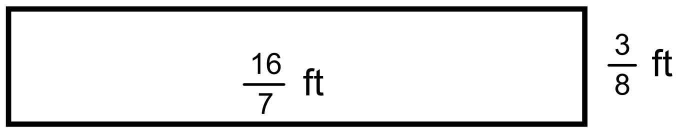
Notice that when raising $\frac{1}{10}$ to a power, we get 1 in the numerator and 1 followed by the power number of zeros in the denominator.

Objective c: Area of a Triangle.

Recall that the area of a rectangle is length times width:

Find the area of the following:

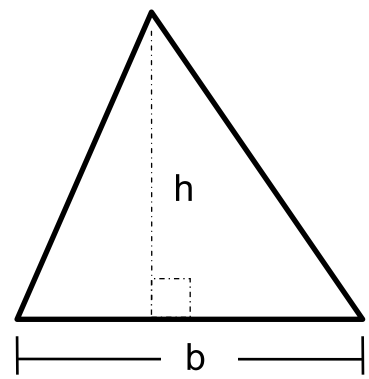
Ex. 9



Solution:
Since the area is equal to the length times the width, then:

$$A = \frac{16}{7} \bullet \frac{3}{8} = \frac{2 \bullet 8}{7} \bullet \frac{3}{1 \bullet 8} = \frac{6}{7} \text{ sq. ft}$$

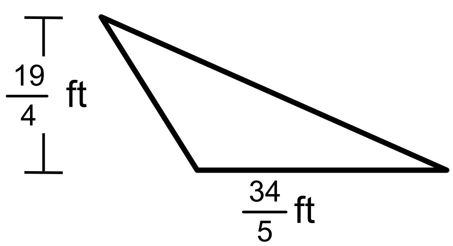
Now, let us consider a triangle.
The length of the bottom of a triangle is referred to as the base of the triangle and is denoted by the letter b. The vertical height of the triangle (sometimes called the altitude of a triangle) is called the height of the triangle and is denoted by the letter h. If you think of a triangle as half of a rectangle, then the area of a triangle is one half times the base times the height.



$$A = \frac{1}{2}bh$$

Find the area of the following:

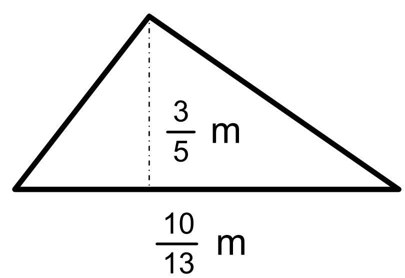
Ex. 10



Solution:

$$A = \frac{1}{2} \bullet \left(\frac{34}{5}\right) \bullet \left(\frac{19}{4}\right)$$
$$= \frac{1}{1 \bullet 2} \bullet \left(\frac{2 \bullet 17}{5}\right) \bullet \left(\frac{19}{4}\right) \quad \text{(Reduce)}$$
$$= \frac{1}{1} \bullet \left(\frac{17}{5}\right) \bullet \left(\frac{19}{4}\right)$$
$$= \frac{323}{20} \text{ or } 16\frac{3}{20} \text{ ft}^2$$

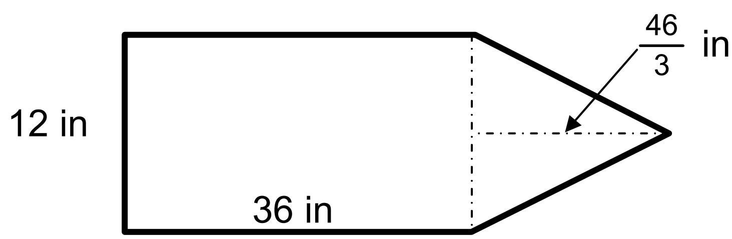
Ex. 11



Solution:

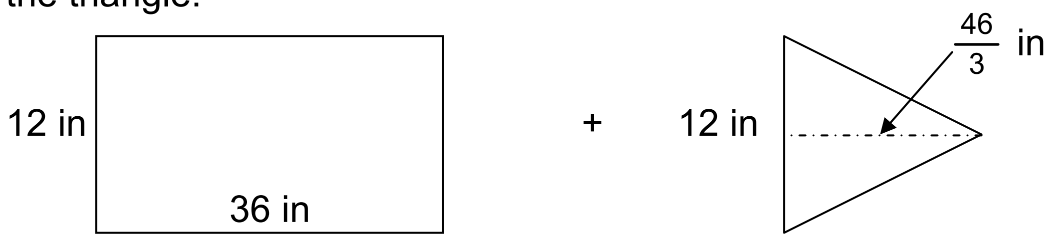
$$A = \frac{1}{2} \bullet \frac{10}{13} \bullet \frac{3}{5}$$
$$= \frac{1}{2} \bullet \frac{2 \bullet 5}{13} \bullet \frac{3}{5} \quad \text{(Reduce)}$$
$$= \frac{1}{1} \bullet \frac{1}{13} \bullet \frac{3}{1}$$
$$= \frac{3}{13} \text{ m}^2.$$

Ex. 12



Solution:

The total area is the sum of the area of the rectangle plus the area of the triangle:



$$Lw \quad + \quad \frac{1}{2}bh$$

$$= 12(36) \quad + \quad \frac{1}{2}(12)(\frac{46}{3}) \quad \text{(Write 12 over 1)}$$

$$= 12(36) \quad + \quad \frac{1}{2}(\frac{12}{1})(\frac{46}{3})$$

$$= 12(36) \quad + \quad \frac{1}{1 \bullet 2}(\frac{2 \bullet 2 \bullet 3}{1})(\frac{46}{3 \bullet 1}) \quad \text{(Reduce)}$$

$$= 12(36) \quad + \quad \frac{1}{1}(\frac{2}{1})(\frac{46}{1}) \quad \text{(Multiply)}$$

$$= 432 \quad + \quad 92 \quad \text{(Add)}$$

$$= 524 \text{ in}^2$$

Objective d: Applications.

Solve the following:

Ex. 13 It is estimated that $\frac{7}{8}$ of the students attending St. Philip's College receive some kind of financial assistance. If 10,616 students were registered for the Spring Semester, how many received financial assistance?

Solution:

Multiply $\frac{7}{8}$ by 10616:

$$\frac{7}{8} \bullet 10616 = \frac{7}{8} \bullet \frac{10616}{1} = \frac{7}{1 \bullet 8} \bullet \frac{8 \bullet 1327}{1} = \frac{9289}{1} = 9289 \text{ students.}$$

Ex. 14 A cake recipes calls for $\frac{3}{4}$ cups of sugar. If Samuel triples the recipe to make enough cake for a family reunion, how much sugar will he need?

Solution:

Multiply 3 by $\frac{3}{4}$:

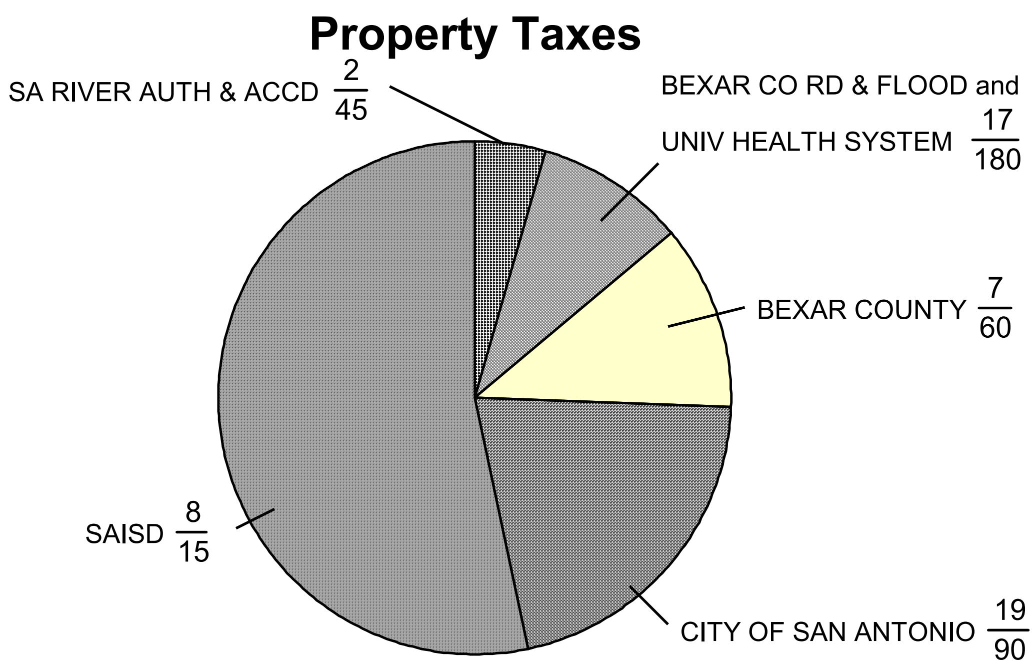
$3\bullet\frac{3}{4} = \frac{3}{1}\bullet\frac{3}{4} = \frac{9}{4}$.

Now, write the answer as a mixed number:

$$\begin{array}{r} 2 \\ 4\overline{)\,9} \\ -8 \\ \hline 1 \end{array}$$

So, he will need is $2\frac{1}{4}$ cups of sugar.

The circle graph shows how property taxes paid in Bexar County were divided among the different categories. Use the graph to answer the following question:



Ex. 15 If the Martinez's paid a total of \$1980 in taxes last year, how much went to the City of San Antonio?

Solution:

Multiply $\frac{19}{90}$ by 1980:

$\frac{19}{90}\bullet 1980 = \frac{19}{90}\bullet\frac{1980}{1} = \frac{19}{1\bullet 90}\bullet\frac{90\bullet 22}{1} = \frac{418}{1}$ = \$418.